HALF- YEARLY EXAMINATION -2K23 DEC
SPB MATRIC. HR.SEC.SCHOOL-SPB COLONY

STD: XII **18.12.2K23**

SUBJECT: MATHEMATICS **ANSWER KEY** **MARKS : 90**

SECTION – I

Q.No	ANSWER KEY	MARKS
1.	(2) $adj(AB) = (adjA)(adjB)$	1
2.	(1) 1	1
3.	(4) (1,1)	1
4.	(2) $\frac{-1}{i+2}$	1
5.	(3) $\frac{4}{5}$	1
6.	(1) one negative and two imaginary zeros	1
7.	(2) $\frac{\pi}{3}$	1
8.	(4) $\frac{x}{\sqrt{1+x^2}}$	1
9.	(4) 12	1
10.	(3) 45°	1
11.	(4) has no points of inflection	1
12.	(2) $\frac{1}{5}$	1
13.	(4) 0	1
14.	(4) $\frac{2}{27}$	1
15.	(4) $\frac{\pi a^3}{6}$	1
16.	(4) 2,1	1
17.	(4) cot x	1
18.	(2) 0.25	1
19.	(2) z	1
20.	(4) $\sqrt{5}$ is an irrational number	1

SECTION – II

Q.No	ANSWER KEY	MARKS
21.	$A^{-1^T} = \frac{1}{5}\begin{bmatrix} 7 & -1 \\ -9 & 2 \end{bmatrix}$ $(A^T)^{-1} = \frac{1}{5}\begin{bmatrix} 7 & -1 \\ -9 & 2 \end{bmatrix}$	1 1
22.	$\left\|\|Z\| - 16 - 8i\right\| \le \|z + 6 - 8i\| \le \left\|\|z\| + \|6 - 8i\|\right\|$ $7 \le \|z + 6 - 8i\| \le 13$	1 1
23.	$\alpha + \beta + \gamma = -p$ and $\alpha\beta\gamma = -r$ $\sum \frac{1}{\beta\gamma} = \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha} + \frac{1}{\alpha\beta} = \frac{p}{r}$	1 1

24.	The distance from (2,1) to the line 2x+4y +10 =0 is Radius = 5 $x^2+y^2-4x-2y-20=0$	1 1
25.	$\frac{f(7)-f(2)}{7-2} = f'(c) \leq 29$ $f(7) \leq 162$	1 1
26.	$L(x) = x_0^{\frac{1}{3}} + \frac{1}{3x_0^{\frac{2}{3}}}(x - x_0)$ $f(27.2) \simeq 1.0074+2=3.0074 \simeq 3.0074$	1 1
27	$I = \frac{4.3.2.1}{9.7.5.3.1} = \frac{8}{315}$	2
28.	$(y-k)^2 = 4a(x-h)$; $y^1 = 2a$ $2ay^{11} + (y^1)^3 = 0$	1 1
29.	$k\int_0^{\infty} xe^{-2x}dx = 1$ $k\left[\frac{1!}{(2)^{1+1}}\right] = 1$; K=4	1 1
30	$\hat{b} = -3\hat{\imath} + \frac{2}{7}m\hat{\jmath} + 2\hat{k}$; $\hat{d} = -\frac{3}{7}m\hat{\imath} + \hat{\jmath} - 5\hat{k}$ $\hat{b}.\hat{d} = 0$ $m = \frac{70}{11}$	1 1
SECTION – III		
31.	$\Delta = 17$, $\Delta_1 = -34$, $\Delta_2 = 51$ $x = \frac{\Delta_1}{\Delta} = -2$, $y = \frac{\Delta_2}{\Delta} = 3$	1 1 1
32.	$z^3 + 2\lvert\bar{z}\rvert = 0$; $\lvert z\rvert(\lvert z\rvert^2 - 2) = 0$ $z^4 + 4 = 0$ has four solution The given equation has five solutions	1 1 1
33.	The domain of $\sin^{-1}x$ is [-1, 1] $-1 \leq \frac{x(x^2+1)}{2x^2} \leq 1$; $-2x^2 \leq x(x^2+1) \leq 2x^2$ The domain of the function is {-5,5}	1 2
34.	Equation of the tangent to parabola $y^2 = 4ax$ at t is , yt = yt = x + at^2 $yt_1 = x + at_1^2$ . . . . (1) $yt_2 = x + at_2^2$ . . . . .(2) $y = a(t_2 + t_1)$, $x = at_1 t_2$ Hence the point of intersection is $[at_1 t_2, a(t_1+t_2)]$.	1 1 1

35.	Given the vectors are coplanar. ∴ $[\vec{a}, \vec{b}, \vec{c}]=0$ a(0 – c) – a (b – c) + c (c – 0) =0 ; c = $\sqrt{ab}$ c is the geometric mean of a and b.	1 1 1
36.	$\frac{\partial U}{\partial x}=\frac{x^2-y^2}{x^2y}$ $\frac{\partial U}{\partial y}=\frac{y^2-x^2}{y^2x}+3z^2$ $\frac{\partial U}{\partial z}=6yz$	1 1 1
37.	$I=\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{1+\sqrt{\tan x}}\,dx \quad I=\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{\sqrt{\tan x}}{1+\sqrt{\tan x}}\,dx$ $2I=\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \left[\frac{1+\sqrt{\tan x}}{1+\sqrt{\tan x}}\right] dx$ $I=\frac{\pi}{8}$	1 1 1
38.	$\frac{dx}{dy}=\frac{xe^{\frac{x}{y}}+y}{ye^{\frac{x}{y}}}$ This is a homogeneous differential equation $x=vy\Rightarrow v=\frac{x}{y}$; $\int e^v dv=\int \frac{dy}{y}$ $e^{\frac{x}{y}}=\log(cy)$	1 1 1
39.	(1) m+n-mn ∈ Z, ∴ m*n∈Z ∴ m*n = n*m, ∀m,n∈Z * is commutative on Z ∴ (m*n) *p = m*(n*p), ∀m,n,p∈Z Associative property is true.	1 1 1
40.	$\lim_{x\to\frac{\pi}{2}^-} \frac{\cos x}{(2x-\pi)^2} \left(\frac{0}{0} form\right)$ $\lim_{x\to\frac{\pi}{2}^-} \frac{-\sin x}{2(2x-\pi)2}$ $=-\infty$	1 1 1

SECTION – IV

41. (a)	$[A\mid B]=\left[\begin{array}{ccc\|c} 2 & 3 & 5 & 9 \\ 7 & 3 & -5 & 8 \\ 2 & 3 & \lambda & \mu \end{array}\right]$	1

$$\left[\begin{array}{ccc|c} 1 & \frac{3}{2} & \frac{5}{2} & \frac{9}{2} \\ 7 & \frac{-15}{2} & \frac{-45}{2} & \frac{-47}{2} \\ 0 & 0 & \lambda-5 & \mu-9 \end{array}\right]$$ 1 1

Case (i) If λ =5 and $\mu \neq 9$ ρ[A] $\neq$ $\rho\left[A|B\right]$ consistent , has no solution.

Case (ii) If $\lambda \neq 5$ and $\mu \in$ R then ρ[A] = $\rho\left[A|B\right]$=n consistent has a unique solution. 1

Case (iii) If λ =5 and μ=9 ρ[A] = $\rho\left[A|B\right]$<n consistent and has infinite number of solution 1

(b)

$$\tan^{-1}\left[\frac{x-1+x+1}{1-(x-1)(x+1)}\right]+\tan^{-1}x=\tan^{-1}(3x)$$ 1

$$\tan^{-1}\left[\frac{2x}{2-x^2}\right]+\tan^{-1}x=\tan^{-1}(3x)$$ 1

$$\tan^{-1}\left[\frac{\frac{2x}{2-x^2}+x}{1-\left(\frac{2x}{2-x^2}\right)x}\right]=\tan^{-1}(3x)$$ 1

$2x(4x^2-1)$=0 1

The number of solution of the given equation is 3 1

42. (a)

$\Delta=\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ = ad − bc; $\Delta_{x^2}\begin{vmatrix} 1 & b \\ 1 & d \end{vmatrix}$ = d − b ; $\Delta_{y^2}=\begin{vmatrix} a & 1 \\ c & 1 \end{vmatrix}$ = a − c 2

$m_1=-\frac{x}{by}$; $m_2=-\frac{cx}{dy}$ 1

$\left(-\frac{ax}{by}\right)\left(-\frac{cx}{dy}\right)$=-1 1

$\therefore \frac{1}{a}-\frac{1}{b}=\frac{1}{c}-\frac{1}{d}$ 1

(b)

p	q	r	¬ q	¬qVr	p→(¬qVr)	¬ p	¬ pV(¬qVr)
T	T	T	F	T	T	F	T
T	T	F	F	F	F	F	F
T	F	T	T	T	T	F	T
T	F	F	T	T	T	F	T
F	T	T	F	T	T	T	T
F	T	F	F	F	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

1 1 1 1 1

43. (a)	$x = \cos\alpha + i\sin\alpha$; $y = \cos\beta + i\sin\beta$	1
	(i) $y - \frac{1}{xy} = cos(\alpha+\beta) + isin(\alpha+\beta) - cos(\alpha+\beta) + isin(\alpha+\beta)$	1
	$xy - \frac{1}{xy} = 2\, i\sin(\alpha+\beta)$	1
	(iii) $\frac{x^m}{y^n} - \frac{y^n}{x^m} = cos(m\alpha - n\beta) + isin(m\alpha - n\beta) - cos(m\alpha - n\beta) + isin(m\alpha - n\beta)$	1
	$\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i\sin(m\alpha - n\beta)$	1
(b)	$\left(x^2+\frac{1}{x^2}\right) - 10\left(x+\frac{1}{x}\right) + 26 = 0$	1
	$(y^2-2) - 10y + 26 = 0$ $y=6,\ y=4$	1
	$x^2 - 6x + 1 = 0$; $x = 3 \pm 2\sqrt{2}$	1
	$x^2 - 4x + 1 = 0$; $x = 2 \pm \sqrt{3}$	1
	The solution are $3+2\sqrt{2}$, $3-2\sqrt{2}$, $2+\sqrt{3}$, $2-\sqrt{3}$	1
44. (a)	Diagram	1
	$x^2 - x - 2 = 0$; The point of intersections are (2,3) and (-1,0)	1
	Area = $= \int_{-1}^{2} [(x+1) - (x^2-1)]\, dx$	1
	$= \frac{9}{2} sq.units$	2
(b)	Diagram	1
	$A = (x+3)(y+2)$	1
	$A' = 2 - \frac{72}{x^2}$ $A'' = \frac{144}{x^3}$	1
	$x = 6$, $A'' = \frac{144}{x^3} = \frac{144}{6^3} > 0$	1
	The dimensions of the paper are Length =6 cm, Breadth= 6cm	1
45. (a)	Condition: $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$	1
	$\text{L.H.S} = \begin{vmatrix} -1 & 1 & 1 \\ 1 & 1 & 3 \\ -3 & 2 & 1 \end{vmatrix} = 0$	1
	Therefore the two lines are coplanar,	1

<table>
<tr><td></td><td>Cartesian equation of the plane
$x + 2y - z - 4 = 0$</td><td>2</td></tr>
<tr><td>(b)</td><td>u is not homogeneous function
$f(x, y) = \sin u = \frac{x+y}{\sqrt{x}+\sqrt{y}}$
$f(\lambda x, \lambda y) = \lambda^{\frac{1}{2}} f(x, y)$ ∴ f is homogeneous function with degree $\frac{1}{2}$
$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = \frac{1}{2}f$
$x \cos u \frac{\partial u}{\partial x} + y\cos u \frac{\partial u}{\partial y} = \frac{1}{2} \sin u$
$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2} \tan u$</td><td>1
1
1
1
1</td></tr>
<tr><td>46.
(a)</td><td>$A = Ce^{kt}$
$A = A_0 e^{kt}$
When $t = 50$; $A = 2A_0$ $k = \frac{1}{50}\log 2$
When $t = ?$; $A = A_0 2^{\frac{t}{50}}$ $t = 50\frac{\log 3}{\log 2}$ years</td><td>1
1
1
2</td></tr>
<tr><td>(b)</td><td>∴ x can takes the values are 40, 10 and -20; $n(s) = 12c_2 = 66$
<table>
<tr><td>x</td><td>40</td><td>10</td><td>-20</td><td>Total</td></tr>
<tr><td>f(x)</td><td>$\frac{6}{66}$</td><td>$\frac{32}{66}$</td><td>$\frac{28}{66}$</td><td>1</td></tr>
</table>
$E(x) = \Sigma x f(x) = 0$
$E(x^2) = \Sigma x^2 f(x) = \frac{4000}{11}$
$Var(x) = E(x^2) - [E(x)]^2 = \frac{4000}{11}$</td><td>1
1
1
1
1</td></tr>
<tr><td>47.
(a)</td><td>Diagram
$\frac{x^2}{a^2} + \frac{y^2}{5^2} = 1$
(8,4) lies on the ellipse
$a = \frac{40}{3}$</td><td>1
1
1
1</td></tr>
</table>

	Width of opening = 2a = $\frac{80}{3} = 26.6\ m$	1
(b)	Diagram $\hat{a} = \cos\alpha\vec{i} + \sin\alpha\vec{j}$ $\hat{b} = \cos\beta\vec{i} - \sin\beta\vec{j}$ $\hat{a}.\hat{b} = \cos(\alpha+\beta)\ldots.(1)$ $\hat{a}.\hat{b} = \cos\alpha\cos\beta - \sin\alpha\sin\beta\ldots.(2)$ $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$	1 1 1 1 1

PREPARED BY

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