ST.ANNE'S ACADEMY
PUNNAI NAGAR, NAGERCOIL – 4.
CLASS – XII – MATHEMATICS
Common II Mid-Term Test (Model Question)
Time : 1.5 Hrs Marks : 45

PART – I

I. Answer ALL questions. 10x1 = 10

1) The point on the curve $6y = x^3 + 2$ at which y-coordinate changes 8 times as fast as x-coordinate is
(1) (4,1 1) (2) (4, –1 1) (3) (–4,1 1) (4) (–4, –1 1)

2) The value of the limit $\lim_{x\to 0}\left(\cot x - \frac{1}{x}\right)$ is
(1) 0 (2) 1 (3) 2 (4) ∞

3) The maximum slope of the tangent to the curve $y = e^{\sin x}, x \in [0, 2\pi]$ is at
(1) $x = \frac{\pi}{4}$ (2) $x = \frac{\pi}{2}$ (3) $x = \pi$ (4) $x = \frac{3\pi}{2}$

4) If $w(x, y) = x^y, x > 0$, then $\frac{\partial w}{\partial x}$ is equal to
(1) $x^y \log x$ (2) $y \log x$ (3) yx^{y-1} (4) $x \log y$

5) If $g(x, y) = 3x^2 - 5y + 2y^2, x(t) = e^t$ and $y(t) = \cos t$, then $\frac{dg}{dt}$ is equal to
(1) $6e^{2t} + 5\sin t - 4\cos t \sin t$
(2) $6e^{2t} - 5\sin t + 4\cos t \sin t$
(3) $3e^{2t} + 5\sin t + 4\cos t \sin t$
(4) $3e^{2t} - 5\sin t + 4\cos t \sin t$

6) If $f(x, y, z) = xy + yz + zx$, then $f_x - f_z$ is equal to
(1) $z - x$ (2) $y - z$ (3) $x - z$ (4) $y - x$

7) The value of
$\int_{-4}^{4}\left[\tan^{-1}\left(\frac{x^2}{x^4+1}\right) + \tan^{-1}\left(\frac{x^4+1}{x^2}\right)\right]dx$ is
(1) π (2) 2π (3) 3π (4) 4π

8) The value of $\int_0^{\pi} \frac{dx}{1+5^{\cos x}}$ is
(1) $\frac{\pi}{2}$ (2) π (3) $\frac{3\pi}{2}$ (4) 2π

9) If $\int_0^a \frac{1}{4+x^2}\,dx = \frac{\pi}{8}$ then a is
(1) 4 (2) 1 (3) 3 (4) 2

10) The number given by the Rolle's theorem for the function $x^3 - 3x, x \in [0, \sqrt{3}]$ is
(1) 1 (2) $\sqrt{2}$ (3) $\frac{3}{2}$ (4) 2

PART – II

II. Answer any seven questions. 3x2 = 6
Question No.15 is Compulsory

11) If the radius of a sphere, with radius 10 cm, has to decrease by 0.1 cm, approximately how much will its volume decrease?

12) Let $V(x, y, z) = xy + yz + zx, x, y, z \in \mathbb{R}$. Find the differential dV.

13) Evaluate : $\int_0^1 \lfloor 2x \rfloor\, dx$

14) Prove that the function $f(x) = x^2 - 2x - 3$ is strictly increasing in $(2, \infty)$.

15) Using mean value theorem prove that for, $a > 0, b > 0, |e^{-a} - e^{-b}| < |a - b|$.

PART – III

III. Answer any seven questions. 3x3 = 9
Question No.40 is Compulsory

16) If $\lim_{\theta\to 0}\left(\frac{1-\cos m\theta}{1-\cos n\theta}\right) = 1$, then prove that $m = \pm n$.

17) Evaluate the following
$\int_0^{\frac{\pi}{2}} \frac{e^{-\tan x}}{\cos^6 x}\,dx$

18) Assuming $\log_{10} e = 0.4343$, find an approximate value of $\log_{10} 1003$.

19) If $U(x, y, z) = \log(x^3 + y^3 + z^3)$, find $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z}$

20 Find the slant (oblique) asymptote for the function $f(x) = \frac{x^2 - 6x + 7}{x + 5}$.

PART – IV

IV. Answer all questions. 4x5 = 20

21) a) Find intervals of concavity and points of inflexion for the following function:
$f(x) = \sin x + \cos x, 0 < x < 2\pi$

Or

b) If the curves $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$ intersect each other orthogonally then, show that $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$.

22) a) Find the volume of the spherical cap of height 'h' cut of from a sphere of radius r.

Or

b) Find the area of the region bounded by the parabola $y^2 = x$ and the line $y = x - 2$.

23) a) If $z(x, y) = x \tan^{-1}(xy)$, $x = t^2$, $y = se^t$.

Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ at $s = t = 1$; $s, t \in \mathbb{R}$

Or

b) If $v(x, y) = \log\left(\frac{x^2 + y^2}{x + y}\right)$

prove that $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = 1$.

24) a) Evaluate the following using properties of integration : $\int_0^{\pi} \frac{x \sin x}{1 + \sin x} dx$

Or

b) Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is 10 metre high?

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