MOUNT CARMEL MAT. H.S. SCHOOL
KALLAKURICHI.

Class : 11

Register Number

COMMON HALFYEARLY EXAMINATION - 2023 - 24

MATHEMATICS

Time Allowed : 3.00 Hours] [Max. Marks : 90

I.Answer all the Questions **PART-A** 20X1 = 20

1. Let $f:\mathbb{R}\to\mathbb{R}$ be defined by $f(x)=1-|x|$. Then the range of f is

(1) $\mathbb{R}$ (2) $(1,\infty)$ (3) $(-1,\infty)$ (4) $(-\infty,1]$

2. The number of parallelograms that can be formed from a set of four parallel lines intersecting another set of three parallel lines.

(1) 6 (2) 9 (3) 12 (4) 18

3. If $\frac{|x-5|}{x-5}\geq 0$, then x belongs to

(1) $[5,\infty)$ (2) $(5,\infty)$ (3) $(-\infty,5)$ (4) $(-5,\infty)$

4. The sum up to n terms of the series $\sqrt{2}+\sqrt{8}+\sqrt{18}+\sqrt{32}+\cdots$ is

(1) $\frac{n(n+1)}{2}$ (2) $2n(n+1)$ (3) $\frac{n(n+1)}{\sqrt{2}}$ (4) 1.

5. Which of the following is not true?

(1) $\sin\theta=-\frac{3}{4}$ (2) $\cos\theta=-1$ (3) $\tan\theta=25$ (4) $\sec\theta=\frac{1}{4}$

6. If $A=\begin{bmatrix} a & x \\ y & a \end{bmatrix}$ and if $xy=1$, then $det(AA^T)$ is equal to

(1) $(a-1)^2$ (2) $(a^2+1)^2$ (3) a^2-1 (4) $(a^2-1)^2$

7. Let A and B be subsets of the universal set $\mathbb{N}$, the set of natural numbers. Then $A'\cup[(A\cap B)\cup B']$ is

(1) A (2) A' (3) B (4) $\mathbb{N}$

8. The value of $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}$ is

(1) $\overrightarrow{AB}$ (2) $\overrightarrow{CA}$ (3) $\vec{0}$ (4) $-\overrightarrow{AC}$

9. If m is a number such that $m\leq 5$, then the probability that quadratic equation $2x^2+2mx+m+1=0$ has real roots is

(1) $\frac{1}{5}$ (2) $\frac{2}{5}$ (3) $\frac{3}{5}$ (4) $\frac{4}{5}$

10. The slope of the line which makes an angle 45° with the line $3x-y=-5$ are

(1) $1,-1$ (2) $\frac{1}{2},-2$ (3) $1,\frac{1}{2}$ (4) $2,-\frac{1}{2}$

11. In 3 fingers, the number of ways four rings can be worn is ______ ways.

(1) 4^3-1 (2) 3^4 (3) 68 (4) 64

12. The value of $\log_3 11 \cdot \log_{11} 13 \cdot \log_{13} 15 \cdot \log_{15} 27 \cdot \log_{27} 81$ is

(1) 1 (2) 2 (3) 3 (4) 4

13. The value of the series $\frac{1}{2}+\frac{7}{4}+\frac{13}{8}+\frac{19}{16}+\cdots$ is

(1) 14 (2) 7 (3) 4 (4) 6.

14. If $\sin\alpha+\cos\alpha=b$, then $\sin 2\alpha$ is equal to

(1) b^2-1, if $b \le \sqrt{2}$ (2) b^2-1, if $b>\sqrt{2}$

(3) b^2-1, if $b \ge 1$ (4) b^2-1, if $b \ge \sqrt{2}$

15. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=2\hat{i}+x\hat{j}+\hat{k}, \vec{c}=\hat{i}-\hat{j}+4\hat{k}$ and $\vec{a}\cdot(\vec{b}\times\vec{c})=70$, then x is equal to

(1) 5 (2) 7 (3) 26 (4) 10

16. Which of the following equation is the locus of $(a\cos\theta, a\sin\theta)$

(1) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ (2) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (3) $x^2+y^2=a^2$ (4) $y^2=4ax$

17. If $A+I=\begin{bmatrix}3 & -2\\ 4 & 1\end{bmatrix}$, then $(A+I)(A-I)$ is equal to

(1) $\begin{bmatrix}-5 & -4\\ 8 & -9\end{bmatrix}$ (2) $\begin{bmatrix}-5 & 4\\ -8 & 9\end{bmatrix}$ (3) $\begin{bmatrix}5 & 4\\ 8 & 9\end{bmatrix}$ (4) $\begin{bmatrix}-5 & -4\\ -8 & -9\end{bmatrix}$

18. $\lim_{x\to 3}\lfloor x\rfloor =$

(1) 2 (2) 3 (3) does not exist (4) 0

19. The number of points in $\mathbb{R}$ in which the function $f(x)=|x-1|+|x-3|+\sin x$ is not differentiable, is

(1) 3 (2) 2 (3) 1 (4) 4

20. $\int e^{\sqrt{x}}dx$ is

(1) $2\sqrt{x}\left(1-e^{\sqrt{x}}\right)+c$ (2) $2\sqrt{x}\left(e^{\sqrt{x}}-1\right)+c$

(3) $2e^{\sqrt{x}}(1-\sqrt{x})+c$ (4) $2e^{\sqrt{x}}(\sqrt{x}-1)+c$

PART- B

7X2= 14

II. Answer for any 7 Questions (Question No 30 Compulsory Question)

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21. If $n(A\cap B)=3$ and $n(A\cup B)=10$, then find $n(\wp(A\Delta B))$.

22. Find the radius of the spherical tank whose volume is $\frac{32\pi}{3}$ units.

23. Prove that $\sin(45°+\theta)-\sin(45°-\theta)=\sqrt{2}\sin\theta$

24. Find the middle terms in the expansion of $(x+y)^7$.

25. If G is the centroid of a triangle ABC, prove that $\overrightarrow{GA}+\overrightarrow{GB}+\overrightarrow{GC}=\vec{0}$.

26. The length of the perpendicular drawn from the origin to a line is 12 and makes an angle 150° with positive direction of the x-axis. Find the equation of the line.

27. If $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$, then compute A^6

28. Differentiate : $y = (x^3 - 1)^{100}$.

29. Given that $P(A) = 0.52$, $P(B) = 0.43$ and $P(A \cap B) = 0.24$, find(i) $P(A \cap \bar{B})$ (ii)$P(\bar{A} \cup \bar{B})$

30. Compute $\lim\limits_{x \to 1} \frac{\sqrt{x}-1}{x-1}$.

PART – C

7X 3 = 21

III. Answer for any 7 Questions (Question No 40 Compulsory Question)

31. Find the range of the function $f(x) = \frac{1}{1-3\cos x}$.

32. Find the number of solutions of $x^2 + |x - 1| = 1$.

33. Simplify : $\frac{\sin 75° - \sin 15°}{\cos 75° + \cos 15°}$.

34. Find the number of ways of forming a committee of 5 members out of 7 Indians and 5 Americans, so that always Indians will be the majority in the committee.

35. Find the distance between the parallel lines $3x - 4y + 5 = 0$ and $6x - 8y - 15 = 0$

36. Show that $\begin{vmatrix} b+c & bc & b^2c^2 \\ c+a & ca & c^2a^2 \\ a+b & ab & a^2b^2 \end{vmatrix} = 0$.

37. Find the vectors of magnitude 9 which are perpendicular to both vectors $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = -2\hat{i} + 3\hat{j} - 2\hat{k}$.

38. Find $\frac{dy}{dx}$ if $x = a(t - \sin t), y = a(1 - \cos t)$.

39. Evaluate : $\int \frac{1}{\sin^2 x \cos^2 x} dx$

40. A problem in Mathematics is given to three students whose chances of solving it are $\frac{1}{3}, \frac{1}{4}$, and $\frac{1}{5}$ (i) What is the probability that the problem is solved? (ii) What is the probability that exactly one of them will solve it?

PART-D

7X5 = 35

IV. Answer all the Questions

41. (a) Rewrite $\sqrt{3}x + y + 4 = 0$ in to normal form.

(OR)

(b) If $f: \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = 2x - 3$ prove that f is a bijection and find its inverse.

42. (a) If $A + B + C = \frac{\pi}{2}$, prove that $\sin 2A + \sin 2B + \sin 2C = 4\cos A \cos B \cos C$

(OR)

(b) Resolve into partial fractions. $\frac{x^3+2x+1}{x^2+5x+6}$

43. (a) Integrate $\frac{5x-2}{2+2x+x^2}$ with respect to x

(OR)

(b) Prove that $\sqrt[3]{x^3+7}-\sqrt[3]{x^3+4}$ is approximately equal to $\frac{1}{x^2}$ when x is large.

44. (a) If $ABCD$ is a quadrilateral and E and F are the midpoints of AC and BD respectively, then prove that $\overrightarrow{AB}+\overrightarrow{AD}+\overrightarrow{CB}+\overrightarrow{CD}=4\overrightarrow{EF}$.

(OR)

(b) If $y=e^{tan^{-1}x}$, show that $(1+x^2)y''+(2x-1)y'=0$.

45. (a) Prove that $log\,2+16\,log\frac{16}{15}+12\,log\frac{25}{24}+7\,log\frac{81}{80}=1$.

(OR)

(b) Show that the vectors $2\hat{i}-\hat{j}+\hat{k}, 3\hat{i}-4\hat{j}-4\hat{k}, \hat{i}-3\hat{j}-5\hat{k}$ form a right angled triangle.

46. (a) Eight coins are tossedonce.Find the probability ofgetting

(i)Exactly three tails (ii)at least three tails (iii)at most three tails

(OR)

(b) Prove that $3^{2n+2}-8n-9$ is divisible by 8 for all $n \geq 1$.

47. (a) Show that the equation $9x^2-24xy+16y^2-12x+16y-12=0$ represents a pair of parallel lines. Find the distance between them.

(OR)

(b) Using Factor Theorem, prove that $\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix}=(x-1)^2(x+9)$.

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COMMON HALF YEARLY EXAM - 2023-24

CLASS: 11 ANSWER KEY

PART - A

ONE MARKS

1. (4) $(-\infty, 1]$
2. (4) 18
3. (2) $(5, \infty)$
4. (3) $\frac{n(n+1)}{\sqrt{2}}$
5. (4) $\sec\theta = \frac{1}{4}$
6. (4) $(a^2-1)^2$
7. (4) $\mathbb{N}$
8. (3) $\vec{0}$
9. (3) $\frac{3}{5}$
10. (2) $\frac{1}{2}, -2$
11. (2) 3^4
12. (4) 4
13. (1) 14
14. (1) b^2-1, if $b \le \sqrt{2}$
15. (3) 26
16. (2) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
17. (1) $\begin{bmatrix} -5 & -4 \\ 8 & -9 \end{bmatrix}$
18. (3) does not exist
19. (1) 3
20. (4) $2e^{\sqrt{x}}(\sqrt{x}-1)+c$

PART - B

2 MARKS

21.

We know that,

$n(A\cup B) = n(A-B) + n(B-A) + n(A\cap B)$

if A and B are not disjoint.

$n(A-B) + n(B-A)$

$n(A\Delta B) = n(A\cup B) - n(A\cap B)$

$= 10 - 3$

$n(A\Delta B) = 7$

$\therefore n(P(A\Delta B)) = 2^7 = 128.$

22.

Let r be the radius of the spherical tank.

Then,

volume of the spherical tank $= \frac{32\pi}{3}$

$\frac{4\pi r^3}{\cancel{3}} = \frac{32\pi}{\cancel{3}}$

$4r^3 = 32$

$r^3 = 32/4 = 8$

$r^3 = 2^3$

$\boxed{r=2}$

$\therefore$ Radius of the spherical tank is 2 units.

23) LHS =

$\sin(45°+\theta) - \sin(45°-\theta)$

$= (\sin 45° \cos\theta + \cos 45° \sin\theta)$

$-(\sin 45° \cos\theta - \cos 45° \sin\theta)$

$= \cancel{\sin 45° \cos\theta} + \cos 45° \sin\theta - \cancel{\sin 45° \cos\theta} + \cos 45° \sin\theta$

$= 2\cos 45° \sin\theta$

$= 2\left(\frac{1}{\sqrt{2}}\right)\sin\theta$ $(\because \cos 45° = \frac{1}{\sqrt{2}})$

$= \frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \sin\theta$

$= \frac{\cancel{2}\sqrt{2}}{\cancel{2}} \sin\theta$ $(\because \sqrt{2}\sqrt{2} = 2)$

$= \sqrt{2}\sin\theta = RHS$

$\therefore \sin(45°+\theta) - \sin(45°-\theta) = \sqrt{2}\sin\theta$.

Hence proved.

24)

$(a+b)^n = nC_0\, a^{n-0} b^0 + nC_1 a^{n-1} b^1 + \dots nC_r\, a^{n-r} b^r$

$T_{r+1} = nC_r\, a^{n-r} b^r$

Here, $n=7$, $a=x$, $b=y$

Total number of terms 8 $(\because n+1$
$7+1=8)$

$T_1, T_2, T_3, \underbrace{T_4, T_5}, T_6, T_7, T_8$

$(\because T_1 \Rightarrow$ Term 1)

T_4, T_5 is middle term

Now,

$T_4 = T_{3+1}$

$= 7C_3\, x^{7-3} y^3$

$(\because n=7, r=3, a=x, b=y)$

$= \frac{7\times\cancel{6}\times 5}{1\times\cancel{2}\times\cancel{3}} x^4 y^3$

$T_4 = 35x^4y^3$

$T_5 = T_{4+1}$

$= 7C_4\, x^{7-4} y^4$

$= \frac{7\times\cancel{6}\times 5\times\cancel{4}}{\cancel{4}\times\cancel{3}\times\cancel{2}\times 1} x^3 y^4$

$T_5 = 35x^3y^4$

$\therefore$ The middle term is $35x^4y^3$ and $35x^3y^4$.

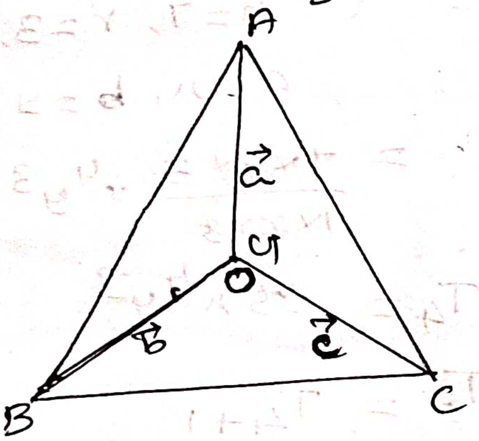
(25)

Let the position vector of the vertices of the ΔABC be $\vec{a}, \vec{b}$ and $\vec{c}$ respectively.

$\therefore \vec{OA} = \vec{a}, \vec{OB} = \vec{b}, \vec{OC} = \vec{c}$

Since G is the centroid of ΔABC, we have

$$\vec{OG} = \frac{\vec{OA} + \vec{OB} + \vec{OC}}{3}$$



$3\vec{OG} = \vec{OA} + \vec{OB} + \vec{OC}$ — (1)

$LHS = \vec{GA} + \vec{GB} + \vec{GC}$

$= \vec{OA} - \vec{OG} + \vec{OB} - \vec{OG} + \vec{OC} - \vec{OG}$

$= (\vec{OA} + \vec{OB} + \vec{OC}) - 3\vec{OG}$

$= \vec{OA} + \vec{OB} + \vec{OC} - (\vec{OA} + \vec{OB} + \vec{OC})$ $(\because$ by (1))

$= \vec{O} = RHS$

$LHS = RHS$

$\therefore \vec{GA} + \vec{GB} + \vec{GC} = \vec{O}$

Hence proved.

(26)

Here, $p = 12$, and $\alpha = 150°$

The equation of the line form

$x\cos\alpha + y\sin\alpha = p$

$x\cos 150° + y\sin 150° = 12$

$x\left(\frac{-\sqrt{3}}{2}\right) + y\left(+\frac{1}{2}\right) = 12$

$-\sqrt{3}x + y = 24$

$\Rightarrow \sqrt{3}x - y + 24 = 0$.

(28)

Take; $u = x^3 - 1$

$y = u^{100}$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= 100u^{100-1}(3x^2 - 0)$

$= 100u^{99} \times 3x^2$

$= 100(x^3-1)^{99} \times 3x^2$

$\frac{dy}{dx} = 300x^2 (x^3-1)^{99}$

(or)

$y = (x^3-1)^{100}$

$\boxed{\frac{d}{dx}(x^n) = nx^{n-1}}$

$\frac{dy}{dx} = 100(x^3-1)^{100-1} \cdot 3x^2$

$\frac{dy}{dx} = 300x^2(x^3-1)^{99}$

(29)

Given: $P(A) = 0.52$

$P(B) = 0.43$

and $P(A \cap B) = 0.24$

To find, (i) $P(A \cap \bar{B})$

(ii) $P(\bar{A} \cup \bar{B})$

(i) $P(A \cap \bar{B}) = P(A) - P(A \cap B)$

$= 0.52 - 0.24$

$\therefore P(A \cap \bar{B}) = 0.28$

(ii) $P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B})$

(By de Morgan's law)

$= 1 - P(A \cap B)$

$= 1 - 0.24$

$= 0.76$

$\therefore P(\bar{A} \cup \bar{B}) = 0.76.$

(30)

Here, $\lim\limits_{x \to 1} (x-1) = 0.$

In such cases, rationalise the numerator.

$$\lim_{x \to 1} \frac{\sqrt{x}-1}{x-1} = \lim_{x \to 1} \frac{\cancel{\sqrt{x}-1}}{(\cancel{\sqrt{x}-1})(\sqrt{x}+1)}$$

$$= \lim_{x \to 1} \frac{1}{\sqrt{x}+1}$$

$$= \frac{1}{\sqrt{1}+1} = \frac{1}{2}$$

$$\therefore \lim_{x \to 1} \frac{\sqrt{x}-1}{x-1} = \frac{1}{2}.$$

PART - C

3 MARKS

(31)

Clearly,

$-1 \le \cos x \le 1$

$\Rightarrow 3 \geq -3\cos x \geq -3$

$-3 \le -3\cos x \le 3$

$1-3 \le 1-3\cos x \le 1+3$

Thus we get $-2 \le 1-3\cos x$

and $1-3\cos x \le 4$

By taking reciprocals we get,

$\frac{1}{1-3\cos x} \le -\frac{1}{2}$

and $\frac{1}{1-3\cos x} \geq \frac{1}{4}$

Hence the range of f is $(-\infty, -\frac{1}{2}] \cup [\frac{1}{4}, \infty)$.

(32)

Case (i)

For $x \geq 1$, $|x-1| = x-1$.

Then the given equation reduces to $x^2+x-2=0$.

Factoring we get,

$(x+2)(x-1) = 0.$

-2, -1, 2, 1

$\Rightarrow x = -2$ or 1

As $x \geq 1$,

We obtain $x = 1$.

Case (ii)

For $x<1$, $|x-1| = 1-x$

Then the given equation becomes $x^2+1-x=1$.

Thus we have $x(x-1)=0$

$\Rightarrow x=0$, or $x=1$.

As $x<1$, we have to choose $x=0$.

Thus the soln is $\{0,1\}$.

Hence the equation has two solutions.

(33)

We have,

$$\frac{\sin 75^\circ - \sin 15^\circ}{\cos 75^\circ + \cos 15^\circ} = \frac{2\cos\left(\frac{75^\circ+15^\circ}{2}\right)\sin\left(\frac{75^\circ-15^\circ}{2}\right)}{2\cos\left(\frac{75^\circ+15^\circ}{2}\right)\cos\left(\frac{75^\circ-15^\circ}{2}\right)}$$

$$\left[\because \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right.$$
$$\left.\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$= \frac{2\cos\left(\frac{90^\circ}{2}\right)\sin\left(\frac{60^\circ}{2}\right)}{2\cos\left(\frac{90^\circ}{2}\right)\cos\left(\frac{60^\circ}{2}\right)}$$

$$= \frac{\cancel{2\cos 45^\circ}\ \sin 30^\circ}{\cancel{2\cos 45^\circ}\ \cos 30^\circ} = \tan 30^\circ$$

$$= \frac{1}{\sqrt{3}}$$

$\left(\because \frac{\sin\theta}{\cos\theta} = \tan\theta\right)$

(35) Given parallel lines are

$3x-4y+5=0$

$\times 2 \Rightarrow 6x-8y+10=0$.

& $6x-8y-15=0$.

Here, $a=6$, $b=-8$, $c_1=10$, $c_2=-15$

Distance b/w Parallel lines $= \left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right|$

$$= \left|\frac{10+15}{\sqrt{36+64}}\right| = \frac{25}{10}$$

$$= 5/2$$

(34) There are 7 Indian's and 5 American's.

A committee of 5 members with majority Indian's can be formed by the following ways.

	Indians (7)	Americans (5)	combination
(i)	4	1	$7C_4 \times 5C_1$
(ii)	3	2	$7C_3 \times 5C_2$
(iii)	5	0	$7C_5 \times 5C_0$

∴ Total number of way of forming the committee

$$= 7C_4 \times 5C_1 + 7C_3 \times 5C_2 + 7C_5 \times 5C_0$$

$$= \frac{7\times 6\times 5}{3\times 2\times 1}\times 5 + \frac{7\times 6\times 5}{3\times 2\times 1}\times \frac{5\times 4}{2\times 1} + \frac{7\times 6}{2\times 1}\times 1$$

$(\because 7C_4 = 7C_3 \ ; \ 7C_5 = 7C_2)$

$= 175 + 350 + 21$

$= 546.$

(36)

Applying $R_1 \to aR_1, R_2 \to bR_2, R_3 \to cR_3$, we get,

$$A = \begin{vmatrix} ab+ac & abc & ab^2c^2 \\ bc+ab & abc & a^2bc^2 \\ ac+bc & abc & a^2b^2c \end{vmatrix}$$

Taking out (abc) common from c_2 and c_3 we get,

$$(abc)^2 \begin{vmatrix} ab+ac & 1 & bc \\ bc+ab & 1 & ac \\ ac+bc & 1 & ab \end{vmatrix}$$

Applying $C_1 \to C_1 + C_3$ we get,

$$(abc)^2 \begin{vmatrix} ab+bc+ca & 1 & bc \\ ab+bc+ca & 1 & ac \\ ab+bc+ca & 1 & ab \end{vmatrix}$$

Taking out $(ab+bc+ca)$ common from C_1, we get

$$A = (abc)^2 (ab+bc+ca) \begin{vmatrix} 1 & 1 & bc \\ 1 & 1 & ac \\ 1 & 1 & ab \end{vmatrix}$$

$$= (abc)^2 (ab+bc+ca)\,(0)$$

$$= 0 \quad (\because C_1 \equiv C_2)$$

Hence proved.

(38)

we have,

$x = a(t - \sin t)$, $y = a(1 - \cos t)$.

Now,

$$\frac{dx}{dt} = a(1-\cos t); \quad \frac{dy}{dt} = a(\sin t)$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \Rightarrow \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= \frac{\cancel{a}\sin t}{\cancel{a}(1-\cos t)} = \frac{a \sin t}{1 - \cos t}$$

$$\therefore \frac{dy}{dx} = \frac{\sin t}{1-\cos t}.$$

(39) $\int \frac{1}{\sin^2 x \cos^2 x} dx$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx$$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$$= \int \frac{\cancel{\sin^2 x}}{\cancel{\sin^2 x}\cos^2 x} dx + \int \frac{\cancel{\cos^2 x}}{\sin^2 x \cancel{\cos^2 x}} dx$$

$$= \int \frac{1}{\cos^2 x} dx + \int \frac{1}{\sin^2 x} dx$$

$$= \int \sec^2 x \, dx + \int \operatorname{cosec}^2 x \, dx$$

$$= \tan x - \cot x + c$$

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PART - D

5 MARKS

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(a) $\sqrt{3}x + y + 4 = 0$.

$\sqrt{3}x + y = -4$

$-\sqrt{3}x - y = 4$

$$\boxed{-\frac{\sqrt{3}}{2}x - \frac{1}{2}y = 2}$$

$x\cos\alpha + y\sin\alpha = P$

[$Ax + By + C = 0$

$A = -\sqrt{3}$; $B = -1$, $C = -4$

$C: -\sqrt{3}x - y = +4$)

$\sqrt{A^2 + B^2} = \sqrt{(-\sqrt{3})^2 + (-1)^2}$

$= \sqrt{3+1} = \sqrt{4} = 2$]

$\cos\alpha = -\frac{\sqrt{3}}{2}$; $\sin\alpha = -\frac{1}{2}$

Angle, $\frac{\pi}{6} \Rightarrow \alpha = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$.

$\therefore x\cos\alpha + y\sin\alpha = P$.

$x\cos\frac{7\pi}{6} + y\sin\frac{7\pi}{6} = 2$

The required equation of Normal form.

(OR)

(b)

Let $y = 2x - 3$. Then $x = \frac{y+3}{2}$.

Let $g(y) = \frac{y+3}{2}$

$(g\circ f)(x) = g(f(x))$

$= g(2x-3)$

$(g\circ f)(x) = \frac{(2x-3)+3}{2} = x$

$(f\circ g)(y) = f(g(y))$

$= f\left(\frac{y+3}{2}\right)$

$= 2\left(\frac{y+3}{2}\right) - 3$

$= y$

$(f\circ g)(y) = f(g(y)) = y$

Thus $g\circ f = I_x$ and $f\circ g = I_y$

This implies that f and g are bijections and inverse to each other.

Hence f is a bijection and $f^{-1}(y) = \frac{y+3}{2}$

Replacing y by x we get,

$f^{-1}(x) = \frac{x+3}{2}$.

42

(a)

LHS $= \sin 2A + \sin 2B + \sin 2C$

$= 2\sin\left(\frac{2A+2B}{2}\right)\cos\left(\frac{2A-2B}{2}\right) + 2\sin C\cos C$

$\left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right.$

$\left.\because \sin 2A = 2\sin A\cos A\right]$

$= 2\sin(A+B)\cos(A-B) + 2\sin C\cos C$

$= 2\sin\left(\frac{\pi}{2} - C\right)\cos(A-B) + 2\sin C\cos C$

$\left[\because A+B+C = \frac{\pi}{2}\right.$

$\Rightarrow \quad A+B = \left.\pi/2 - C\right]$

$= 2\cos C\cos(A-B) + 2\sin C\cos C$

$\left(\because \sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta\right)$

$= 2\cos C\left[\cos(A-B) + \sin\left(\frac{\pi}{2} - (A+B)\right)\right]$

$\left[\because A+B+C = \frac{\pi}{2}\right.$

$\left. C = \pi/2 - (A+B)\right]$

$= 2\cos C\left[\cos(A-B) + \cos(A+B)\right]$

$= 2\cos C\left[2\cos A\cos B\right]$

$= 4\cos A\cos B\cos C$

LHS = RHS

Hence proved

(OR)

(b)

$$\begin{array}{r|l} & x - 5 \\ \hline x^2+5x+6 & x^3 + 0x^2 + 2x + 1 \\ & x^3 + 5x^2 + 6x \\ & (-) \quad (-) \quad (-) \\ \hline & -5x^2 - 4x + 1 \\ & -5x^2 - 25x - 30 \\ & (+) \quad (+) \quad (+) \\ \hline & 21x + 31 \end{array}$$

$$\therefore \frac{x^3+2x+1}{x^2+5x-6} = (x-5) + \frac{21x+31}{x^2+5x+6} \quad \text{—(1)}$$

Consider,

$$\frac{21x+31}{x^2+5x+6} = \frac{21x+31}{(x+3)(x+2)}$$

$$\frac{21x+31}{(x+3)(x+2)} = \frac{A}{x+3} + \frac{B}{(x+2)} \quad \text{—(2)}$$

$$\frac{21x+31}{\cancel{(x+3)(x+2)}} = \frac{A(x+2) + B(x+3)}{\cancel{(x+3)(x+2)}}$$

$$21x+31 = A(x+2) + B(x+3) \quad \text{—(3)}$$

putting $x=-2$ in (3) we get

$$-11 = B(1)$$

$$\boxed{B = -11}$$

Putting $x=-3$ in (3) we get

$$-32 = A(-1)$$

$$\boxed{A = 32}$$

$$\therefore \frac{21x+31}{x^2+5x+6} = \frac{32}{x+3} - \frac{11}{x+2}.$$

Sub (1)

$$\frac{x^3+2x+1}{x^2+5x+6} = (x-5) + \frac{32}{x+3} - \frac{11}{x+2}.$$

43

(a)

Let $I = \int \frac{(5x-2)}{2+2x+x^2}\,dx$

$$5x-2 = A\frac{d}{dx}(x^2+2x+2) + B$$

$$5x-2 = A(2x+2) + B$$

Equating the co-efficients of x term and constant term we get

$$5 = 2A$$

$$\boxed{A = \frac{5}{2}}$$

$$-2 = 2A + B$$

$$-2 = 2(5/2) + B$$

$$-2 = 5 + B$$

$$\boxed{B = -7}$$

$$\therefore 5x-2 = \frac{5}{2}(2x+2) - 7$$

$$\therefore I = \int \frac{(5x-2)}{x^2+2x+2}\,dx = \frac{5}{2}\int \frac{2x+2}{x^2+2x+2}\,dx - 7\int \frac{dx}{x^2+2x+2}$$

$$= \frac{5}{2}\log|x^2+2x+2| - 7\int \frac{dx}{x^2+2x+1-1+2}$$

$$= \frac{5}{2}\log|x^2+2x+2| - 7\int \frac{dx}{(x+1)^2+1^2}$$

$$= \frac{5}{2}\log|x^2+2x+2| - 7\tan^{-1}\left(\frac{x+1}{1}\right)+c.$$

$$\left(\because \int \frac{dx}{x^2+a^2} = \tan^{-1}\left(\frac{x}{a}\right)+c.\right)$$

$$= \frac{5}{2}\log|x^2+2x+2| - 7\tan^{-1}(x+1)+c.$$

(OR)

(b)

$$\sqrt[3]{x^3+7} = (x^3+7)^{\frac{1}{3}}$$

$$= \left[x^3\left(1+\frac{7}{x^3}\right)\right]^{1/3}$$

$$\Rightarrow x\left(1+\frac{7}{x^3}\right)^{1/3}$$

$$= x\left(1+\frac{1}{3}\times\frac{7}{x^3} + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}\left(\frac{7}{x^3}\right)^2\right.$$

$$\left[\because (1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3+\cdots\right]$$

$$= x\left(1+\frac{7}{3}\times\frac{1}{x^3} - \frac{49}{9}\times\frac{1}{x^6}+\cdots\right)$$

$$= x + \frac{7}{3}\times\frac{1}{x^2} - \frac{49}{9}\times\frac{1}{x^5}+\cdots$$

$$\sqrt[3]{x^3+4} = (x^3+4)^{1/3}$$

$$= \left[x^3\left(1+\frac{4}{x^3}\right)\right]^{1/3}$$

$$= x\left(1+\frac{4}{x^3}\right)^{1/3}$$

$$= x\left(1+\frac{1}{3}\times\frac{4}{x^3} + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}\left(\frac{4}{x^3}\right)^2+\cdots\right)$$

$$= x + \frac{4}{3}\times\frac{1}{x^2} - \frac{16}{9}\times\frac{1}{x^5}+\cdots$$

since x is large, $\frac{1}{x^2}$ is very small and hence higher powers of $\frac{1}{x}$ are negligible.

Thus $\sqrt[3]{x^3+7} = x + \frac{7}{3} \times \frac{1}{x^2}$

and $\sqrt[3]{x^3+4} = x + \frac{4}{3} \times \frac{1}{x^2}$.

Therefore,

$$\sqrt[3]{x^3+7} - \sqrt[3]{x^3+4} = \left(x + \frac{7}{3} \times \frac{1}{x^2}\right) - \left(x + \frac{4}{3} \times \frac{1}{x^2}\right)$$

$$= \cancel{x} + \frac{7}{3x^2} - \cancel{x} - \frac{4}{3x^2}$$

$$= \frac{7}{3x^2} - \frac{4}{3x^2}$$

$$= \frac{7-4}{3x^2} = \frac{3}{3x^2}$$

$$= \frac{1}{x^2}$$

$\therefore \sqrt[3]{x^3+7} - \sqrt[3]{x^3+4} = \frac{1}{x^2}$.

Hence proved

44 (a)

Let the position vector of the vertices of the Quadrilateral ABCD be $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ respectively.

$\vec{OA} = \vec{a}$, $\vec{OB} = \vec{b}$, $\vec{OC} = \vec{c}$ and $\vec{OD} = \vec{d}$

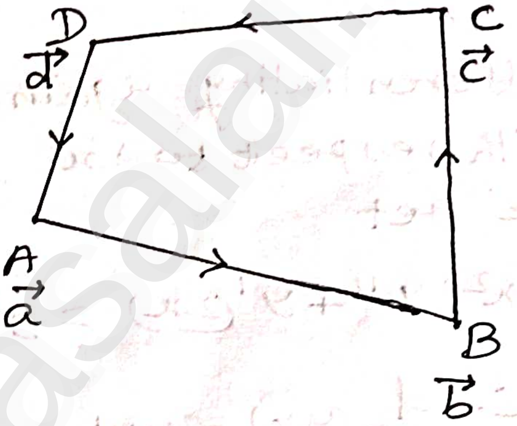
Since E and F are the midpoints of AC and BD respectively, we have,

$$\vec{OE} = \frac{\vec{a}+\vec{c}}{2} \text{ and}$$

$$\vec{OF} = \frac{\vec{b}+\vec{d}}{2} \rightarrow (1)$$

To prove that

$$\vec{AB} + \vec{AD} + \vec{CB} + \vec{CD} = 4\vec{EF}$$



$LHS = \vec{AB} + \vec{AD} + \vec{CB} + \vec{CD}$

$= \vec{OB} - \vec{OA} + \vec{OD} - \vec{OA} + \vec{OB} - \vec{OC} + \vec{OD} - \vec{c}$

$= \vec{b} - \vec{a} + \vec{d} - \vec{a} + \vec{b} - \vec{c} + \vec{d} - \vec{c}$

$= -2\vec{a} + 2\vec{b} - 2\vec{c} + 2\vec{d}$

$= 2\left[(\vec{b}+\vec{d}) - (\vec{a}+\vec{c})\right]$

$= 2\left[2\vec{OF} - 2\vec{OE}\right]$ [From (1)]

$= 4\left[\vec{OF} - \vec{OE}\right]$

$= 4\vec{EF} = RHS$

Hence proved.

(44) (OR)

(b)

Given $y = e^{\tan^{-1}x}$

$y' = e^{\tan^{-1}x} \cdot \frac{d}{dx}\tan^{-1}(x)$

$y' = e^{\tan^{-1}x}\left(\frac{1}{1+x^2}\right)$

$\Rightarrow (1+x^2)\,y' = y$

$(\because y = e^{\tan^{-1}x})$

Differentiating again with respect ~~to~~ 'x' we get

$(1+x^2)y'' + y'(2x) = y'$

$[\because \frac{d}{dx}(uv) = uv' + vu']$

$(1+x^2)y'' + 2xy' - y' = 0$

$\therefore (1+x^2)y'' + (2x-1)y' = 0.$

Hence proved

(45)

(a)

LHS $= \log 2 + 16\log\frac{16}{15} + 12\log\frac{25}{24} + 7\log\frac{81}{80}$

$= \log 2 + \log\left(\frac{16}{15}\right)^{16} + \log\left(\frac{25}{24}\right)^{12} + \log\left(\frac{81}{80}\right)^{7}$

$= \log\left[2 \times \left(\frac{2^4}{3\times5}\right)^{16} \times \frac{(5^2)^{12}}{(2^3\times 3)^{12}} \times \frac{(3^4)^7}{(2^4\times5)^7}\right]$

$= \log\left[2^1 \times \frac{2^{64}}{3^{16}\times5^{16}} \times \frac{5^{24}}{2^{36}\times3^{12}} \times \frac{3^{28}}{2^{28}\times5^7}\right]$

$= \log\frac{2^{1+64}\cdot 5^{24}\cdot 3^{28}}{3^{16+12}\cdot 5^{16+7}\cdot 2^{36+28}}$

$\left[\because (a^m)^n = a^{mn},\ \frac{a^m}{a^n} = a^{m-n}\right]$

$= \log\frac{2^{65}\cdot 5^{24}\cdot \cancel{3^{28}}}{\cancel{3^{28}}\cdot 5^{23}\cdot 2^{64}}$

$= \log(2^{65-64} \times 5^{24-23})$

$= \log(2^1 \times 5^1)$

$= \log_{10} 10 = 1.$

LHS = RHS.

Hence proved.

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(OR)

46

(b)

Let $P(n) = 3^{2n+2} - 8n - 9$ is divisible by 8.

Substituting the value of $n=1$ in the statement we get,

$P(1) = 3^{2+2} - 8(1) - 9$
$= 64$

which is divisible by 8.

Hence, $P(1)$ is true.

Let us assume that the statement is true for $n=k$.

Then $P(k) = 3^{2k+2} - 8k - 9$ is divisible by 8.

We can write

$P(k) = 3^{2k+2} - 8k - 9$
$= 8k_1, \ k_1 \in N$

$3^{2k+2} = 8k_1 + 8k + 9$

We need to show that

$P(k+1) = 3^{2(k+1)+2} - 8(k+1) - 9$

is divisible by 8.

$P(k+1) = 3^{2(k+1)+2} - 8(k+1) - 9$

$= 3^2 \, 3^{2k+2} - 8k - 8 - 9$

$= 3^2 (8k_1 + 8k + 9) - 8k - 17$

$= 72k_1 + 61k\,k + 64$

$= 8(9k_1 + 8k + 8)$

$= 8k_2, \ k_2 = 9k_1 + 8k + 8 \in N.$

which is divisible by 8.

$\Rightarrow P(k+1)$ is true.

This means that the validity of $P(k+1)$ follows from that of $P(k)$.

$\therefore$ The principle of Mathematical induction, $3^{2n+2} - 8n - 9$ is divisible by 8 for all $n \geq 1$.

47

(a)

Given equation of line is $9x^2 - 24xy + 16y^2 - 12x + 16y - 12 = 0$.

Here,

$a = 9, \ 2h = -24$
$b = 16, \ 2g = -12$
$2f = 16, \ c = -12$
$h = -12, \ g = -6,$
$f = 8$

The condition to represent a pair of parallel lines is $h^2 - ab = 0$.

$(-12)^2 - 9(16) = 0$.

$\Rightarrow 144 - 144 = 0$.

Hence they are parallel lines.

Distance b/w the parallel lines

$$= 2\sqrt{\frac{g^2 - ac}{a(a+b)}}$$

$$= 2\sqrt{\frac{(-6)^2 - 9(-12)}{9(9+16)}}$$

$$= 2\sqrt{\frac{36+108}{9(25)}}$$

$$= 2\sqrt{\frac{144}{9(25)}}$$

$$= 2 \times \frac{\cancel{12}^{4}}{\cancel{3} \times 5} = \frac{8}{5} \text{ units.}$$

(47) (OR)
(b)

Let $|A| = \begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix}$

Put $x = 1$, we get

$$|A| = \begin{vmatrix} 2 & 3 & 5 \\ 2 & 3 & 5 \\ 2 & 3 & 5 \end{vmatrix} = 0$$

Since all the three rows are identical $(x-1)^2$ is a factor of $|A|$.

Putting $x = -9$ in $|A|$

$$|A| = \begin{vmatrix} -8 & 3 & 5 \\ 2 & -7 & 5 \\ 2 & 3 & -5 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 3 & 5 \\ 0 & -7 & 5 \\ 0 & 3 & -5 \end{vmatrix} = 0$$

$[\because C_1 \to C_1 + C_2 + C_3]$

Therefore $x+9$ is a factor of $|A|$.

Now, determinant is a cubic polynomial in x.

Therefore the remaining factor must be a constant 'k'.

$$\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = k(x-1)^2(x+9)$$

Equating x^3 term on both sides, we get $k = 1$. Thus $|A| = (x-1)^2(x+9)$

ONE MARK Solution

(2) Required number of parallelograms $= 4C_2 \times 3C_2$

$= \frac{4\times3}{2\times1} \times \frac{3\times2}{2\times1}$

$= 6\times3$

$= 18$

(9) $x^2 + mx + \frac{1}{2} + \frac{m}{2} = 0$

$D = m^2 - 4\left(\frac{1}{2} + \frac{m}{2}\right)$

$(\because D = b^2 - 4ac)$

Discriminant D of the Quadratic equation

$(\because ax^2 + bx + c)$

$= m^2 - 2m - 2$

$= (m-1)^2 - 3$

$D \geq 0$.

$m = 3, 4, 5$.

Also total number of ways choosing m is 5 ways.

Prob. of the req. event $= \frac{3}{5}$.

(10) $3x - y - 5 = 0$.

$y = 3x - 5$

slope $(m_1) = 3$.

Angle b/w two lines

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\tan 45° = \left|\frac{3 - m_2}{1 + 3m_2}\right|$

$1 = \frac{3 - m_2}{1 + 3m_2}$

$1 + 3m_2 = 3 - m_2$

$4m_2 = 2$

$\boxed{m_2 = \frac{1}{2}}$

$1 = \frac{-(3 - m_2)}{1 + 3m_2}$

$1 + 3m_2 = -3 + m_2$

$2m_2 = -4$

$\boxed{m_2 = -2}$

$\left(\frac{1}{2}, -2\right)$.

(13) $\frac{1}{2} + \frac{7}{4} + \frac{13}{8} + \frac{19}{16} + \ldots$

It is an arithmetic Geometric series

Here, $a = 1$, $d = 7 - 1 = 6$

$r = \frac{1}{2}$.

$S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$

$= \frac{1}{1 - \frac{1}{2}} + 6\frac{\left(\frac{1}{2}\right)}{\left(1 - \frac{1}{2}\right)^2}$

$= \frac{1}{\frac{1}{2}} + \frac{3}{(\frac{1}{2})^2}$

$= 2 + (3 \times 4) = 2 + 12$

$= 14.$

(14) $\sin\alpha + \cos\alpha = b \rightarrow (1)$

Squaring,

$(\sin\alpha + \cos\alpha)^2 = b^2$

$\sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha = b^2$

$1 + 2\sin\alpha\cos\alpha = b^2$

$1 + \sin2\alpha = b^2$

$\sin2\alpha = b^2 - 1$

$(\because \sin2\theta = 2\sin\theta\cos\theta)$

$-1 \leq \sin\theta \leq 1 \quad \forall \theta \in R.$

$-1 \leq \sin2\alpha \leq 1$

$-1 \leq b^2 - 1 \leq 1$

$-1 + 1 \leq b^2 \leq 1 + 1$

$0 \leq b^2 \leq 2$

$0 \leq b^2, \quad b^2 \leq 2$

$0 \leq b, \quad b \leq \sqrt{2}$

$\therefore b^2 - 1$, if $b \leq \sqrt{2}$

(16) Let $x = a\cos\theta$

$y = b\sin\theta$

$\frac{x}{a} = \cos\theta \rightarrow (1)$

$\frac{y}{b} = \sin\theta \rightarrow (2)$

Squaring,

$(\frac{x}{a})^2 + (\frac{y}{b})^2 = \sin^2\theta + \cos^2\theta$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(27) $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$

$A^6 = A^2 \times A^2 \times A^2$

$A^2 = A \times A$

$A^2 = \begin{bmatrix} 1 & 2a \\ 0 & 1 \end{bmatrix}$

$A^4 = \begin{bmatrix} 1 & 4a \\ 0 & 1 \end{bmatrix}$

$A^6 = \begin{bmatrix} 1 & 6a \\ 0 & 1 \end{bmatrix}$ //.

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