



V12M

Virudhunagar District
Second Mid Term Test - 2023

Standard 12
MATHEMATICS
Part - I

Time: 1.30 Hours

Marks: 50

Choose the correct answer:

10×1=10

- Angle between $y^2 = x$ and $x^2 = y$ at the origin is
 - $\tan^{-1}\left(\frac{4}{3}\right)$
 - $\frac{\pi}{4}$
 - $\frac{\pi}{2}$
 - $\tan^{-1}\left(\frac{3}{4}\right)$
- The maximum value of the product of two positive numbers, when their sum of the squares is 200, is
 - 100
 - $25\sqrt{7}$
 - 28
 - $24\sqrt{14}$
- The number given by the mean value theorem for the function $\frac{1}{x}$, $x \in [1, 9]$ is
 - 2.5
 - 3
 - 2
 - 3.5
- The slope of the line normal to the curve $f(x) = 2 \cos 4x$ at $x = \frac{\pi}{12}$ is
 - $-4\sqrt{3}$
 - 4
 - $\frac{\sqrt{3}}{12}$
 - $4\sqrt{3}$
- If $w(x, y) = x^y$, $x > 0$, then $\frac{\partial w}{\partial x}$ is equal to
 - $x^y \log x$
 - $y \log x$
 - yx^{y-1}
 - $x \log y$
- The approximate change in the volume V of a cube of side x metres caused by increasing the side by 1% is
 - $0.3 \ xdx \ m^3$
 - $0.03 \ xm^3$
 - $0.03 \ x^3m^3$
 - $0.03x^2 \ m^3$
- If $w(x, y, z) = x^2(y-z) + y^2(z-x) + z^2(x-y)$, then $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z}$ is
 - $xy + yz + zx$
 - $x(y+z)$
 - $y(z+x)$
 - 0
- The value of $\int_0^{\frac{2}{3}} \frac{dx}{\sqrt{4-9x^2}}$ is
 - $\frac{\pi}{2}$
 - π
 - $\frac{\pi}{4}$
 - $\frac{\pi}{6}$
- The value of $\int_0^1 x(1-x)^{99} dx$ is
 - $\frac{1}{11000}$
 - $\frac{1}{10100}$
 - $\frac{1}{10010}$
 - $\frac{1}{10001}$
- The value of $\int_0^{\pi} \sin^4 x \ dx$ is
 - $\frac{3\pi}{10}$
 - $\frac{3\pi}{8}$
 - $\frac{3\pi}{4}$
 - $\frac{3\pi}{2}$

Part - II

Answer any four of the following:

4×2=8

- If the volume of a cube of side length x is $V=x^3$. Find the rate of change of the volume with respect to x when $x = 5$ units.
- Prove that the function $f(x) = x^2-2x - 3$ is strictly increasing in $(2, \infty)$
- Let $V(x, y, z) = xy + yz + zx$, $x, y, z \in \mathbb{R}$. Find the differential dv .
- Show that $F(x, y) = \frac{x^2 + 5xy - 10y^2}{3x + 7y}$ is a homogeneous function of degree 1

V12M

2

15) Evaluate: $\int_0^{\pi/2} \sin^{10} x \, dx$

16) Evaluate: $\int_{-\pi/2}^{\pi/2} x \cos x \, dx$

Part - III**Answer any four of the following:****4×3=12**

17) Find the value in the interval $\left(\frac{1}{2}, 2\right)$ satisfied by the Rolle's theorem for the function $f(x) = x + \frac{1}{x}, x \in \left[\frac{1}{2}, 2\right]$

18) Evaluate: $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$

19) If the radius of a sphere, with radius 10cm, has to increase by 0.1 cm approximate how much will its volume decrease?

20) If $U(x, y, z) = \log(x^2 + y^2 + z^2)$, then find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$

21) Evaluate: $\int_0^a \frac{f(x)}{f(x) + f(a-x)} \, dx$

22) Evaluate: $\int_0^1 x^3 e^{-2x} \, dx$

Part - IV**Answer any four of the following:****4×5=20**

23) Find the angle between the curves $y=x^2$ and $y=(x-3)^2$

24) Find the local extrema of the function $f(x) = 4x^6 - 6x^4$.

25) Prove that $g(x,y) = x \log \left(\frac{y}{x}\right)$ is homogenous, what is the degree? verify Euler's theorem for g.

26) Let $w(x, y) = xy + \frac{e^y}{y^2 + 1}$ for all $x, y \in \mathbb{R}^2$ calculate $\frac{\partial^2 w}{\partial y \partial x}$ and $\frac{\partial^2 w}{\partial x \partial y}$

27) Evaluate $\int_0^1 x^2 \, dx$, as the limit of a sum.

28) Prove that $\int_0^{\pi/4} \log(1 + \tan x) \, dx = \frac{\pi}{8} \log 2$
