

# HALF YEARLY EXAMINATION - 2023

10 - STD

Time: 3.00 Hrs.

Mathematics

Maximum Marks - 100

PART- I (Marks - 14)

Note: i) Answer All the 14 questions

ii) Choose the most suitable answer from given the four alternatives and write the option code with the corresponding answers. 14 x 1 = 14

1. Let  $A = \{1, 2, 3, 4\}$  and  $B = \{4, 8, 9, 10\}$ . A function  $f: A \rightarrow B$  given by  $f = \{(1, 4), (2, 8), (3, 9), (4, 10)\}$  is a  
 (A) Many-one function (B) Identity function  
 (C) One-to-one function (D) Into function
2. If  $f$  is a constant function of the value  $\frac{1}{8}$ , then the value of  $f(1) + f(2) + \dots + f(8)$  is  
 (A)  $\frac{1}{8}$  (B) 1 (C) 8 (D) 0
3. If  $A = 2^{65}$  and  $B = 2^{64} + 2^{63} + 2^{62} + \dots + 2^0$  which of the following is true?  
 (A)  $B$  is  $2^{64}$  more than  $A$  (B)  $A$  and  $B$  are equal  
 (C)  $B$  is larger than  $A$  by 1 (D)  $A$  is larger than  $B$  by 1
4. If the HCF of 65 and 117 is expressible in the form of  $65m - 117$ , then the value of  $m$  is  
 (A) 4 (B) 2 (C) 1 (D) 3
5. The solution of the system  $x + y - 3z = -6$ ,  $-7y + 7z = 7$ ,  $3z = 9$  is  
 (A)  $x = 1, y = 2, z = 3$  (B)  $x = -1, y = 2, z = 3$   
 (C)  $x = -1, y = -2, z = 3$  (D)  $x = 1, y = -2, z = 3$
6. If the roots of the equation  $q^2x^2 + p^2x + r^2 = 0$  are the squares of the roots of the equation  $qx^2 + px + r = 0$ , then  $q, p, r$  are in \_\_\_\_\_  
 (A) A.P (B) G.P (C) Both A.P and G.P (D) None of these
7. If  $A = \begin{pmatrix} 4 & -2 \\ 6 & -3 \end{pmatrix}$ , then  $A^2$  is  
 (A)  $\begin{pmatrix} 16 & 4 \\ 36 & 9 \end{pmatrix}$  (B)  $\begin{pmatrix} 4 & -2 \\ 6 & -3 \end{pmatrix}$  (C)  $\begin{pmatrix} 8 & -4 \\ -12 & -6 \end{pmatrix}$  (D)  $\begin{pmatrix} -4 & 2 \\ -6 & 3 \end{pmatrix}$
8. If  $\triangle ABC$  is an isosceles triangle with  $\angle C = 90^\circ$  and  $AC = 5$  cm, then  $AB$  is  
 (A) 2.5 cm (B) 5 cm (C) 10 cm (D)  $5\sqrt{2}$  cm



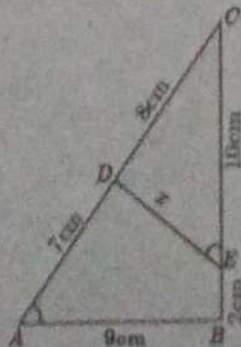
9. The inclination of a line  $y = \sqrt{3}x + 11$  is  
 (A)  $30^\circ$  (B)  $45^\circ$  (C)  $60^\circ$  (D)  $90^\circ$
10. If  $x = a \tan \theta$  and  $y = b \sec \theta$  then  
 (A)  $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$  (B)  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  (C)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  (D)  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$
11. The height of a right circular cone whose radius is 5 cm and slant height is 13 cm will be  
 (A) 12 cm (B) 10 cm (C) 13 cm (D) 5 cm
12. The ratio of the volumes of a cylinder, a cone and a hemisphere, if each has the same diameter and same height is  
 (A) 1:2:3 (B) 2:1:3 (C) 1:3:2 (D) 3:1:2
13. If the standard deviation of  $x, y, z$  is  $t$  then the standard deviation of  $x+5, y+5, z+5$  is \_\_\_\_  
 (A)  $\frac{t}{3}$  (B)  $t+5$  (C)  $t$  (D)  $xyz$
14. Which of the following is incorrect?  
 (A)  $P(A) > 1$  (B)  $0 \leq P(A) \leq 1$  (C)  $P(\phi) = 0$  (D)  $P(A) + P(\bar{A}) = 1$

## PART - II (Marks - 20)

Note: Answer TEN questions. Question Number 28 is compulsory.

10 x 2 = 20

15. A function  $f$  is defined by  $f(x) = 2x - x^2$ . Find the value of  $f(x+1)$ .
16.  $f(x) = 3x - 2$ ,  $g(x) = 2x + k$  and  $f \circ g = g \circ f$  then find the value of  $k$
17. 'a' and 'b' are two positive integers such that  $a^b \times b^a = 800$ . Find 'a' and 'b'.
18. Find the sum  $3 + 1 + \frac{1}{3} + \dots \infty$
19. Determine the quadratic equations, whose sum and product of roots are  $\frac{5}{3}, 4$
20. If  $A = \begin{pmatrix} \cos^2 \theta & 0 \\ 0 & \cos^2 \theta \end{pmatrix}$  and  $B = \begin{pmatrix} \sin^2 \theta & 0 \\ 0 & \sin^2 \theta \end{pmatrix}$  then, prove that  $A + B = I_2$
21. In the Figure, If  $\angle A = \angle CED$  and  $\Delta CAB \sim \Delta CED$ , then find the value of  $x$



22. Prove that  $(\sin^6 \theta + \cos^6 \theta) = 1 - 3\sin^2 \theta \cos^2 \theta$ .
23. Find the equation of a straight line which is parallel to the line  $3x - 7y = 12$  and passing through the point  $(6, 4)$ .
24. A garden roller whose length is 3 m long and whose diameter is 2.8 m is rolled to level a garden. How much area will it cover in 8 revolutions?
25. If total surface area of a solid hemisphere is  $675\pi$  sq.cm then, find its curved surface area.
26. The standard deviation and coefficient of variation of a data are 6.84 and 57 respectively. Find the value of mean.
27. Express the sample space for rolling two dice using tree diagram.
28. Find the domain of the function  $f(x) = \sqrt{1 + \sqrt{1 - \sqrt{1 - x^2}}}$ .

## PART - III (Marks - 50)

Note: Answer TEN questions. Question Number 42 is compulsory.

10 x 5 = 50

29.  $A = \{x \in \mathbb{N} / 1 < x < 4\}$ ,  $B = \{x \in \mathbb{W} / 0 \leq x < 2\}$  and  $C = \{x \in \mathbb{N} / x < 3\}$ , check if  $A \times (B \cap C) = (A \times B) \cap (A \times C)$  is true?
30. The function 't' which maps temperature in Celsius (C) into temperature in Fahrenheit (F) is defined by  $t(C) = F$ , where  $F = \frac{9}{5}C + 32$ . Find,  
 (i)  $t(0)$  (ii)  $t(28)$  (iii)  $t(-10)$   
 (iv) the value of C when  $t(C) = 212$   
 (v) the temperature when the Celsius value is equal to the Fahrenheit value.
31. In an A.P., sum of four consecutive terms is 28 and their sum of their squares is 276. Find the four numbers.
32. Find the sum to n terms of the series  $7 + 77 + 777 + \dots$ .
33. If  $P = \frac{x}{x+y}$ ;  $Q = \frac{y}{x+y}$ , then find the value of  $\frac{1}{P-Q} - \frac{2Q}{P^2-Q^2}$ .
34. If  $9x^4 + 12x^3 + 28x^2 + ax + b$  is a perfect square, find the values of a and b.
35. If  $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{pmatrix}$  and  $B = \begin{pmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{pmatrix}$  then, verify that  $(AB)^T = B^T A^T$ .
36. If vertices of a quadrilateral are at  $A(-5, 7)$ ,  $B(-4, k)$ ,  $C(-1, -6)$  and  $D(4, 5)$  its area is 72 sq.units. Find the value of k.



37. State and Prove Pythagoras theorem.
38. A solid iron cylinder has total surface area of 1848 sq. m. Its curved surface area is five – sixth of its total surface area. Find the radius and height of the iron cylinder.
39. A right circular cylinder having radius 6 cm and height 15 cm is full of ice cream. The ice cream is to be filled in cones of height 9 cm and radius 3 cm, having a hemispherical shape on top. Find the number of such cones which can be filled with the ice cream available.
40. The marks scored by the students in a slip test are given below. Find the standard deviation of their marks.

$x$	4	6	8	10	12
$f$	7	3	5	9	5

41. In a class of 50 students, 28 opted for NCC, 30 opted for NSS and 18 opted both NCC and NSS. One of the students is selected at random. Find the probability that
- The student opted for NCC but not NSS
  - The students opted for NSS but not NCC
  - The students opted for exactly one of them.
42. From a window ( $h$  meters high above the ground) of a house in a street, the angles of elevation and depression of the top and the foot of another house on the opposite side of the street are  $\theta_1$  and  $\theta_2$  respectively. Show that the height of the opposite house is  $h \left( 1 + \frac{\cot \theta_2}{\cot \theta_1} \right)$

#### PART - IV (Marks- 16)

Note: Answer both questions.

2 x 8 = 16

43. (A). Draw a triangle  $\Delta PQR$  of base  $QR = 5$  cm,  $\angle P = 30^\circ$  and the altitude from  $P$  to  $QR$  is of length 4.2 cm.

OR

(B). Draw a circle of diameter 6 cm from a point  $P$ , which is 8 cm away from its centre. Draw the two tangents  $PA$  and  $PB$  to the circle and measure their lengths.

44. (A). A bus is travelling at a uniform speed of 50 km/hr. Draw the distance-time graph and hence find

- the constant of variation
- how far will it travel in  $1\frac{1}{2}$  hr
- the time required to cover a distance of 300 km from the graph.

OR

(B). Draw the graph of  $y = 2x^2$  and use it to solve  $2x^2 - x - 6 = 0$ .