Common Half Yearly exam - 2023

XI - Mathematics

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Answer Key [Thiruvallur District]

I Choose the Correct Answer:

(1) (b) 512 (2) (c) 3
(3) (a) 18 (4) (b) 2
(5) (c) 0 (6) (c) $2\cos^2\frac{x}{2}$
(7) (a) $r!$ (8) (b) $a \geq 9$
(9) (c) 37 (10) (c) $(-3,-2)$
(11) (d) $k=3$ (12) (b) 0
(13) (c) 3 (14) (b) $4\hat{i}+5\hat{j}$
(15) (c) $\frac{15}{4}$ (16) (b) $2(\log 2)^2$
(17) (a) -1 (18) (d) $\frac{x^3}{3}+c$
(19) (a) $\sin^{-1}\left(\frac{x}{a}\right)+c$ (20) (c) $1/2$

II PART - B

(21) $(g\circ f)(x) = g(f(x)) = g(3x-4)$
$= (3x-4)^2+3$
$= 9x^2-24x+19$

$(f\circ g)(x) = f(g(x)) = f(x^2+3)$
$= 3(x^2+3)-4$
$= 3x^2+5$

(22) $\frac{7+\sqrt{6}}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}} = \frac{21+7\sqrt{2}+3\sqrt{6}+\sqrt{12}}{9-2}$

$= \frac{21+7\sqrt{2}+3\sqrt{6}+2\sqrt{3}}{7}$

(23) -450 — IVth quadrant
1150 — Ist quadrant

(24) $\frac{n!}{r!(n-r)!}$ $n=50, r=47$

$= \frac{50!}{47!(50-47)!}$

$= \frac{50!}{47! \times 3!}$

$= \frac{50\times 49\times \cancel{48}^{16}\times \cancel{47!}}{\cancel{47!}\times \cancel{3}\times \cancel{2}\times 1}$

$= 19600$

(25) (i) Intercept form : $\frac{x}{a}+\frac{y}{b}=1$

(ii) Two point form

$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

(26) $r = \sqrt{x^2+y^2+z^2} = \sqrt{3^2+(-3)^2+4^2}$
$r = \sqrt{9+9+16} = \sqrt{34}$

Direction ratios are $3, -3, 4$

Direction Cosines are $\frac{3}{\sqrt{34}}, \frac{-3}{\sqrt{34}}, \frac{4}{\sqrt{34}}$

(27) $x = a\cos t$, $y = a\sin t$

$\frac{dx}{dt} = -a\sin t$, $\frac{dy}{dt} = a\cos t$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{a\cos t}{-a\sin t} = -\cot t$

(28) $\frac{\cos x}{\sin^2 x} = \frac{\cos x}{\sin x}\times\frac{1}{\sin x}$
$= \operatorname{cosec} x \cot x$

$\int \frac{\cos x}{\sin^2 x}dx = \int \operatorname{cosec} x \cot x\, dx$
$= -\operatorname{cosec} x + C$

(29) $P(\bar{A}\cup\bar{B}) = P(\overline{A\cap B})$
$= 1 - P(A\cap B)$
$= 1 - 0.15$

$P(\bar{A}\cup\bar{B}) = 0.85$

30) $\begin{vmatrix} 3 & 2 & 1 \\ 1 & 2 & 3 \end{vmatrix} = -3$

$2(6-2) - x(9-1) + 4(6-2) = -3$

$2(4) - x(8) + 4(4) = -3$

$8 - 8x + 16 = -3$

$24 - 8x = -3$

$24 + 3 = 8x$

$27 = 8x \Rightarrow \boxed{x = \frac{27}{8}}$

III **Part-C**

31) $f(x) = \dfrac{\sqrt{4-x^2}}{\sqrt{x^2-9}}$

when $x = 2, \quad f(x) = 0$

$x = -2, \quad f(x) = 0$

For all the other values we get negative value in the square root which is not possible

$\therefore$ Domain $= \phi$

32) $\tan\left(\frac{\pi}{4}+\theta\right) - \tan\left(\frac{\pi}{4}-\theta\right)$

$= \dfrac{\tan\left(\frac{\pi}{4}\right) + \tan\theta}{1 - \tan\frac{\pi}{4} \times \tan\theta} - \dfrac{\tan\frac{\pi}{4} - \tan\theta}{1 + \tan\frac{\pi}{4} \cdot \tan\theta}$

$= \dfrac{1+\tan\theta}{1-\tan\theta} - \dfrac{1-\tan\theta}{1+\tan\theta}$

$= \dfrac{(1+\tan\theta)^2 - (1-\tan\theta)^2}{(1-\tan\theta)(1+\tan\theta)}$

$= \dfrac{\cancel{1} + \cancel{\tan^2\theta} + 2\tan\theta - \cancel{1} - \cancel{\tan^2\theta} + 2\tan\theta}{1-\tan^2\theta}$

$= \dfrac{2 \times 2\tan\theta}{1-\tan^2\theta} = 2\tan 2\theta$

33) $\left(2x - \frac{1}{2x}\right)^4 = 4C_0 (2x)^4 \left(-\frac{1}{2x}\right)^0 + 4C_1 (2x)^3 \times \left(-\frac{1}{2x}\right)^1 + 4C_2 (2x)^2 \left(-\frac{1}{2x}\right)^2 + 4C_3 (2x)^1 \left(-\frac{1}{2x}\right)^3 + 4C_4 (2x)^0 \left(-\frac{1}{2x}\right)^4$

$= (2x)^4 - 4(2x)^3\left(\frac{1}{2x}\right) + 6(2x)^2\left(\frac{1}{2x}\right)^2 - 4(2x)\left(\frac{1}{2x}\right)^3 + \left(\frac{1}{2x}\right)^4$

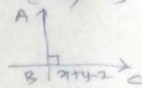
$= 16x^4 - 16x^2 + 6 - \frac{1}{x^2} + \frac{1}{16x^4}$

34) Given equation of line is

$x + y - 2 = 0 \longrightarrow (1)$

Any ... will be of the form $x - y + k = 0$

It is passes through $(-10, -2)$



$-10 + 2 + k = 0$

$\boxed{k = 8}$

$\therefore$ Equation of AB $\Rightarrow x - y + 8 = 0 \longrightarrow (2)$

Solve (1) & (2) $\Rightarrow \boxed{x = -3}$

Sub $x = -3$ in (1) $\Rightarrow \boxed{y = 5}$

$\therefore$ Coordinate of the foot of the $\perp$ B is $(-3, 5)$

Length of the $\perp$ from $(-10, -2)$ to the line $x + y - 2 = 0$ is

$\pm\left(\dfrac{-10-2-2}{\sqrt{1^2+1^2}}\right) = \pm\left(\dfrac{-14}{\sqrt{2}}\right) = \dfrac{14}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{14\sqrt{2}}{2} = 7\sqrt{2}$

35) $A = \begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & -6 & -4 \\ 3 & 8 & 6 \\ 5 & 3 & 5 \end{bmatrix}$

Let $P = \frac{1}{2}[A + A^T] = \frac{1}{2}\begin{bmatrix} 2 & -3 & 1 \\ -3 & 16 & 9 \\ 1 & 9 & 10 \end{bmatrix}$

$P^T = \frac{1}{2}\begin{bmatrix} 2 & -3 & 1 \\ -3 & 16 & 9 \\ 1 & 9 & 10 \end{bmatrix} = P$

$\therefore$ P is a symmetric matrix

Let $Q = \frac{1}{2}[A - A^T]$

$= \frac{1}{2}\begin{bmatrix} 0 & 9 & 9 \\ -9 & 0 & -3 \\ -9 & 3 & 0 \end{bmatrix}$

$Q^T = \frac{1}{2}\begin{bmatrix} 0 & -9 & -9 \\ 9 & 0 & 3 \\ 9 & -3 & 0 \end{bmatrix} = -Q$

$\therefore$ Q is a skew symmetric matrix

$A = P + Q = \frac{1}{2}\begin{bmatrix} 2 & -3 & 1 \\ -3 & 16 & 9 \\ 1 & 9 & 10 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 9 & 9 \\ -9 & 0 & -3 \\ -9 & 3 & 0 \end{bmatrix}$

36) Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{a} \times \hat{i} = (a_1\hat{i} + a_2\hat{j} + a_3\hat{k}) \times \hat{i}$

$= a_2(\hat{j} \times \hat{i}) + a_3(\hat{k} \times \hat{i})$

$= a_3\hat{j} - a_2\hat{k}$

$|\vec{a} \times \hat{i}| = \sqrt{a_3^2 + a_2^2}$

$|\vec{a} \times \hat{i}|^2 = a_3^2 + a_2^2 \longrightarrow (1)$

$\therefore |\vec{a}\times\hat{j}|^2 = a_1^2 + a_3^2 \longrightarrow (2)$

$|\vec{a}\times\hat{k}|^2 = a_1^2 + a_2^2 \longrightarrow (3)$

Adding (1), (2), (3)

$|\vec{a}\times\hat{i}|^2 + |\vec{a}\times\hat{j}|^2 + |\vec{a}\times\hat{k}|^2$

$= a_3^2 + a_2^2 + a_1^2 + a_3^2 + a_1^2 + a_2^2$

$= 2(a_1^2 + a_2^2 + a_3^2)$

$= 2|\vec{a}|^2$

37) $\lim_{t\to 0} \dfrac{\sqrt{t^2+9}-3}{t^2}$

Rationalising the numerator

$\dfrac{\sqrt{t^2+9}-3}{t^2} \times \dfrac{\sqrt{t^2+9}+3}{\sqrt{t^2+9}+3}$

$= \dfrac{t^2+9-9}{t^2[\sqrt{t^2+9}+3]}$

$= \dfrac{1}{\sqrt{t^2+9}+3}$

$\lim_{t\to 0} \dfrac{1}{\sqrt{t^2+9}+3} = \dfrac{1}{\sqrt{0+9}+3}$

$= \dfrac{1}{\sqrt{9}+3}$

$= \dfrac{1}{6}$

38) Given $y = \sqrt{x+\sqrt{x}}$

Squaring on both sides

$y^2 = x+\sqrt{x}$

Differentiate with respect to 'x'

$2y\dfrac{dy}{dx} = 1 + \dfrac{1}{2\sqrt{x}}$

$2y\dfrac{dy}{dx} = \dfrac{2\sqrt{x}+1}{2\sqrt{x}}$

$\dfrac{dy}{dx} = \dfrac{1}{2y}\left[\dfrac{2\sqrt{x}+1}{2\sqrt{x}}\right]$

$= \dfrac{1}{2\sqrt{x+\sqrt{x}}}\left[\dfrac{2\sqrt{x}+1}{2\sqrt{x}}\right]$

$\dfrac{dy}{dx} = \dfrac{2\sqrt{x}+1}{4\sqrt{x}\sqrt{x+\sqrt{x}}}$

39) $\int x^2 e^{5x}\,dx$

Let $u = x^2$, $u' = 2x$, $u'' = 2$, $u''' = 0$

$\int dv = e^{5x}dx$, $v = \dfrac{e^{5x}}{5}$, $v_1 = \dfrac{e^{5x}}{25}$, $v_2 = \dfrac{e^{5x}}{125}$

Applying Bernoulli's formula

$\int u\,dv = uv - u'v_1 + u''v_2 - \cdots$

$\int x^2 e^{5x}dx = x^2\left(\dfrac{e^{5x}}{5}\right) - 2x\left(\dfrac{e^{5x}}{25}\right) + \dfrac{2e^{5x}}{125} - (0)\cdots$

$= \dfrac{x^2e^{5x}}{5} - \dfrac{2xe^{5x}}{25} + \dfrac{2e^{5x}}{125} + C$

40) $f(x) = |2x+5|$

$f(x)$ is defined by

$f(x) = \begin{cases} 2x+5 & x \geq -2.5 \text{ or } -5/2 \\ -2x-5 & x < -2.5 \text{ or } -5/2 \end{cases}$

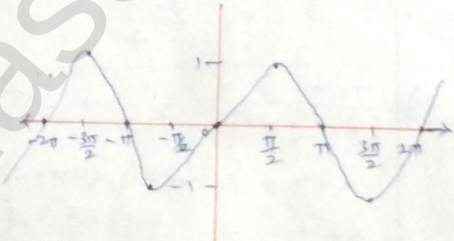
$f(-3) = -2(-3)+5 = 6+5 = 11$

$f(0) = 2(0)+5 = 5$

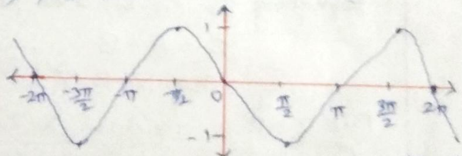
$\therefore f(-3) = 11, \quad f(0) = 5$

PART-D

IV 41) a) $y = \sin x$

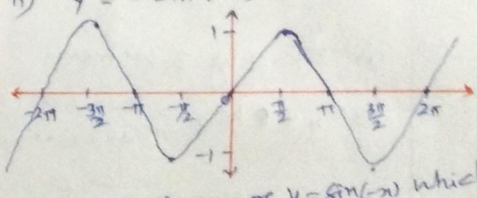


i) $y = \sin(-x) = -\sin x$



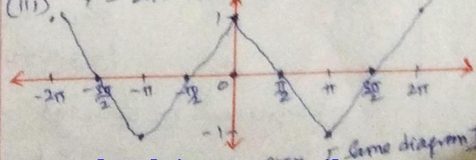
$y = \sin(-x)$ is the reflection of the graph of $\sin x$ about y axis.

ii) $y = -\sin(-x) = +(+\sin x) = \sin x$



It is the reflection of $y = \sin(-x)$ which is same as $y = \sin x$.

(iii) $y = \sin(\pi/2 + x) = \cos x$



[Same diagram]

41) (b) $5\theta = 90^\circ$

$3\theta + 2\theta = 90^\circ \Rightarrow 2\theta = 90^\circ - 3\theta$

$\sin 2\theta = \sin(90^\circ - 3\theta) = \cos 3\theta$

$2\sin\theta\cos\theta = 4\cos^3\theta - 3\cos\theta$

$2\sin\theta = 4\cos^2\theta - 3$

$= 4[1-\sin^2\theta] - 3$

$\Rightarrow 4\sin^2\theta + 2\sin\theta - 1 = 0$

$$\sin\theta = \frac{-2 \pm \sqrt{4 - 4(4)(-1)}}{2(4)}$$

$$= \frac{-2\pm\sqrt{20}}{8} = \frac{-2\pm\sqrt{4\times5}}{8}$$

$$= \frac{\cancel{2}[-1\pm\sqrt{5}]}{\cancel{8}}$$

$$= \frac{-1\pm\sqrt{5}}{8}$$

$$\therefore \sin 18^\circ = \frac{\sqrt{5}-1}{8}$$

$\cos 36^\circ = 1 - 2\sin^2 18^\circ$

$$= 1 - 2\left[\frac{\sqrt{5}-1}{4}\right]^2$$

$$= \frac{\sqrt{5}+1}{4}$$

42) (a) $$\frac{7+x}{(1+x)(1+x^2)} = \frac{A}{1+x} + \frac{Bx+C}{x^2+1}$$

$x + 7 = A(x^2+1) + (Bx+C)(x+1)$

$x=-1, \Rightarrow 6 = 2A \Rightarrow \boxed{A=3}$

Equating the co-efficient of x^2,

$0 = A+B$

$0 = 3+B \Rightarrow \boxed{B=-3}$

Put $x=0$, $7 = A+C \Rightarrow 7 = 3+C$ $\boxed{C=4}$

$$\therefore \frac{7+x}{(1+x)(x^2+1)} = \frac{3}{1+x} + \frac{-3x+4}{x^2+1}$$

(b) (i) Exactly 3 women

$$8C_4 \times 4C_3 = \frac{8\times7\times6\times5}{4\times3\times2\times1} \times \frac{4\times3\times2}{3\times2\times1}$$

$= 280$

(ii) at least 3 women

Men (8)	Women (4)	combinations
4	3	$8C_4 \times 4C_3 = 280$
3	4	$8C_3 \times 4C_4 = 56$

Required no. of ways $= 336$

(iii) At most 3 women

	Men (8)	Women (4)	Combination
i)	4	3	$8C_4 \times 4C_3 = 280$
ii)	5	2	$8C_5 \times 4C_2 = 336$
iii)	6	1	$8C_6 \times 4C_1 = 112$
iv)	7	0	$8C_7 \times 4C_0 = 8$

No. of ways $= 280 + 336 + 112 + 8$

$= 736$

43) (a) Consider LHS: $\left(\frac{p}{q}\right)^{1/n}$

$$\left(\frac{2p}{2q}\right)^{1/n} = \left[\frac{p+p+q-q}{q+q+p-p}\right]^{1/n}$$

$$= \left[\frac{(p+q)+(p-q)}{(p+q)-(p-q)}\right]^{1/n} \quad \div\ p+q \text{ both Nr \& Dr}$$

$$= \left[\frac{\frac{p+q}{p+q} + \frac{p-q}{p+q}}{\frac{p+q}{p+q} - \frac{p-q}{p+q}}\right]^{1/n} = \left[\frac{1+\left(\frac{p-q}{p+q}\right)}{1-\left(\frac{p-q}{p+q}\right)}\right]^{1/n}$$

$$= \frac{1+\frac{1}{n}\left(\frac{p-q}{p+q}\right)}{1-\frac{1}{n}\left(\frac{p-q}{p+q}\right)} = \frac{\frac{n(p+q)+(p-q)}{n(p+q)}}{\frac{n(p+q)-(p-q)}{n(p+q)}}$$

$$= \frac{np+nq+p-q}{np+nq-p+q} = \frac{p(n+1)+q(n-1)}{p(n-1)+q(n+1)}$$

$n = 8, p = 15, q = 16$

$$\sqrt[n]{\frac{15}{16}} = \frac{(8+1)15+(8-1)16}{(8-1)15+(8+1)16} = \frac{135+112}{105+144}$$

$$= \frac{247}{249} \simeq 0.99196$$

(b) General equation is

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

$\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$

$a = \lambda,\ b = 12,\ c = -3,\ h = -5,\ g = 5/2,\ c = -8$

Condition for pair of straight lines

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$\lambda(12)(-3) + 2(-8)(5/2)(-5) - \lambda(-8)^2 - 12(5/2)^2 - (-3)(-5)^2 = 0$

$\Rightarrow -36\lambda + 200 - 64\lambda - \cancel{75} + \cancel{75} = 0$

$-100\lambda = -200$

$\boxed{\lambda = 2}$

(ii) The angle between the lines

$$\tan\theta = \left|\frac{2\sqrt{h^2-ab}}{a+b}\right| = \frac{2\sqrt{25-24}}{2+12}$$

$$= \frac{2}{14} = \frac{1}{7}$$

$\tan\theta = 1/7, \quad \theta = \tan^{-1}(1/7)$

44) (a) $|A| = \begin{vmatrix} (q+r)^2 & p^2 & p^2 \\ q^2 & (r+p)^2 & q^2 \\ r^2 & r^2 & (p+q)^2 \end{vmatrix}$

Taking $p=0$,

$|A| = \begin{vmatrix} (q+r)^2 & 0 & 0 \\ q^2 & r^2 & q^2 \\ r^2 & r^2 & q^2 \end{vmatrix} = 0$

$\therefore (p-0)$ is a factor. Hence p is a factor.

$|A|$ is in cyclic symmetric form in p, q, r & hence q & r also factors

Putting $p+q+r=0 \Rightarrow q+r=-p$, $r+p=-q$, $p+q=-r$

$\therefore |A| = \begin{vmatrix} p^2 & p^2 & p^2 \\ q^2 & q^2 & q^2 \\ r^2 & r^2 & r^2 \end{vmatrix} = 0$

$\therefore (p+q+r)^2$ is a factor of $|A|$

$m = 6-5 = 1$

required factor is $k(p+q+r)$

$\begin{vmatrix} (q+r)^2 & p^2 & p^2 \\ q^2 & (r+p)^2 & q^2 \\ r^2 & r^2 & (p+q)^2 \end{vmatrix} = k(p+q+r)(p+q+r)^2 \times pqr$

Taking $p=1, q=1, r=1$

$\begin{vmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{vmatrix} = k(1+1+1)^3(1)(1)(1)$

$\Rightarrow k=2$

$\therefore |A| = 2pqr(p+q+r)^3$

(b) $f(x) = \begin{cases} 0 & \text{for } x<0 \\ x & \text{for } 0 \le x < 1 \\ -x^2+4x-2 & \text{for } 1 \le x < 3 \\ 4-x & \text{for } x \ge 3 \end{cases}$

(i) At the point $x=0$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} 0 = 0$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} 0 = 0$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$

$\therefore f(x)$ is continuous at $x=0$

(ii) At the point 1

$\lim_{x\to 1^-} f(x) = x = 1$

$\lim_{x\to 1^+} f(x) = -x^2+4x-2 = -1+4-2 = 1$

$f(1) = -1+4-2 = 1$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1) = 1$

$\therefore f(x)$ is continuous at $x=1$.

(iii) At the point $x=3$

$\lim_{x\to 3^-} -x^2+4x-2 = -(3)^2+4(3)-2 = 1$

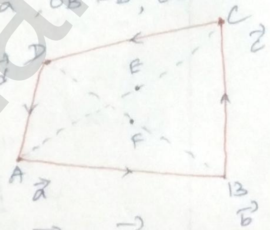
$\lim_{x\to 3^+} 4-x = 4-3 = 1$

$f(3) = 4-3 = 1$

$\therefore \lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = f(3) = 1$

$\therefore f(x)$ is continuous at $x=3$

(45) a) Let $\vec{OA} = \vec{a}$, $\vec{OB} = \vec{b}$, $\vec{OC} = \vec{c}$ and $\vec{OD} = \vec{d}$



Let $\vec{OE} = \frac{\vec{a}+\vec{c}}{2}$

$\vec{OF} = \frac{\vec{b}+\vec{d}}{2}$

LHS: $\vec{AB} + \vec{AD} + \vec{CB} + \vec{CD}$

$= \vec{OB} - \vec{OA} + \vec{OD} - \vec{OA} + \vec{OB} - \vec{OC} + \vec{OD} - \vec{OC}$

$= \vec{b} - \vec{a} + \vec{d} - \vec{a} + \vec{b} - \vec{c} + \vec{d} - \vec{c}$

$= -2\vec{a} + 2\vec{b} - 2\vec{c} + 2\vec{d}$

$= 2[(\vec{b}+\vec{d}) - (\vec{a}+\vec{c})]$

$= 2[2\vec{OF} - 2\vec{OE}] = 4\vec{EF}$

Hence proved.

(b) Let $I = \int \frac{3x+5}{x^2+4x+7}\,dx$

$3x+5 = A\frac{d}{dx}(x^2+4x+7) + B$

$3x+5 = A(2x+4)+B$

Comparing the co-efficients of like terms

$2A=3 \Rightarrow A=3/2$, $4A+B=5 \Rightarrow B=-1$

$I = \int \frac{\frac{3}{2}(2x+4)-1}{x^2+4x+7}\,dx$

$I = \frac{3}{2}\int \frac{2x+4}{x^2+4x+7}\,dx - \int \frac{dx}{x^2+4x+7}$

$= \frac{3}{2}\log|x^2+4x+7| - \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x+2}{\sqrt{3}}\right) + C$

$\left[\because x^2+4x+7 = (x+2)^2+(\sqrt{3})^2\right]$

46) (a) $y = (\cos^{-1}x)^2$

Diff w.r.to 'x'

$y' = 2\cdot\cos^{-1}x \cdot \frac{d}{dx}(\cos^{-1}x)$

$y' = 2\cdot\cos^{-1}x \cdot \frac{-1}{\sqrt{1-x^2}}$

$\Rightarrow y'\sqrt{1-x^2} = -2\cos^{-1}x$ — (1)

Squaring on both sides

$(y')^2(1-x^2) = 4(\cos^{-1}x)^2$

Diff again w.r.to 'x'

$-2x(y')^2 + 2(1-x^2)y'\cdot y'' = 8\cos^{-1}x \times -\frac{1}{\sqrt{1-x^2}}$

$-2x(y')^2 + 2(1-x^2)y'\cdot y'' = 4\left(\frac{-2\cos^{-1}x}{\sqrt{1-x^2}}\right)$

$-2x(y')^2 + 2(1-x^2)y'\cdot y'' = 4y'$ [From (1)]

$\div 2y'$

$-xy' + (1-x^2)y'' = 2$

$(1-x^2)y'' - xy' - 2 = 0$

when $x=0$,

$y'' - 2 = 0$

$\boxed{y'' = 2}$

(b) Let A_1, A_2, & A_3 be the events of X, Y and Z becoming managers of the company respectively. Let B be the event that the bonus scheme will be introduced.

$P(A_1) = \frac{4}{9}$ | $P(B/A_1) = 0.3$

$P(A_2) = \frac{2}{9}$ | $P(B/A_2) = 0.5$

$P(A_3) = \frac{3}{9}$ | $P(B/A_3) = 0.4$

$P(A_3/B) = \frac{P(A_3)\,P(B/A_3)}{P(A_1)P(B/A_1) + P(A_2)P(B/A_2) + P(A_3)\cdot P(B/A_3)}$

$= \frac{\frac{3}{9}\times 0.4}{\frac{4}{9}\times 0.3 + \frac{2}{9}(0.5) + \left(\frac{3}{9}\right)(0.4)}$

$= \frac{12}{34}$

$= \frac{6}{17}$

(47)(a) $\sin\theta = \frac{11}{12}$

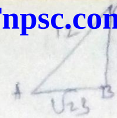
$\cos\theta = \frac{\sqrt{23}}{12}$

$AC^2 = AB^2 + BC^2$

$12^2 - 11^2 = AB^2$

$23 = AB^2$

$AB = \sqrt{23}$



$\sec\theta = \frac{12}{\sqrt{23}}$,

$\tan\theta = \frac{11}{\sqrt{23}}$

The value of

$\sec(360°-\theta)\tan(180°-\theta) + \cot(90°+\theta) \times \sin(270°+\theta)$

$= \sec\theta \times -\tan\theta + -\tan\theta \times -\cos\theta$

$= -\frac{12}{\sqrt{23}} \times \frac{11}{\sqrt{23}} + \frac{11}{\sqrt{23}} \times \frac{\sqrt{23}}{12}$

$= -\frac{132}{23} + \frac{11}{12} = \frac{-1584+253}{276}$

$= \frac{-1331}{276}$

(b) Let $y = \left[\log(\sin(x^2+5))\right]^2$

Diff w.r.to 'x'

$\frac{dy}{dx} = 2\left[\log(\sin(x^2+5))\right]^{2-1} \times \frac{d}{dx}\left[\log(\sin(x^2+5))\right]$ → (1)

Consider

$\frac{d}{dx}\left[\log(\sin(x^2+5))\right]$

Let $u = x^2+5$, $\frac{du}{dx} = 2x$ | $v = \sin u$, $\frac{dv}{du} = \cos u$

$y = \log v$

$\frac{dy}{dv} = \frac{1}{v}$

$\frac{dy}{dx} = \frac{dy}{dv} \times \frac{dv}{du} \times \frac{du}{dx} = 2x\cos u \times \frac{1}{v}$

$= 2x\cos(x^2+5) \times \frac{1}{\sin u}$

$= 2x\cos(x^2+5) \times \frac{1}{\sin(x^2+5)}$

$= 2x\cot(x^2+5)$

(1) $\Rightarrow$

$\frac{dy}{dx} = 2\left[\log(\sin(x^2+5))\right] \times 2x\cot(x^2+5)$

$\frac{dy}{dx} = 4x\cot(x^2+5)\left[\log(\sin(x^2+5))\right]$

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