HALF-YEARLY EXAMINATION - 2023

STD - XI **MATHEMATICS** **MARKS : 90**

TIME : 3.00 Hrs

PART - I

20 x 1 = 20

Answer all the questions.

1. The range of the function $f(x)=\lfloor x \rfloor - x,\ x \in R$ is
   a) [0, 1] b) [0, ∞) c) [0, 1) d) (0, 1)
2. The number of reflexive relations on a set containing n elements is
   a) 2^{mn} b) 2^{n^2+n} c) 2^{n^2-n} d) 2^{n^2}
3. If a and b are the real roots of the equation x2 - kx + c = 0, then the distance between the points (a,0) and (b,0) is
   a) $\sqrt{k^2-4c}$ b) $\sqrt{4k^2-c}$ c) $\sqrt{4c-k^2}$ d) $\sqrt{k-8c}$
4. Principal value of cot θ = $\sqrt{3}$ a) $\frac{\pi}{3}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{2}$ d) 0
5. $\cos 2\theta \cos 2\phi + \sin^2(\theta-\phi) - \sin^2(\theta+\phi)$ is equal to
   a) $\sin 2(\theta+\phi)$ b) $\cos 2(\theta+\phi)$ c) $\sin 2(\theta-\phi)$ d) $\cos 2(\theta-\phi)$
6. If $n_{c_4}, n_{c_5}, n_{c_6}$ are in A.P the value of n can be a) 14 b) 9 c) 5 d) 11
7. The value of $\int x \cos x\, dx =$
   a) x cos x + sin x + c b) x sin x + cos x + c c) x cos x - sin x + c d) x sin x - cos x + c
8. $\int e^{\sqrt{x}} dx =$ a) $2\sqrt{x}(1-e^{\sqrt{x}})+c$ b) $2\sqrt{x}(e^{\sqrt{x}}-1)+c$ c) $2e^{\sqrt{x}}(1-\sqrt{x})+c$ d) $2e^{\sqrt{x}}(\sqrt{x}-1)+c$
9. The equation of the line with slope 2 and the length of the perpendicular from the origin equal to $\sqrt{5}$ is
   a) $x + 2y = \sqrt{5}$ b) $2x + y = \sqrt{5}$ c) 2x - y + 5 = 0 d) x - 2y - 5 = 0
10. If $2X - \begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix} = \begin{bmatrix}3 & 8\\ 7 & 2\end{bmatrix}$, then X = a) $\begin{bmatrix}3 & 5\\ 5 & 2\end{bmatrix}$ b) $\begin{bmatrix}5 & 3\\ 2 & 5\end{bmatrix}$ c) $\begin{bmatrix}2 & 5\\ 5 & 3\end{bmatrix}$ d) $\begin{bmatrix}2 & -5\\ -5 & 3\end{bmatrix}$
11. If f(x) = x $\tan^{-1}$ x then f'(x) = ... a) $1+\frac{\pi}{4}$ b) $\frac{1}{2}+\frac{\pi}{4}$ c) $\frac{1}{2}-\frac{\pi}{4}$ d) 2
12. $\lim_{x\to 0} \frac{8^x - 4^x - 2^x + 1^x}{x^2} =$ a) 2 log 2 b) 2 $(\log 2)^2$ c) log 2 d) 3 log 2
13. a and b be any two positive real numbers then Harmonic mean is a) $\frac{a+b}{2}$ b) $\sqrt{ab}$ c) ab d) $\frac{2ab}{a+b}$
14. Ten coins are tossed. The probability of getting at least 8 heads is a) $\frac{7}{64}$ b) $\frac{7}{32}$ c) $\frac{7}{16}$ d) $\frac{7}{128}$
15. If $y = e^{-\alpha x}$ then a) $\frac{dy}{dx}+\alpha y=0$ b) $\frac{dy}{dx}-\alpha y=0$ c) $\alpha\frac{dy}{dx}+y=0$ d) $\frac{dy}{dx}+\alpha x=0$
16. The value of $\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^x =$ a) e^x b) e^2 c) 2^x d) $\frac{1}{e}$
17. If $|\vec{a}+\vec{b}|=60$, $|\vec{a}-\vec{b}|=40$ and $|\vec{b}|=46$ then the value of $|\vec{a}|$ is a) 42 b) 12 c) 22 d) 32

18. What is the area of a triangle having the points (1, 0, 0), (0, 1, 0) and (0, 0, 1) as its vertices

a) $\frac{2}{\sqrt{3}}$ b) $\frac{\sqrt{3}}{2}$ c) $\sqrt{3}$ d) $\frac{4}{\sqrt{3}}$

19. Let A and B be two symmetric matrices of same order. Then which one of the following statement is not true?

a) AB is a symmetric matrix b) $AB = (BA)^T$ c) A + B is a symmetric matrix d) $A^TB = AB^T$

20. What is coordinates of the equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

a) (a cos θ, b sin θ) b) (a cos θ, a sin θ) c) (a sec θ, b tan θ) d) (a tan θ, b sec θ)

PART - II

Answer any Seven Questions. Q.No.30 is compulsory **7 x 2 = 14**

21. Let f = {(1,4), (2, 5), (3, 5) } and g = { (4,1), (5,2), (6, 4) } Find g of. Can you find fog?

22. Solve : $x^{\log_3 x} = 9$

23. Find the distinct permutations of the letters of the word MISSISSIPPI?

24. Write the first 4 terms of the logasithmic series log(1 - 2x).

25. Write the equation of the line through the point (1, -1) and parallel to x + 3y - 4 = 0.

26. Find the value of $\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$

27. If D is the midpoint of the side BC of a triangle ABC, prove that $\overrightarrow{AB} + \overrightarrow{AC} = 2\overrightarrow{AD}$

28. Find the derivative of $y = \sqrt{x + \sqrt{x}}$

29. Evaluate $\int \frac{1}{x \log x} dx$

30. Do the limits of following function $\frac{x\lfloor x \rfloor}{Sin|x|}$ exist as $x \to 0$? State reasons for your answer.

PART - III

Answer any Seven Questions. Q.No.40 is compulsory **7 x 3 = 21**

31. From the curve y = x draw i) y = -x ii) y = 2x iii) y = x + 1

32. If $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 = \frac{9}{2}$ then find the value of $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)$, for x > 1

33. If θ is an acute angle then find $\sin\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$, when $\sin\theta = \frac{1}{25}$

34. Find coefficient of x^{15} of the expansion $\left(x^2 + \frac{1}{x^3}\right)^{10}$

35. The slope of one of the straight lines $ax^2 + 2hxy + by^2 = i$ is three times the other, show that $3h^2 = 4ab$

36. If $y = \tan^{-1}\left(\frac{1+x}{1-x}\right)$ then find y'.

37. Prove that $24_{C_4} + \sum_{r=0}^{4} (28-r)_{C_3} = 29_{C_4}$

38. Find the constant b that makes g continuous on R $g(x)=\begin{cases} x^2-b^2 \ ; \ if \ x<4 \\ bx+20 \ ; \ if \ x\geq 4 \end{cases}$

39. If A and B are two independent events such that P(A) = 0.4 and P (A ∪ B) = 0.9. Find P(B).

40. Evaluate $\int \frac{dx}{x^2+2x+10}$

PART - IV

Answer all the questions. **7 x 5 = 35**

41. a) On the set of natural numbers let R be the relation define by a R b if $a+b\leq 6$. Write down the relation by listing all the palts cheek whether it is i) reflexive ii) symmetric iii) transitive iv) equivalence (OR)

b) If P(A) = 0.52, P(B) = 0.43 and P (A ∩ B) = 0.24 then find

i) P (A ∩ $\bar{B}$) ii) P (A ∪ B) iii) P ($\bar{A}$ ∩ $\bar{B}$) iv) P ($\bar{A}$ ∪ $\bar{B}$) v) P ($\bar{A}$ ∩ B)

42. a) Resolve into partial fractions $\frac{x+12}{(x+1)^2(x-2)}$ (OR)

b) Evaluate $\int \frac{3x+5}{x^2+4x+7}dx$

43. a) If $\theta+\phi=\alpha$ and $\tan\theta = k\tan\phi$ then prove that $\sin(\theta-\phi)=\frac{k-1}{k+1}\sin\alpha$ (OR)

b) Prove that $\lim_{x\to 0^+} x\left[\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+.....+\left[\frac{15}{x}\right]\right]=120$

44. a) Express the matrix $A=\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrices. (OR)

b) By the principle of mathematical induction, prove that, for all intergers $n\geq 1$,

$1^2+2^2+3^2+.....+n^2=\frac{n(n+1)(2n+1)}{6}$

45. a) If $y=\frac{\sin^{-1}x}{\sqrt{1-x^2}}$ then show that $(1-x^2)y_2-3xy_1-y=0$. (OR)

b) Show that the equation $4x^2+4xy+y^2+-6x-3y-4=0$ represents a pair of parallel lines. Find the distance between them.

46. a) If ABCD is a quadrilateral and E and F are mid points of AC and BD respectively, then prove that $\vec{AB}+\vec{AD}+\vec{CB}+\vec{CD}=4\vec{EF}$ (OR)

b) Prove that $\sqrt{\frac{1-x}{1+x}}$ is approximately equal to $1-x+\frac{x^2}{2}$ when x is very small.

47. a) If $n_{p_r}=11880$ and $n_{c_r}=495$ then find the values of n and r. (OR)

b) If R is any point on the x-axis and Q is any point on the y-axis and P is a variable point on RQ with RP = b, PQ = a, then find the equation of locus of P.