HSL

HALF YEARLY EXAMINATION - 2023

10 - STD **MATHS** Time : 3.00 Hrs

Marks : 100

I Part - I Answer all the questions. 14 X 1 = 14

1. The range of the relation R = {(x, x^2) / x is a prime number less than 13} is
a) {2, 3, 5, 7} b) {2,3,5,7,11} c) {4,9,25,49,121} d) {1,4,9,25,49,121}

2. Let f(x) = $\sqrt{1+x^2}$ then
a) f(xy) = f(x) . f(y) b) f(xy) $\geq$ f(x).f(y) c) f(xy) $\leq$ f(x) . f(y) d) none of these

3. If 6 times of 6th term of an A.P. is equal to 7 times the 7th term, then the 13th term of the A.P is a) 0 b) 6 c) 7 c) 13

4. The sum of first n terms of a G.P, when r = 1 is a) a^n b) n c) na d) a

5. Which of the following should be added to make $x^4 + 64$ a perfect square
a) $4x^2$ b) $16x^2$ c) $8x^2$ d) $-8x^2$

6. The square root of $4m^2 - 24m + 36 = 0$ is a) 4(m-3) b) 2(m-3) c) $(2m - 3)^2$ d) 4(m+ 3)

7. In a ΔABC, AD is the bisector of ∠BAC, If AB = 8cm, BD = 6cm and DC = 3cm. The length of the side AC is a) 6cm b) 4 cm c) 3 cm d) 8 cm

8. The point of intersection of 3x - y = 4 and x + y = 8 is
a) (5, 3) b) (2,4) c) (3,5) d) (4,4)

9. A straight line PQ cuts the X axis at A and Y axis at B. If the mid point of AB is (2a, 2b) then the cor-ordinates of A and B are
a) (a, 0), (0, b) b) (2a, 0), (0, 2b) c) (0, b), (a, 0) d) (0, 2b), (2a, 0)

10. If sin θ + cos θ = a and sec θ + cosec θ = b, then the value of $b(a^2-1)$ is equal to
a) 2a b) 3a c) 0 d) 2ab

11. If two solid hemispheres of same base radius r units are joined together along their bases, then curved surface area of this new solid is
a) $4\pi r^2$ sq. units b) $6\pi r^2$ sq. units c) $3\pi r^2$ sq. units d) $8\pi r^2$ sq. units

12. The curved surface area of a right circular cone of height 15cm and base diameter 16cm is a) 60π cm² b) 66π cm² c) 120π cm² d) 136π cm²

13. The range of the data 8, 8, 8, 8, 8, ... 8 is a) 0 b) 1 c) 8 d) 3

14. A page is selected at random from a book. The probability that the digit at units place of the page number chosen is less than 7 is
a) $\frac{3}{10}$ b) $\frac{7}{10}$ c) $\frac{3}{9}$ d) $\frac{7}{9}$

II Part - II Answer any 10 questions. Question No. 28 is compulsory. 10 X 2 = 20

15. If B X A = {(-2, 3), (-2, 4), (0, 3), (0, 4), (3,3), (3,4)} find A and B.
16. Find k if fof(k) = 5 where f(k) = 2k-1.
17. Find the number of terms in the A.P. 3,6,9,12,.......... 111.
18. If $1^3+2^3+3^3+\dots\dots+k^3 = 44100$ then find 1 + 2 + 3 + ... + k.
19. Find the excluded values of the following expression $\frac{x^2+6x+8}{x^2+x-2}$.
20. What length of ladder is needed to reach a height of 7ft along the wall when the base of the ladder is 4ft from the wall? Round off your answer to the next tenth place.
21. If the three points (3, -1), (a, 3) and (1, - 3) are collinear, find the value of a.
22. Find the equation of a line through the given pair of points (2,3) and (-7, -1).

23. Prove that $\frac{\sec\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} = \cot\theta$.
24. The volume of a solid right circular cone is 11088cm³. If its height is 24cm then find the radius of the cone.
25. A right circular cylinder just enclose a sphere of radius r units. Calculate i) the surface area of the sphere. ii) the curved surface area of the cylinder.
26. Find the standard deviation of first 21 natural numbers.
27. A coin is tossed thrice. What is the probability of getting two consecutive tails?
28. If $(5\ x\ 1)\begin{pmatrix}2\\-1\\3\end{pmatrix} = (20)$ find the value of x?

III Part - III Answer any 10 questions. Question No. 42 is compulsory. 10 X 5 = 50

29. Let $A = \{x \in W / x < 2\}$, $B = \{x \in N / 1 < x \leq 4\}$ and $C = \{3,5\}$ Verify that $AX(B \cap C) = (AXB) \cap (AXC)$.
30. If f(x) = x-1, g(x) = 3x + 1 and h(x) = x^2 show that (fog) oh = fo(goh).
31. The product of three consecutive terms of a Geometric progression is 343 and their sum is $\frac{91}{3}$. Find the three terms.
32. Rekha has 15 square colour papers of sizes 10cm, 11cm, 12cm, ... 24cm. How much area can be decorated with these colour papers?
33. If $ax^4 + bx^3 + 361x^2 + 220x + 100$ is a perfect square, find the values of a and b.
34. If α, β are the roots of the equation $2x^2 - x - 1 = 0$ then form the equation whose roots are i) $\frac{1}{\alpha}, \frac{1}{\beta}$ ii) $2\alpha + \beta$, $2\beta + \alpha$.
35. If $A = \begin{pmatrix}1 & 2 & 1\\2 & -1 & 1\end{pmatrix}$, $B = \begin{pmatrix}2 & -1\\-1 & 4\\0 & 2\end{pmatrix}$ show that $(AB)^T = B^T A^T$.
36. State and prove angle bisector theorem.
37. Find the area of the quadrilateral whose vertices are at (-9, 0), (-8, 6), (-1, -2) and (-6, -3).
38. Find the equation of a straight line parallel to y axis and passing through the point of intersection of the line 4x + 5y = 13, x - 8y + 9 = 0.
39. A man is watching a boat speeding away from the top of a tower. The boat makes an angle of depression of 60° with the man's eye when at a distance of 200m from the tower. After 10 seconds, the angle of depression becomes 45°. What is the approximate speed of the boat (in Km/hr), assuming that it is sailing in still water? ($\sqrt{3} = 1.732$)
40. A conical container is fully filled with petrol. The radius is 10m and the height is 15m. If the container can release the petrol through its bottom at the rate of 25cu. meter per minute, in how many minutes the container will be emptied. Round off your answer to the nearest minutes.
41. Two dice are rolled together. Find the probability of getting a doublet or sum of face as 4.
42. A chess board contains 64 equal squares and the area of each square is 6.25cm². A border around the board is 2cm wide. Find the length of the side of the chess board?

IV Part - IV Answer all questions. 2 X 8 = 16

43. Draw a triangle ABC of base BC = 5.6cm, $\angle A = 40^0$ and the bisector of $\angle A$ meets BC at D such that CD = 4cm. ***(OR)*** b) Draw tangent to the circle from the point P having radius 3.6cm and the centre at O. Point P is at a distance 7.2cm from the centre.
44. a) Graph the following linear function y= ½ x. Identify the constant of variation and verify it with the graph. Also i) find y when x= 9. ii) find x when y = 7.5. ***(OR)***
    b) Draw the graph of $y = x^2 + 3x + 2$ and use it to solve $x^2 + 2x + 1 = 0$.

BRINDHAVAN HR SEC SCHOOL,SUKKIRANPATTI

HALF YEARLY EXAMINATION DEC 2023

10th Standard

Date : 19-Dec-23

Reg.No. :

Maths

Time : 03:00:00 Hrs

Total Marks : 100

PART- I 14 x 1 = 14

ANSWER ALL THE QUESTIONS

1) The range of the relation R = {(x, x^2) |x is a prime number less than 13} is

(a) {2,3,5,7} (b) {2,3,5,7,11} **(c) {4,9,25,49,121}** (d) {1,4,9,25,49,121}

2) Let f(x) = $\sqrt{1+x^2}$ then

(a) f(xy) = f(x).f(y) (b) f(xy) ≥ f(x).f(y) **(c) f(xy) ≤ f(x).f(y)** (d) None of these

3) If 6 times of 6^{th} term of an A.P. is equal to 7 times the 7^{th} term, then the 13^{th} term of the A.P. is

(a) 0 (b) 6 (c) 7 (d) 13

4) The sum of first n terms of a G.P.when r=1 is

(a) a^n (b) n **(c) na** (d) a

5) Which of the following should be added to make x^4 + 64 a perfect square

(a) $4x^2$ **(b) $16x^2$** (c) $8x^2$ (d) $-8x^2$

6) The square root of $4m^2$ -24m+36=0 is

(a) 4(m-3) **(b) 2(m-3)** (c) $(2m-3)^2$ (d) 4(m+3)

7) In a △ABC, AD is the bisector ∠BAC. If AB = 8 cm, BD = 6 cm and DC = 3 cm. The length of the side AC is

(a) 6 cm **(b) 4 cm** (c) 3 cm (d) 8 cm

8) The point of intersection of 3x − y = 4 and x + y = 8 is

(a) (5, 3) (b) (2, 4) **(c) (3, 5)** (d) (4, 4)

9) Is straight line PQ cuts the X axis at A and Y axis at B. If the midpoint of AB is (a,b) then the coordinates of A and B are

(a) (a,0),(0,b) **(b) (2a,0),(0,2b)** (c) (0,b),(a,0) (d) (0,2b),(2a,0)

10) If sin θ + cosθ = a and sec θ + cosec θ = b, then the value of b(a^2 - 1) is equal to

(a) 2a (b) 3a (c) 0 (d) 2ab

11) If two solid hemispheres of same base radius r units are joined together along their bases, then curved surface area of this new solid is

(a) $4\pi r^2$ sq.units (b) $6\pi r^2$ sq.units (c) $3\pi r^2$ sq.units (d) $8\pi r^2$ sq.units

12) The curved surface area of a right circular cone of height 15 cm and base diameter 16 cm is

(a) 60π cm^2 (b) 68π cm^2 (c) 120π cm^2 **(d) 136π cm^2**

13) The range of the data 8, 8, 8, 8, 8. . . 8 is

(a) 0 (b) 1 (c) 8 (d) 3

14) A page is selected at random from a book. The probability that the digit at units place of the page number chosen is less than 7 is

(a) $\frac{3}{10}$ **(b) $\frac{7}{10}$** (c) $\frac{3}{9}$ (d) $\frac{7}{9}$

PART -II 10 x 2 = 20

ANSWER ANY 10 QUESTIONS QUESTION NO.28 IS COMPULSORY

15) If B x A = {(-2,3), (-2,4),(0,3), (0,4),(3,3),(3,4)} find A and B.

Answer : From B x A, All the first entries belong to the set B and all the second entries belong to A.

A = {3,4} and

B = {-2,0,3}

16) Find k if f o f(k) = 5 where f(k) = 2k - 1.

Answer : f o f(k) = f(f(k))

= 2(2k - 1) -1 = 4k - 3

Thus, f o f(k) = 4k - 3

But, it is given that f o f(k) = 5

Therefore 4k - 3 = 5 ⇒ k = 2

17) Find the number of terms in the A.P. 3, 6, 9, 12,..., 111.

Answer : First term a = 3; common difference d = 6 - 3 = 3 ; last term l = 111

We know that, n = $\left(\frac{l-a}{d}\right)+1$

n = $\left(\frac{111-3}{3}\right)+1$ = 37

Thus the A.P. contain 37 terms

18) If $1^3 + 2^3 + 3^3 + \ldots k^3 = 44100$ then find 1 + 2 + 3 +...+ k

Answer : $1^3 + 2^3 + 3^3 + \ldots K^3 = \left[\frac{k(k+1)}{2}\right]^2 = 44100 = (210)^2$

1 + 2 + 3 +...+ k = $\frac{k(k+1)}{2} = 210$

1 + 2 + 3 +...+ k = 210

19) Find the excluded values of the following expressions.

$\frac{x^2+6x+8}{x^2+x-2}$

Answer : $\frac{x^2+6x+8}{x^2+x-2} = \frac{(x+2)(x+4)}{(x+2)(x-1)} = \frac{(x+4)}{(x-1)}$ (x-1) = 0.is undefined ∴ The excluded value is .1.

20) What length of ladder is needed to reach a height of 7 ft along the wall when the base of the ladder is 4 ft from the wall? Round off your answer to the next tenth place.

Answer : Let x be the length of the ladder. BC = 4 ft, AC = 7 fit.

By Pythagoras theorem we have, $AB^2 = AC^2 + BC^2$

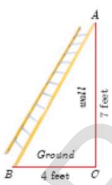
$x^2 = 7^2 + 4^2$ gives $x^2 = 49 + 16$

$x^2 = 65$, Hence $x = \sqrt{65}$

The number $\sqrt{65}$ is between 8 and 8.1.

82 = 64 < 65.61 = 8.1^2

Therefore, the length of the ladder is approximately 8.1ft



21) If the three points (3, - 1) , (a, 3) and (1, - 3) are collinear, find the value of a.

Answer : Given points (3, - 1), (a, 3) and (1, - 3)

Let the points be A (3, - 1), B (a, 3) and C (1,- 3)

Slope of AB = $\frac{y_1-y_2}{x_1-x_2} = \frac{-1-3}{3-a} = \frac{-4}{3-a}$

Slope of BC = $\frac{y_1-y_2}{x_1-x_2} = \frac{3+3}{a-1} = \frac{6}{a-1}$

Since, the points A, B and C are collinear.

Slope of AB = Slope of BC

$\frac{-4}{3-a} = \frac{6}{a-1}$

-2 (a - 1) = 3(3 - a)

-2a + 2 = 9 - 3a

3a - 2a = 9 - 2

a = 7

22) Find the equation of a line through the given pair of points (2, 3) and (-7, -1)

Answer : Given points (2, 3) and (- 7, - 1)

Equation of the line passing through (x_1 , y_1) and (x_2, y_2) is

$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

$\frac{y-3}{-1-3} = \frac{x-2}{-7-2}$

9 (y - 3) - 4 (x - 2)

9y - 27 = 4 x - 8

4 x - 9y +19 = 0

23) prove that $\frac{sec\theta}{sin\theta} - \frac{sin\theta}{cos\theta} = cot\theta$

Answer : $\frac{sec\theta}{sin\theta} - \frac{sin\theta}{cos\theta} = \frac{\frac{1}{cos\theta}}{sin\theta} - \frac{sin\theta}{cos\theta} = \frac{1}{sin\theta cos\theta} - \frac{sin\theta}{cos\theta}$

$= \frac{1-sin^2\theta}{sin\theta cos\theta} = cot\theta$

24) The volume of a solid right circular cone is 11088 cm^3. If its height is 24 cm then find the radius of the cone.

Answer : Let r and h be the radius and height of the cone respectively.

Given that, volume of the cone = 11088 cm^3

$\frac{1}{3}\pi r^2 h = 11088$

$\frac{1}{3} \times \frac{22}{7} \times r^2 \times 24 = 11088$

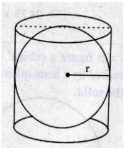
$r^2 = 441$

Therefore, radius of the cone r = 21 cm.

25) A right circular cylinder just enclose a sphere of radius r units. Calculate

(i) the surface area of the sphere

(ii) the curved surface area of the cylinder

Answer : 

(i) Surface area of a sphere Radius of sphere = r

Surface area = $4r^2$ sq. units

(ii) Curved surface area of cylinder

Radius of cylinder = r

Height of cylinder = r + r = 2r

Curved surface area $= 2\pi rh\ sq.units$

$= 2\pi r$(2r)

$= 4\pi r^2$ sq. units

26) Find the standard deviation of first 21 natural numbers.

Answer : Standard deviation of first n natural numbers

$= \sqrt{\frac{n^2-1}{12}}$

SD of first 21 natural numbers

$= \sqrt{\frac{21^2-1}{12}}$

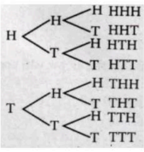
$= \sqrt{\frac{441-1}{12}} = \sqrt{\frac{440}{12}}$

$= \sqrt{36.6666} = 6.05$

Standard deviation of first 21 natural numbers = 6.05

27) A coin is tossed thrice. What is the probability of getting two consecutive tails?

Answer : When a coin is tossed thrice, the outcome will be



The sample space s = {(HHH), (THH), (HTH), (HHT), (HTT), (THT), (TTH), (TTT)}

n(S) = 3

Let A be the event of getting two consecutive tails

4 = {HTT, TTH, TTT}

n(A) = 3

$\Rightarrow P = \frac{n\{F\}}{n\{O\}} = \frac{3}{8}$

Probability of getting two consecutive tails = $\frac{3}{8}$

28) Find the values of x

$$\begin{bmatrix} 5 & x & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 20 \end{bmatrix}$$

Answer : $\begin{bmatrix} 5 & x & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 20 \end{bmatrix}$

⇒(10-x+3) = (20)

⇒ 13-x = 20 x = 13-20 x= -7

PART -III 14 x 5 = 70

ANSWER ANY 10 QUESTIONS QUESTION NO.42 IS COMPULSORY

29) Let A ={x ∈ W | x < 2}, B={x ∈ N | 1 < x ≤ 4} and C = (3,5). Verify that
A x (B ∩ C) = (A x B) ∩ (A x C)

Answer : A x (B ∩ C) = (A x B) ∩ (A x C)

A = {0,1} , B = {2,3,4} , C = {3,5} , B∩C = {3}

$A \cap (B \cap C) = \{0,1\} \times \{3\}$

= {(0,3),(1, 3)} ...(1)

A x B = {(0,2),(0,3),(0,4),(1,2),(1,3),(1,4)}

A x C = {(0,3),(0,5),(1,3),(1,5)}

$(A \times B) \cap (A \times C)$ = {(0,3),(1,3)} ...(2)

From (1) and (2), it is clear that

A x (B ∩ C) = (A x B) ∩ (A x C)

Hence verified

30) Show that (f o g) o h = f o (g o h) if f(x) = x - 1, g(x) = 3x + 1 and h(x) = x^2

Answer : (i) f o g = f [g(x)] = f[3x + 1]

= (3x + 1) - 1 = 3x

(f o g) o h = (f o g) [h(x)] = (f o g)[x^2] = $3x^2$

g o h = g [h(x)] = g [x^2] = $3x^2 + 1$

f o(g o h) = f [g (h(x))] = f [g(x^2)] = f [$3x^2 + 1$]

= ($3x^2 + 1$) - 1 = $3x^2$

(f o g) o h = f o(g o h)

Hence proved.

31) The product of three consecutive terms of a Geometric Progression is 343 and their sum is $\frac{91}{3}$. Find the three terms.

Answer : Since the product of 3 consecutive terms is given.

we can take them as $\frac{a}{r}$, a, ar

Product of the terms = 343

$\frac{a}{r}$ x a x ar = 343

a^3 = 73 gives a = 7

Sum of the terms = $\frac{91}{3}$

Hence $a\left(\frac{1}{r}+1+r\right)=\frac{91}{3}$ $7\left(\frac{1+r+r^2}{r}\right)=\frac{91}{3}$

3 + 3r + $3r^2$ = 13r gives $3r^2$ -10r + 3 = 0

(3r - 1)(r - 3) = 0 gives r = 3 or r = $\frac{1}{3}$

if a = 7, r = 3 then the three terms are $\frac{7}{3}$, 7, 21

If a = 7, r = $\frac{1}{3}$ then the three terms are 21, 7, $\frac{7}{3}$.

32) Rekha has 15 square colour papers of sizes 10 cm, 11 cm, 12 cm,..., 24 cm. How much area can be decorated with these colour papers?

Answer : $1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$

With the square colour papers are decorated

= $10^2 + 11^2 + 12^2 + ... + 24^2$

$10^2 + 11^2 + 12^2 + ... + 24^2 = (1^2 + 2^2 + 3^2 + ... + 24^2) - (1^2 + 2^2 + ... + 9^2)$

$=\frac{24\times(24+1)[2(24)+1]}{6}-\frac{9\times(9+1)[2(9)+1]}{6}$

$=(4\times25\times49)-\frac{9\times10\times19}{6}$

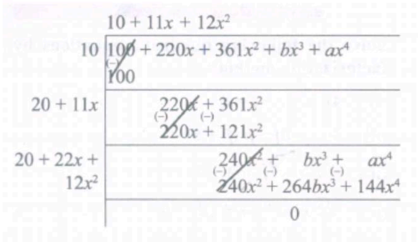
= 4900 - 285 = 4615

4615 cm^2 area can be decorated

33) Find the values of a and b if the following polynomials are perfect squares

$ax^4 + bx^3 + 361x^2 + 220x + 100$

Answer : $ax^4 + bx^3 + 361x^2 + 220x + 100$



$\therefore$ a = 144

b = 264

34) a) If α, β are the roots of the equation $2x^2$ - x - 1 = 0, then form the equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}$

Answer : $2x^2$ - x - 1 = 0 here, a = 2, b = -1, c = -1

$\alpha + \beta = \frac{-b}{a}=\frac{-(-1)}{2}=\frac{1}{2}$, $\alpha\beta = \frac{c}{a}=-\frac{1}{2}$

Given roots are $\frac{1}{\alpha}, \frac{1}{\beta}$

Sum of the roots = $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{\frac{1}{2}}{-\frac{1}{2}}$ = -1

Product of the roots = $\frac{1}{\alpha}\times\frac{1}{\beta}=\frac{1}{\alpha\beta}=\frac{1}{-\frac{1}{2}}$ = -2

The required equation is x^2 – (Sum of the roots)x + (Product of the roots) = 0

x^2 - (-1)x - 2 = 0 gives x^2 + x - 2 = 0

b)

2α + β, 2β + α

Answer : $2x^2 - x - 1 = 0$ here, a = 2, b = -1, c = -1

$\alpha + \beta = \frac{-b}{a} = \frac{-(-1)}{2} = \frac{1}{2}$, $\alpha\beta = \frac{c}{a} = -\frac{1}{2}$

$2\alpha + \beta$, $2\beta + \alpha$

Sum of the roots $2\alpha + \beta + 2\beta + \alpha = 3(\alpha + \beta) = 3\left(\frac{1}{2}\right) = \frac{3}{2}$

Product of the roots = $(2\alpha + \beta)(2\beta + \alpha) = 4\alpha\beta + 2\alpha^2 + 2\beta^2 + \alpha\beta$

$= 5\alpha\beta + 2(\alpha^2 + \beta^2) = 5\alpha\beta + 2[(\alpha + \beta)^2 - 2\alpha\beta]$

$= 5\left(-\frac{1}{2}\right) + 2\left[\frac{1}{4} - 2 \times -\frac{1}{2}\right] = 0$

The required equation is x^2 - (Sum of the roots)x + (Product of the roots) = 0

$x^2 - \frac{3}{2}x + 0 = 0$ gives $2x^2 - 3x = 0$

35) If A = $\begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{bmatrix}$ show that $(AB)^T = B^TA^T$

Answer : LHS = $(AB)^T$

AB = $\begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{bmatrix}_{2\times3} \times \begin{bmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{bmatrix}_{3\times2}$

$= \begin{bmatrix} 2-2+0 & -1+8+2 \\ 4+1+0 & -2-4+2 \end{bmatrix} = \begin{bmatrix} 0 & 9 \\ 5 & -4 \end{bmatrix}$

$(AB)^T = \begin{bmatrix} 0 & 9 \\ 5 & -4 \end{bmatrix}^T = \begin{bmatrix} 0 & 5 \\ 9 & -4 \end{bmatrix}$ ....(1)

RHS = (B^TA^T)

$B^T = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \end{bmatrix}$, $A^T = \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 1 & 1 \end{bmatrix}$

$B^TA^T = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \end{bmatrix}_{2\times3} \times \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 1 & 1 \end{bmatrix}_{3\times2}$

$= \begin{bmatrix} 2-2+0 & 4+1+0 \\ -1+8+2 & -2-4+2 \end{bmatrix}$

$B^TA^T = \begin{bmatrix} 0 & 5 \\ 9 & -4 \end{bmatrix}$ ...(2)}

From (1) and (2), $(AB)^T = B^TA^T$.

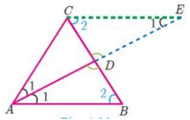
Hence proved.

36) State and Prove - Angle Bisector Theorem

Answer : Statement :

The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the corresponding sides containing the angle

Proof



Given : In ΔABC,AD is the internal bisector

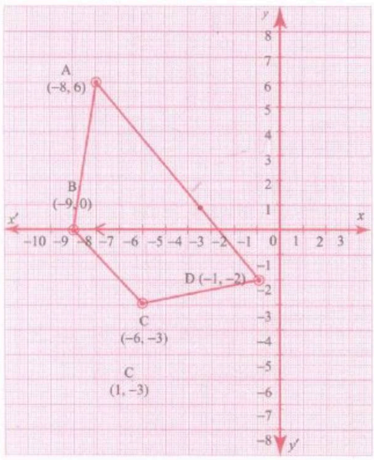
To prove : $\frac{AB}{AC} = \frac{BD}{CD}$

Construction : Draw a line through C parallel to AB. Extend AD to meet line through C at E

NO	STATEMENT	REASON
1.	∠AEC =∠BAE =∠1	Two parallel lines cut by a transversal make alternate angles equal
2.	ΔACE is isosceles AC = CE ... (1)	In ΔACE,∠CAE = ∠CEA
3.	ΔABD~ΔECD $\frac{AB}{CE} = \frac{BD}{CD}$	By AA Similarity
4.	$\frac{AB}{AC} = \frac{BD}{CD}$	From (1) AC = CE Hence proved

37) Find the area of the quadrilateral whose vertices are at (–9, 0), (–8, 6), (–1, –2) and (–6, –3)

Answer :



Given vertices are (- 9, 0), (- 9, 6), (- 1,- 2) and (-6, -3)

Area of quadrilateral ABCD

$= \frac{1}{2}[(x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1) - (x_2y_1 + x_3y_2 + x_4y_3 + x_1y_4)]$

$(or)\frac{1}{2} = [(\dot{x}_1 - x_3)(y_2 - y_4) - (x_2 - x_4)(y_1 - y_3)]\ sq.\,units$

= $\frac{1}{2}$ [(-9 + 1) (- 3 - 6) - (- 6 + 8) (0 + 2)]

= $\frac{1}{2}$ [(- 8) (- 9) - (2) (2)]

= $\frac{1}{2}$ [72 - 4]

$= \frac{68}{2} = 34$

Area of quadrilateral = 34 sq. units

38) Find the equation of a straight line parallel to Y axis and passing through the point of intersection of the lines 4x + 5y = 13 and x - 8y + 9 = 0

Answer : Given lines 4x + 5y − 13 = 0 .....(1)

x − 8y + 9 = 0 .....(2)

To find the point of intersection, solve equation (1) and (2)

x y 1
5 −13 4 5
−8 9 1 −8

$\frac{x}{45-104} = \frac{y}{-13-36} = \frac{1}{-32-5}$

$\frac{x}{-59} = \frac{y}{-49} = \frac{1}{-37}$

$x = \frac{59}{37}, y = \frac{49}{37}$

Therefore, the point of intersection (x, y) = $\left(\frac{59}{37}, \frac{49}{37}\right)$

The equation of line parallel to Y axis is x = c.

It passes through (x, y) = $\left(\frac{59}{37}, \frac{49}{37}\right)$. Therefore, c = $\frac{59}{37}$

The equation of the line x = $\frac{59}{37}$ gives 37x - 59 = 0

39) A man is watching a boat speeding away from the top of a tower. The boat makes an angle of depression of 60° with the man's eye when at a distance of 200 m from the tower. After 10 seconds, the angle of depression becomes 45°. What is the approximate speed of the boat (in km / hr), assuming that it is sailing in still water ?($\sqrt{3}$ = 1.732)

Answer : Let AB be the tower.

Let C and D be the positions of the boat.

From the diagram,

∠XAC = 60° = ∠ACB and ∠ XAD = 45° = ∠ ADB, BC = 200 m

In right triangle ABC, tan60° = $\frac{AB}{BC}$

gives $\sqrt{3}$ $\frac{AB}{200}$

we get AB = 200$\sqrt{3}$ ... (1)

In right triangle ABD, tan45° = $\frac{AB}{BD}$

gives = $\frac{200\sqrt{3}}{BD}$ [by (1)]

we get, BD = 200$\sqrt{3}$

now, CD = BD - BC

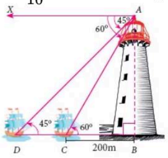
CD = 200$\sqrt{3}$ - 200 = 200($\sqrt{3}$ -1) = 146.4

It is given that the distance CD is covered in 10 seconds.

That is, the distance of 146.4 m is covered in 10 seconds.

Therefore, speed of the boat = $\frac{distance}{time}$

= $\frac{146.4}{10}$ = 14.64 m/s gives 14.64$\times\frac{3600}{100}$ km/hr = 52.704 km/hr



40) A conical container is fully filled with petrol. The radius is 10 m and the height is 15 m. If the container can release the petrol through its bottom at the rate of 25 cu.meter per minute, in how many minutes the container will be emptied. Round off your answer to the nearest minute.

Answer : Radius of conical container = 10 m

Height of conical container = 15 m

Volume $= \frac{1}{3}\pi r^2 h$ cu. units

$= \frac{1}{3} \times \frac{22}{7} \times 10 \times 10 \times 15$

$= \frac{11000}{7} m^3$

water is released at the rate of 25 m^3 / min

Time required to empty the container

$= \frac{11000}{7}$

$= \frac{11000}{25} = 62.85$

= 63 minutes (approx)

41) Two dice are rolled together. Find the probability of getting a doublet or sum of faces as 4.

Answer : When two dice are rolled together, there will be 6 x 6 = 36 outcomes. Let S be the sample space. Then n(S) = 36.

Let A be the event of getting a doublet and B be the event of getting face sum 4.

Then A = {(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)}

B = {(1,3),(2,2),(3,1)}

Therefore, A∩B = {(2,2)}

Then, n(A) = 6, n(B) = 3, n(A∩B) = 1

P(A) = $\frac{n(A)}{n(S)} = \frac{6}{36}$

P(B) = $\frac{n(B)}{n(S)} = \frac{3}{36}$

P(A∩B) = $\frac{n(A\cap B)}{n(S)} = \frac{1}{36}$

Therefore, P (getting a doublet or a total of 4) = P(AUB)

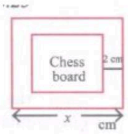
P(AUB) = P(A) + P(B) - P(A∩B)

=$\frac{6}{36} + \frac{3}{36} - \frac{1}{36} = \frac{8}{36} = \frac{2}{9}$

Hence, the required probability is $\frac{2}{9}$.

42) A chess board contains 64 equal squares and the area of each square is 6.25 cm^2, A border round the board is 2 cm wide.

Answer :



Let the length of the side of the chess board be x cm. Then

Area of 64 squares = $(x - 4)^2$

$(x - 4)^2 = 64 \times 6.25$

$\Rightarrow (x - 4)^2 = 400 \Rightarrow (x - 4) = \sqrt{400}$

$\Rightarrow x - 4 = \pm 20$

$\Rightarrow x = 24$ or $x = -16$ is not possible

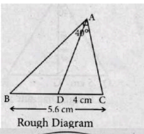
$\Rightarrow$ **x=24 cm.**

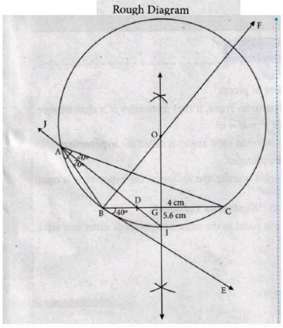
PART -IV 2 x 8 = 16

ANSWER ALL THE QUESTIONS

43) a) Draw a triangle ABC of base BC = 5.6 cm, $\angle A = 40^\circ$ and the bisector of $\angle A$ meets BC at D such that CD = 4 cm.

Answer :





Construction:

Steps (1) Draw a line segment BC = 5.6 cm

Steps (2) At B, draw BE such that $\angle CBE = 60^0$

Steps (3) At B draw BF such that $\angle EBF = 90^0$

Steps (4) Drawn the perpendicular bisector to BC, which intersects BF at O and BC at G.

Steps (5) With O as centre and OB as radius draw a circle

Steps (6) From B, marked an arc of 4 cm on BC at D.

Steps (7) The perpendicular bisector intersects the circle at I. Joined ID.

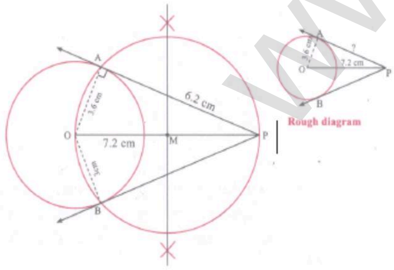
Steps (8) ID produced meets the circle at A. Now joined AB and AC. Then $\triangle$ABC is the required triangle.

(OR)

b) Draw a tangent to the circle from the point P having radius 3.6 cm, and centre at O. Point P is at a distance 7.2 cm from the centre.

Answer : Given radius r = 3.6 cm

Length of the tangents PA = PB = 6.2cm



Construction:

Steps

(1) with centre at o, drawn a circle of radius 3.6 cm.

(2) Draw a line OP = 7.2 cm,

(3) Draw a perpendicular bisector of OP, which cuts OP at M.

(4) With M as centre and MO as radius, draw a circle which cuts previous circle at A and B.

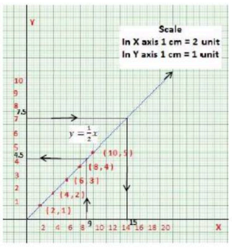
(5) Join AP and BP. AP and BP are the required tangents. Thus length of the tangents are PA = PB = 6.2 cm.

44) Graph the following linear function $y = \frac{1}{2}x$. Identify the constant of variation and verify it with the graph. Also

(i) find y when x = 9

(ii) find x when y = 7.5.

Answer :



1.Table :

x	2	4	6	8	10
y	1	2	3	4	5

2.Variation :

Direct Variation

3. Equation

y = kx

$k = \frac{y}{x} = \frac{1}{2} = \frac{2}{4} = \ldots\ldots \frac{1}{2}$

$y = \frac{1}{2}x$

4. Points :

(2,1),(4,2),(6,3),(8,4),(1,5)

5. Solution

From the graph

(i) If x = 9 then, y = 4.5

(ii) if y = 7.5 then, x = 15

[redacted] Draw the graph of $y = x^2 + 3x + 2$ and use it to solve $x^2 + 2x + 1 = 0$

Answer : -1

[redacted]