10 ^R   **Half Yearly Examination - 2023**   Reg. No. | 1 | 0 | 2 | 4 | 3 | 3 |

Time : 3.00 hrs.   **MATHEMTAICS**   Max. Marks : 100

PART - I

                                                                            14x1=14

I. Choose the best answer.

1. If there are 1024 relations from a set A={1,2,3,4,5} to set B then the number of elements in B is
    a) 3   b) 2   c) 4   d) 8

2. The range of the relation R= {(x, x^2)/x is a prime number less than 13} is
    a) {2,3,5,7}   b) {2,3,5,7,11}   c) {4,9,25,49,121}   d) {1,4,9,25,49,121}

3. The sum of the exponent of the prime factors in the prime factorization of 1729
    a) 1   b) 2   c) 3   d) 4

4. $y^2 + 1/y^2$ is not equal to   a) $y^4 + \dfrac{1}{y^2}$ b) $(y + \dfrac{1}{y})^2$ c) $(y - \dfrac{1}{y})^2 + 2$ d) $(y - \dfrac{1}{y})^2 - 2$

5. Which of the following should be added make $x^4 + 64$ a perfect square?
    a) $4x^2$ b) $16x^2$ c) $8x^2$ d) $-8x^2$

6. If $\triangle ABC$ is an isosceles triangle with $\angle C = 90°$ and AC =5cm then AB is
    a) 2.5cm   b) 5 cm   c) 10 cm   d) $5\sqrt{2}$ cm

7. If (5,7) (3,p) and (6,6) are collinear then the value of P is
    a) 3   b) 6   c) 9   d) 12

8. The slope of the line which is perpendicular to a line joining the points (0,0) and (-8,8) is
    a) -1   b) 1   c) $\dfrac{1}{3}$ d) -8

9. If the ratio of the height of a tower and the length of its shadow is $\sqrt{3}$: 1. Then the angle of elevation of the sun has measure   a) 45°   b) 30°   c) 90°   d) 60°

10. If $\tan\theta + \cot\theta = 2$ then the value of $\tan^2\theta + \cot^2\theta$ is   a) 0   b) 1   b) 2   d) 4

11. The height of a right circular cone whose radius is 5cm and slant height is 13cm will be
    a) 12cm   b) 10 cm   c) 13cm   d) 5cm

12. The total surface area of a hemisphere is how many times the square of its radius
    a) π b) 4π c) 3π d) 2π

13. The mean of 100 observations is 40 and their standard deviation is 3 Sum of Square of all observation
    a) 40000   b) 160900   c) 160000   d) 30000

14. If a letter is chosen at random from the English alphabets {a,b,c....z} then the probability that the letters
    chosen precedes x is   a) $\dfrac{12}{13}$ b) $\dfrac{1}{13}$ c) $\dfrac{23}{26}$ d) $\dfrac{3}{26}$

PART –II

Answer any 10 questions. Question No.28 is compulsory.   10x2=20

15. If A x B = {(3,2) (3,4) (5,2) (5,4) } then find A and B.

16. Show that the function f: N → N defined by $f(m) = m^2 + m + 3$ ias one –one function.

17. If 3+k, 18–k, 5k+1 are in A.P then find k.

18. Determine the quadratic equations, whose sum and product of roots are -9 and 20.

19. Find the 19th term of an AP -11, -15, -19.

20. If A = $\begin{pmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{pmatrix}$ then find the transpose of -A

21. If A = $\begin{pmatrix} 1 & 2 & 0 \\ 3 & 1 & 5 \end{pmatrix}$ B= $\begin{pmatrix} 8 & 3 & 1 \\ 2 & 4 & 1 \\ 5 & 3 & 1 \end{pmatrix}$ then find AB.

22. AB = 5cm and Ac = 10cm BD = 1.5cm and CD= 3.5cm check whether AD is bisector of $\angle A$ of $\triangle ABC$.
23. Find the slope of a line joining the points (14, 10) and (14, -6).
24. Prove $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}} = \sec\theta + \tan\theta$
25. Find the diameter of sphere whose surface area is 154m².
26. If the base area of a hemispherical solid is 1386 sq. metres then find its total surface area.
27. Find the range and coefficient of range of the data
    63, 89, 98, 125, 79, 108, 117, 68
28. Find the volume of the iron used to make a hollow cylinder of height 9cm and whose internal and external radii are 3cm and 5cm respectively.

PART-III

Answer any 10 questions. Question No 42 is compulsory.   10 x 5 = 50

29. Let A = The set of all natural numbers less than 8, B= The set of all prime numbers. C = The set of even prime numbers. Verify that $(A \cap B) \times C = (A \times C) \cap (B \times C)$
30. Let A = {1,2,3,4} and B = {2,5,8,11,14} be two sets. Let f: A → B be a function given by f(x) = 3x − 1
    Represent this function
    i) by Arrow diagram   ii) in a table form   iii) as a set of ordered pairs   iv) in a graphical form
31. Find the sum of all natural numbers between 100 and 1000 which are divisible by 11.
32. Solve x+y+z = 5, 2x − y + z = 9, x − 2y + 3z = 16
33. Find the square root of $64x^4 − 16x^3 + 17x^2 − 2x + 1$
34. State and prove pythagoras theorem
35. Show that in a triangle, the medians are concurrent.

36. If $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{pmatrix}$ show that $(AB)^T = B^T A^T$

37. Find the area of the quadrilateral formed by the points (8,6) (5,11) (-5,12) and (-4,3)
38. Two ships are sailing in the sea on either sides of a lighthouse. The angle of elevation of the top of the lighthouse as observed from the ships are 30° and 45° respectively. If the lighthouse is 200m height find the distance between the two ships ($\sqrt{3} = 1.732$)
39. If the radii of the circular ends of a frustum which is 45 cm heights are 28cm and 7cm. Find the volume of the frustum.
40. A toy is in the shape of a cylinder surmounted by a hemisphere the height of the toy is 25cm. Find the total of the toy is 25cm. Find the total surface area of the toy if its common diameter is 12cm.
41. Two unbiased dice are rolled once. Find the probability of getting
    i) a doublet (equal numbers on both dice) ii) The product as a prime number
    iii) The sum as a prime number   iv) the sum as 1
42. A straight line AB cuts the co-ordinates areas at A and B. If the mid points of AB is (2,3) find the equation of AB.

PART- IV

Answer all the questions.   8x2=16

43. Construct a triangle similar to given triangle ABC with its sides equal to $\dfrac{6}{5}$ of the corresponding sides of the
    $\triangle ABC$ (Scale factor $\dfrac{6}{5} > 1$) **(OR)**
    Draw two tangents from a point which is 10 cm away from the centre of a circle of radius 5cm. Also measure the lengths of the tangents
44. Draw the graph of xy = 24, x,y>0, using the graph find i) y when x=3 and ii) x when y=6
    (OR)
    Draw the graph of $y = x^2 − 4$ and hence solve $x^2 − x − 12 = 0$