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Time : 3.00 hrs.

Reg. No. 1 1 1 6 2

Half Yearly Examination - 2023

MATHEMATICS

Max. Marks : 90

PART - A

I. **Answer all the questions.** **(20x1=20)**

1. If $A = \{(x, y) : y = e^x, x \in R\}$ and $B = \{(x, y) : y = e^{-x}, x \in R\}$ then $n(A \cap B)$ is a) Infinity b) 0 c) 1 d) 2
2. The number of constant functions from a set containing m elements to a set containing n elements is
a) mn b) m c) n d) m + n
3. The number of solutions of $x^2 + |x - 1| = 1$ is a) 1 b) 0 c) 2 d) 3
4. The triangle of maximum area with constant perimeter 12 m
a) is an equilateral triangle with side 4m b) is an isosceles triangle with sides 2m, 5m, 5m
c) is a triangle with sides 3m, 4m, 5m d) does not exist
5. A wheel is spinning at 2 radians/second. How many seconds will it take to make 10 complete rotations?
a) 10π seconds b) 20π seconds c) 5π seconds d) 15π seconds
6. The number of 5 digit numbers all digits of which are odd is
a) 25 b) 5^5 c) 5^6 d) 625
7. Everybody in a room shakes hands with everybody else. The total number of shake hands is 66. The number of persons in the room is a) 11 b) 12 c) 10 d) 6
8. If a, 8, b are in AP, a, 4, b are in GP, and if a, x, b are in HP then x is a) 2 b) 1 c) 4 d) 16
9. If ${}^nC_{10} > {}^nC_r$ for all possible r, then a value of n is a) 10 b) 21 c) 19 d) 20
10. The equation of the locus of the point whose distance from y-axis is half the distance from origin is
a) $x^2 + 3y^2 = 0$ b) $x^2 - 3y^2 = 0$ c) $3x^2 + y^2 = 0$ d) $3x^2 - y^2 = 0$
11. The image of the point (5, 4) in the line $y = -x$ is a) (–4, –5) b) (–4, 5) c) (–5, –4) d) (4, 5)
12. If A and B are two matrices such that A + B and AB are both defined, then
a) A and B are two matrices not necessarily of same order b) A and B are square matrices of same order
c) Number of columns of A is equal to the number of rows of B d) A = B
13. Let A and B be two symmetric matrices of same order. Then which one of the following statement is not true?
a) A + B is a symmetric matrix b) AB is a symmetric matrix c) $AB = (BA)^T$ c) $A^TB = AB^T$
14. If $\vec{BA} = 3\hat{i} + 2\hat{j} + \hat{k}$ and the position vector of B is $\hat{i} + 3\hat{j} - \hat{k}$ then the position vector A is
a) $4\hat{i} + 2\hat{j} + \hat{k}$ b) $4\hat{i} + 5\hat{j}$ c) $4\hat{i}$ d) $-4\hat{i}$
15. Let f be a continuous function on [2, 5]. If f takes only rational values for all x and f(3) = 12, then f(4.5) is equal to
a) $\frac{f(3) + f(4.5)}{7.5}$ b) 12 c) 17.5 d) $\frac{f(4.5) - f(3)}{1.5}$
16. If $f(x) = x^2 - 3x$, then the points at which $f(x) = f'(x)$ are
a) both positive integers b) both negative integers c) both irrational d) one rational and another irrational
17. The number of points in R in which the function $f(x) = |x - 1| + |x + 3| + \sin x$ is not differentiable, is
a) 3 b) 2 c) 1 d) 4
18. $\int x^2 \cos x \, dx$ is
a) $x^2 \sin x + 2x \cos x - 2\sin x + c$ b) $x^2 \sin x - 2x \cos x - 2\sin x + c$ c) $-x^2 \sin x + 2x \cos x + 2\sin x + c$
d) $-x^2 \sin x - 2x \cos x + 2 \sin x + c$
19. Four persons are selected at random from a group of 3 men, 2 women and 4 children. The probability that exactly two of them are children is a) $\frac{3}{4}$ b) $\frac{10}{23}$ c) $\frac{1}{2}$ d) $\frac{10}{21}$
20. Ten coins are tossed. The probability of getting at least 8 heads is a) $\frac{7}{64}$ b) $\frac{7}{32}$ c) $\frac{7}{16}$ d) $\frac{7}{128}$

II. **Answer any seven questions. Question No.30 is compulsory** **7 x 2 = 14**

21. If $n(A \cap B) = 3$ and $n(A \cup B) = 10$, then find $n[P(A \Delta B)]$.
22. If $f : R \rightarrow R$ is defined by $f(x) = 3x - 5$, prove that f is a bijection and find its inverse.
23. Solve $x^{\log_3 x} = 9$
24. Solve $3\cos^2\theta = \sin^2\theta$ (Example 3.45)
25. Evaluate the following : (i) ${}^{10}C_3$ (ii) ${}^{15}C_{13}$
26. The length of the perpendicular drawn from the origin to a line is 12 and makes an angle 150° with positive direction of the

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x-axis. Find the equation of the line.

27. Evaluate $\begin{vmatrix} 2014 & 2017 & 0 \\ 2020 & 2023 & 1 \\ 2023 & 2026 & 0 \end{vmatrix}$

28. Show that the points (2, −1, 3), (4, 3, 1) and (3, 1, 2) are collinear.
29. A year is selected at random. What is the probability that it contains 53 Sundays?
30. Write any two basic rules of integration.

III. **Answer any seven questions. Question No.40 is compulsory.** **7 x 3 = 21**

31. Find the domain of $\frac{1}{1-2\sin x}$
32. Draw the graph of the cosine function.
33. If $\frac{1}{7!}+\frac{1}{8!}=\frac{A}{9!}$ then find the value of A.
34. Prove that ${}^{10}C_2 + 2 \times {}^{10}C_3 + {}^{10}C_4 = {}^{12}C_4$
35. Find λ, when the projection of $\vec{a} = \lambda\hat{i}+\hat{j}+4\hat{k}$ on $\vec{b} = 2\hat{i}+6\hat{j}+3\hat{k}$ is 4 units.
36. Evaluate : $\lim_{x \to 0} (1 + \sin x)^{2\operatorname{cosec} x}$
37. How many ways can a team of 3 boys, 2 girls and 1 transgender be selected from 5 boyes, 2 girls and 2 transgenders?
38. Differentiate : $y = (x^3 - 1)^{100}$.
39. The probability that a girl, preparing for competitive examination will get a State Government service is 0.12, the probability that she will get a Central Government job is 0.25, and the probability that she will get both is 0.07. Find the probability that (i) she will get atleast one of the two jobs (ii) she will get only one of the two jobs.
40. State Bayes' Theorem.

IV. **Answer all questions.** **7 x 5 = 35**

41. *a) Write the value of f at −3, 5, 2, −1, 0 if* $f(x) = \begin{cases} -x+4 & \text{if } -\infty < x \le -3 \\ x+4 & \text{if } -3 < x < -2 \\ x^2 - x & \text{if } -2 \le x < 1 \\ x - x^2 & \text{if } 1 \le x < 7 \\ 0 & \text{otherwise} \end{cases}$ **(OR)**

    b) In the set Z of integers, define mRn if m − n is a multiple of 12. Prove that R is an equivalence relation.

42. a) If f, g : R → R are defined by f(x) = |x| + x and g(x) = |x| − x, find gof and fog. **(OR)**

    b) Prove that $\log_{10} 2 + 16 \log_{10}\frac{16}{15} + 12\log_{10}\frac{25}{24} + 7\log\frac{81}{80} = 1$

43. a) Resolve into partial fractions : $\frac{2x}{(x^2+1)(x-1)}$ **(OR)**

    b) If $A + B + C = \pi$, prove that $\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2\cos A \cos B \cos C$.

44. a) Prove that for any natural number n, $a^n - b^n$ is divisible by a − b, where a > b. **(OR)**

    b) Separate the equations $5x^2 + 6xy + y^2 = 0$

45. a) Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ **(OR)**

    b) If $\vec{a}, \vec{b}$ are unit vectors and is the angle between them, then show that $\tan\frac{\theta}{2} = \frac{|\vec{a}-\vec{b}|}{|\vec{a}+\vec{b}|}$

46. a) If $y = \sin^{-1}\frac{1}{2}(\sqrt{1+x}+\sqrt{1-x})$ then prove that $\frac{dy}{dx} = \frac{-1}{2\sqrt{1-x^2}}$ **(OR)**

    b) There are two identical urns containing respectively 6 black and 4 red balls., 2 black and 2 red balls. An urn is chosen at random and a ball is drawn from it. (i) find the probability that the ball is black (ii) if the ball is black, what is the probability that it is from the first urn?

47. a) Evaluate : $\int \frac{3x+5}{x^2+4x+7}$ **(OR)**

    b) A factory has two Machines - I and II. Machine - I produces 60% of items and Machine - II produces 40% of the items of the total output. Further 2% of the items produced by Machine - I are defective whereas 4% produced by Machine - II are defective. If an item is drawn at random what is the probability that it is defective?