Salem District

HSL **HALF YEARLY EXAMINATION - 2023** Time : 3.00 Hrs

10 - STD **MATHS** Marks : 100

I Part - I Answer all the questions. 14 X 1 = 14

1. The range of the relation R = {(x, x^2) / x is a prime number less than 13} is
   a) {2, 3, 5, 7} b) {2,3,5,7,11} c) {4,9,25,49,121} d) {1,4,9,25,49,121}
2. Let f(x) = $\sqrt{1+x^2}$ then
   a) f(xy) = f(x) . f(y) b) f(xy) $\geq$ f(x).f(y) c) f(xy) $\leq$ f(x) . f(y) d) none of these
3. If 6 times of 6^{th} term of an A.P. is equal to 7 times the 7^{th} term, then the 13^{th} term of the A.P is a) 0 b) 6 c) 7 c) 13
4. The sum of first n terms of a G.P, when r = 1 is a) a^n b) n c) na d) a
5. Which of the following should be added to make $x^4 + 64$ a perfect square
   a) $4x^2$ b) $16x^2$ c) $8x^2$ d) $-8x^2$
6. The square root of $4m^2 - 24m + 36 = 0$ is a) 4(m-3) b) 2(m-3) c) $(2m - 3)^2$ d) 4(m+ 3)
7. In a ΔABC, AD is the bisector of $\angle$BAC, If AB = 8cm, BD = 6cm and DC = 3cm. The length of the side AC is a) 6cm b) 4 cm c) 3 cm d) 8 cm
8. The point of intersection of 3x - y = 4 and x + y = 8 is
   a) (5, 3) b) (2,4) c) (3,5) d) (4,4)
9. A straight line PQ cuts the X axis at A and Y axis at B. If the mid point of AB is (2a, 2b) then the cor-ordinates of A and B are
   a) (a, 0), (0, b) b) (2a, 0), (0, 2b) c) (0, b), (a, 0) d) (0, 2b), (2a, 0)
10. If sin θ + cos θ = a and sec θ + cosec θ = b, then the value of $b(a^2-1)$ is equal to
    a) 2a b) 3a c) 0 d) 2ab
11. If two solid hemispheres of same base radius r units are joined together along their bases, then curved surface area of this new solid is
    a) $4\pi r^2$ sq. units b) $6\pi r^2$ sq. units c) $3\pi r^2$ sq. units d) $8\pi r^2$ sq. units
12. The curved surface area of a right circular cone of height 15cm and base diameter 16cm is a) 60π cm^2 b) 66π cm^2 c) 120π cm^2 d) 136π cm^2
13. The range of the data 8, 8, 8, 8, 8, ... 8 is a) 0 b) 1 c) 8 d) 3
14. A page is selected at random from a book. The probability that the digit at units place of the page number chosen is less than 7 is
    a) $\frac{3}{10}$ b) $\frac{7}{10}$ c) $\frac{3}{9}$ d) $\frac{7}{9}$

II Part - II Answer any 10 questions. Question No. 28 is compulsory. 10 X 2 = 20

15. If B X A = {(-2, 3), (-2, 4), (0, 3), (0, 4), (3,3), (3,4)} find A and B.
16. Find k if fof(k) = 5 where f(k) = 2k-1.
17. Find the number of terms in the A.P. 3,6,9,12,.......... 111.
18. If $1^3+2^3+3^3+........+k^3 = 44100$ then find 1 + 2 + 3 + ... + k.
19. Find the excluded values of the following expression $\frac{x^2+6x+8}{x^2+x-2}$.
20. What length of ladder is needed to reach a height of 7ft along the wall when the base of the ladder is 4ft from the wall? Round off your answer to the next tenth place.
21. If the three points (3, -1), (a, 3) and (1, - 3) are collinear, find the value of a.
22. Find the equation of a line through the given pair of points (2,3) and (-7, -1).

23. Prove that $\frac{\sec\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} = \cot\theta$.
24. The volume of a solid right circular cone is $11088cm^3$. If its height is 24cm then find the radius of the cone.
25. A right circular cylinder just enclose a sphere of radius r units. Calculate i) the surface area of the sphere. ii) the curved surface area of the cylinder.
26. Find the standard deviation of first 21 natural numbers.
27. A coin is tossed thrice. What is the probability of getting two consecutive tails?
28. If $(5\ x\ 1)\begin{pmatrix}2\\-1\\3\end{pmatrix} = (20)$ find the value of x?

$10 - x + 3 = 20$
$-x = 20 - 13$
$x = -17$

III Part - III Answer any 10 questions. Question No. 42 is compulsory. 10 X 5 = 50

29. Let $A = \{x \in W / x < 2\}$, $B = \{x \in N / 1 < x \le 4\}$ and $C = \{3,5\}$ Verify that $AX(B \cap C) = (AXB) \cap (AXC)$.
30. If f(x) = x-1, g(x) = 3x + 1 and h(x) = x^2 show that (fog) oh = fo(goh).
31. The product of three consecutive terms of a Geometric progression is 343 and their sum is $\frac{91}{3}$. Find the three terms.
32. Rekha has 15 square colour papers of sizes 10cm, 11cm, 12cm, ... 24cm. How much area can be decorated with these colour papers?
33. If $ax^4 + bx^3 + 361x^2 + 220x + 100$ is a perfect square, find the values of a and b.
34. If α, β are the roots of the equation $2x^2 - x - 1 = 0$ then form the equation whose roots are i) $\frac{1}{\alpha}, \frac{1}{\beta}$ ii) $2\alpha + \beta$, $2\beta + \alpha$.
35. If $A = \begin{pmatrix}1 & 2 & 1\\2 & -1 & 1\end{pmatrix}$, $B = \begin{pmatrix}2 & -1\\-1 & 4\\0 & 2\end{pmatrix}$ show that $(AB)^T = B^T A^T$.
36. State and prove angle bisector theorem.
37. Find the area of the quadrilateral whose vertices are at (-9, 0), (-8, 6), (-1, -2) and (-6, -3).
38. Find the equation of a straight line parallel to y axis and passing through the point of intersection of the line 4x + 5y = 13, x - 8y + 9 = 0.
39. A man is watching a boat speeding away from the top of a tower. The boat makes an angle of depression of 60° with the man's eye when at a distance of 200m from the tower. After 10 seconds, the angle of depression becomes 45°. What is the approximate speed of the boat (in Km/hr), assuming that it is sailing in still water? ($\sqrt{3} = 1.732$)
40. A conical container is fully filled with petrol. The radius is 10m and the height is 15m. If the container can release the petrol through its bottom at the rate of 25cu. meter per minute, in how many minutes the container will be emptied. Round off your answer to the nearest minutes.
41. Two dice are rolled together. Find the probability of getting a doublet or sum of face as 4.
42. A chess board contains 64 equal squares and the area of each square is $6.25cm^2$. A border around the board is 2cm wide. Find the length of the side of the chess board?

IV Part - IV Answer all questions. 2 X 8 = 16

43. Draw a triangle ABC of base BC = 5.6cm, $\angle A = 40^O$ and the bisector of $\angle A$ meets BC at D such that CD = 4cm. ***(OR)*** b) Draw tangent to the circle from the point P having radius 3.6cm and the centre at O. Point P is at a distance 7.2cm from the centre.
44. a) Graph the following linear function y= ½ x. Identify the constant of variation and verify it with the graph. Also i) find y when x= 9. ii) find x when y = 7.5. ***(OR)***
b) Draw the graph of $y = x^2 + 3x + 2$ and use it to solve $x^2 + 2x + 1 = 0$.

Half Yearly Examination 2023 (Mathematics)

CLASS : 10^{th} std — 18. 12. 2023

Answer key — 100 MARKS

I choose it

1.	(c) $\{4, 9, 25, 49, 121\}$	6.	(b) $2(m-3)$	11.	(a) $4\pi r$ sq.units
2.	(c) $f(xy) \leq f(x).f(y)$	7.	(b) 4 cm	12	(d) $136\pi cm^2$
3.	(a)	8.	(c) $(3, 5)$	13	(a) 0
4.	(c) na	9.	$(4a, 0)\ (0, 4b)$	14.	(b) $\frac{7}{10}$
5	(b) $16x^2$	10.	(a) $2a$		

PART – II

15. $A = \{3, 4\}$

$B = \{-2, 0, 3\}$

16. Given $f \circ f(k) = 5$

$f \circ f(k) = 2(2k-1) - 1$

$= 4k - 2 - 1$

$= 4k - 3$

$4k - 3 = 5 \quad k = \frac{8}{4}$ $\boxed{k = 2}$

17. $a = 3 \quad d = 6 - 3 = 3 \quad l = 111$

$n = \left[\frac{l-a}{d}\right] + 1 = \left[\frac{111-3}{3}\right] + 1$

$= \left[\frac{108}{3} + 1\right] = 37$ $\boxed{n = 37}$

18. Given

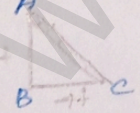
$1^3 + 2^3 + 3^3 + \dots k^3 = 44100$

$1 + 2 + 3 + \dots k = \sqrt{44100} = 210$

19. $\frac{x^2+6x+8}{x^2+x-2} = \frac{(x+4)(x+2)}{(x-1)(x+2)}$

$= \frac{x+4}{x-1}$ $\therefore$ $\boxed{x = 1}$ excluded value

20.



Using Pyth. the

$AC^2 = AB^2 + BC^2$

$AC^2 = (7)^2 + (4)^2$

$= 49 + 16 = 65$

$AC = \sqrt{65}$ is between 8 and 8.1

$8^2 = 64 < 65 < 65.61 = 8.1^2$

The length of approximately is $\boxed{8.1 ft}$

21. $A(3, -1) \quad B = (a, 3) \quad C = (1, -3)$

Slope of $AB = \frac{3+1}{a-3} = \frac{4}{a-3}$

Slope of $AC = \frac{-3+1}{1-3} = \frac{-2}{-2} = 1$

$\frac{4}{a-3} = 1$

$4 = a - 3 \quad a = 4 + 3$ $\boxed{a = 7}$

22. $x_1 = 2 \quad y_1 = 3$

$x_2 = -7 \quad y_2 = -1$

$\frac{y-3}{-1-3} = \frac{x-2}{-7-2}$

$\frac{y-3}{\cancel{-}4} = \frac{x-2}{\cancel{-}9}$

$9y - 27 = 4x - 8$

$\therefore$ The required equation is

$4x - 9y + 19 = 0$

23. $\frac{\frac{1}{\cos\theta}}{\sin\theta} - \frac{\sin\theta}{\cos\theta}$

$= \frac{1}{\cos\theta \sin\theta} - \frac{\sin^2\theta}{\cos\theta \sin\theta}$

$= \frac{1 - \sin^2\theta}{\cos\theta \sin\theta} = \frac{\cos^2\theta}{\cos\theta \sin\theta}$

$= \frac{\cos\theta}{\sin\theta} = \cot\theta$ Hence Proved.

24. The vol of solid right circular cone $= \frac{1}{3}\pi r^2 h$ = 11088 cm³

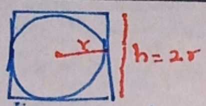
$\frac{1}{3} \times \frac{22}{7} \times r^2 \times 24 = 11088$

$r^2 = \frac{\cancel{11088} \times \cancel{3} \times 7}{1 \times \cancel{22} \times \cancel{24}}$ (1008, 504, 63 ; 2, 8)

$r^2 = 63 \times 7$ $r = \sqrt{9 \times 7 \times 7}$

$r = 3 \times 7 = \boxed{21\ cm}$

25.



(i) The C.S.A of Sphere $= 4\pi r^2$ sq. units — ①

(ii) The C.S.A of Cylinder $= 2\pi r h$

$= 2\pi r \times 2r$

$= 4\pi r^2$ sq. units

26. $\sigma = \sqrt{\frac{n^2-1}{12}} = \sqrt{\frac{(21)^2-1}{12}}$

$= \sqrt{\frac{441-1}{12}} = \sqrt{\frac{440}{12}}$

$= \sqrt{36.67} \approx \boxed{6.05}$

27. $S = \left\{ \begin{matrix} HHH, HHT, HTH, HTT \\ THH, THT, TTH, TTT \end{matrix} \right\}$

$n(S) = 8$

$n(A) = \{HTT, TTH, TTT\} = 3$

$P(A) = \frac{3}{8}$

28.

$\begin{bmatrix} 5 & x & 1 \end{bmatrix} \times \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = 20$

$(5 \times 2) + (x \times -1) + (1 \times 3) = 20$

$10 - x + 3 = 20$

$-x = 20 - 13$

$\boxed{x = -17}$

PART - III

29.

$A = \{0, 1\}$

$B = \{2, 3, 4\}$

$C = \{3, 5\}$

$A \times (B \cap C)$

$B \cap C = \{3\}$

$A \times (B \cap C) = \{(0,3)(1,3)\}$ — ①

$A \times B = \{(0,2)(0,3)(0,4)(1,2)(1,3)(1,4)\}$

$A \times C = \{(0,3)(0,5)(1,3)(1,5)\}$

$(A \times B) \cap (A \times C) = \{(0,3)(1,3)\}$ — ②

① = ② H. proved

30. $f(x) = x - 1$ $g(x) = 3x + 1$

$h(x) = x^2$

$(f \circ g) \circ h$

$fg(x) = f(3x+1)$

$f(x) = x - 1$

$f(3x+1) = (3x + \cancel{1} - \cancel{1}) = 3x$

$f \circ g\ h(x) = f \circ g(x^2)$

$f \circ g(3(x)^2) = 3x^2$ — ①

$f \circ (g \circ h) = g(h(x)) = g(x^2)$

$g(x) = 3x + 1$

$g(x^2) = 3x^2 + 1$

$f \circ (g \circ h) = f(3x^2 + 1)$

$f(x) = x - 1$

$f(3x^2+1) = 3x^2 + 1 - 1$

$= 3x^2$ — ②

① = ②

31. Con. 1

The product of $\frac{a}{r} \times a \times ar = 343$

$a^3 = 343$

$a = 7$

Con: 2

Sum of $\frac{a}{r} + a + ar = \frac{91}{3}$

$a\left[\frac{1+r+r^2}{r}\right] = \frac{91}{3}$

$3r^2 - 10r + 3 = 0$ $r = 3$ $r = \frac{1}{3}$

If $a = 7$ $r = 3$ three terms $\frac{7}{3}, 7, 21$

If $a = 7$ $r = \frac{1}{3}$ " " $21, 7, \frac{7}{3}$

33.

$$\begin{array}{l|l} & 10 + 11x + 12x^2 \\ 10 & 100 + 220x + 361x^2 + bx^3 + ax^4 \\ & 100 \\ \hline 20 + 11x & 220x + 361x^2 \\ & 220x + 121x^2 \\ \hline 20 + 22x + 12x^2 & 240x^2 + bx^3 + ax^4 \\ & 240x^2 + 264x^3 + 144x^4 \\ \hline & 0 \end{array}$$

$\boxed{b = 264}$

$\boxed{a = 144}$

32.

$10^2 + 11^2 + 12^2 + \dots + 24^2$

$(1^2 + 2^2 + 3^2 + \dots 24^2) - 1^2 + 2^2 + \dots 9^2$

$= \frac{24 \times 25 \times 49}{6} - \frac{9 \times 10 \times 19}{6}$

$= 4900 - 285$

$= 4615$ cm^2

34. $2x^2 - x - 1 = 0$

$\alpha + \beta = +\frac{1}{2}$

$\alpha\beta = \frac{1}{2}$

(i) Given roots $\frac{1}{\alpha}, \frac{1}{\beta}$

Sum of $= \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta}$

$= \frac{\frac{1}{2}}{-\frac{1}{2}} = -1$

P. of roots $= \frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta}$

$= -2$

The. re. eq $= x^2 + 1x - 2 = 0$

(ii) $2\alpha + \beta$, $2\beta + \alpha$

S. of. $2\alpha + \beta + 2\beta + \alpha = 3(\alpha + \beta)$

$= 3 \times +\frac{1}{2} = +\frac{3}{2}$

P. of $= (2\alpha + \beta) \times (2\beta + \alpha)$

$= 4\alpha\beta + 2\alpha^2 + 2\beta^2 + \alpha\beta$

$= 5\alpha\beta + 2(\alpha^2 + \beta^2)$

$= 5\alpha\beta + 2[(\alpha + \beta)^2 - 2\alpha\beta]$

$= 5 \times -\frac{1}{2} + 2 \times \left(\frac{1}{2}\right)^2 - 2 \times \frac{1}{2}$

$= 0$

The req. eq $= x^2 - \frac{3}{2}x + 0 = 0$

$2x^2 - 3x = 0$

35.

$AB = \begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 2 - 2 + 0 & -1 + 8 + 2 \\ 4 + 1 + 0 & -2 - 4 + 2 \end{bmatrix} = \begin{bmatrix} 0 & 9 \\ 5 & -4 \end{bmatrix}$

$(AB)^T = \begin{bmatrix} 0 & 5 \\ 9 & -4 \end{bmatrix}$ — (1)

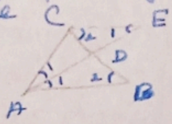
$B^T A^T = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2 - 2 + 0 & 4 + 1 + 0 \\ -1 + 8 + 2 & -2 - 4 + 2 \end{bmatrix} = \begin{bmatrix} 0 & 5 \\ 9 & -4 \end{bmatrix}$ (2)

(1) = (2) H. verified

36. Statement

The internal bisector of an angle of a triangle divides the opposite side internally the ratio of the corresponding sides containing angle



To prove $\frac{AB}{AC} = \frac{BD}{CD}$

$\angle AEC = \angle BAE = 1$ alternate angle

$\triangle ACE$ is isosceles $AC = CE$

$\triangle ABD \sim \triangle ECD$

$\frac{AB}{CE} = \frac{BD}{CD}$ $AC = CE$ sub

$\frac{AB}{AC} = \frac{BD}{CD}$ Hence proved.

38. $4x+5y=13$ — ①

$x-8y=-9$ — ②

① & ② solving

$4x+5y=13$

$4x-32y=-36$

(−) (+) (+)

$37y=49$

$$\boxed{y=\frac{49}{37}}$$

eq ② sub

$x-8\times\frac{49}{37}=-9$

$x-\frac{392}{37}=-9$

$x=-9+\frac{392}{37}$

$x=\frac{-333+392}{37}$

$$\boxed{x=\frac{59}{37}}$$

The point of intersection $\frac{59}{37}, \frac{49}{37}$

Parallel to y axis $x=c$

$x=\frac{59}{37}$

The required equation is

$37x-59=0$

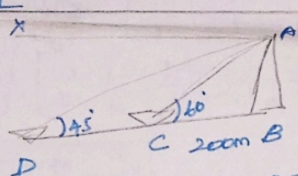
37.

Area of quadrilateral $=\frac{1}{2}\begin{vmatrix} -8 & -9 & -6 & -1 & -8 \\ 6 & 0 & -3 & -2 & 6 \end{vmatrix}$

$=\frac{1}{2}\left[(0+27+12-6)-(-54+0+3+16)\right]$

$=\frac{1}{2}\left[33+35\right]=\frac{1}{2}\times 68=34$ sq units

39.



Given BC = 200

In ΔACB $\tan 60^\circ=\frac{AB}{200}$

$200\sqrt{3}=AB$ — ①

In ΔADB $\tan 45^\circ=\frac{AB}{BD}$

$1=\frac{AB}{BD}$

$BD=200\sqrt{3}$

$CD=BD-CB$

$=200\sqrt{3}-200$

$=200(\sqrt{3}-1)=200(1.732-1)$

$=200\times 0.732=146.4\,m$

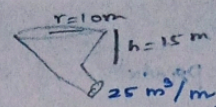
The distance of 146.4 m is covered 10 seconds

$=\frac{146.4}{10}=14.64\,m/s$ (convert km/hr)

$=14.64\times\frac{3600}{1000}$ km/hr

$=52.704$ km/hr

40.



Minutes $=\frac{\text{V. of. conical container}}{\text{V. of. Petrol. rel per minute}}$

$=\frac{\frac{1}{3}\times\frac{22}{7}\times 10\times 10\times 15}{25}$

$=\frac{22\times 10\times 10\times 15}{3\times 7\times 25}=\frac{440}{7}=62.8$

$\simeq 63$ minutes

41. $n(S)=36$

$P(A)=\frac{6}{36}\Rightarrow\{(1,1)(2,2),(3,3)(4,4)(5,5)(6,6)\}$

$P(B)=\frac{3}{36}\Rightarrow\{(1,3),(2,2)(3,1)\}$

$P(A\cap B)=\frac{1}{36}=\{(2,2)\}$

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=\frac{6}{36}+\frac{3}{36}-\frac{1}{36}=\frac{8}{36}=\frac{2}{9}$ Ans.

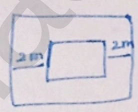
42.

The area of each square is $=6.25\,cm^2$

∴ A chess board Contains 64 squares $=64\times 6.25$

$a^2=400\,cm^2$

a side $=\sqrt{400}=20\,cm$



The length of the side of chess board $=l+2w$

$=20+2\times 2$

$=\boxed{24\ cm}$ Ans.

P. Uma D.TEd, M.SC, B.Ed,

(Asst. Teacher)