12

Register No

Half-Yearly Examination - 2023
MATHEMATICS

Time : 3.00 Hrs Marks : 90

PART - 1

Answer all the questions. Choose the correct answer from the given four alternatives. **20 x 1 = 20**

1. If $\rho(A) = \rho([A|B])$ = number of unknowns then the system $AX = B$ of linear equations
a) consistent and has unique solution b) consistent c) consistent and has infinitely many solutions d) Inconsistent

2. If $(AB)^{-1} = \begin{pmatrix} 12 & -17 \\ -19 & 27 \end{pmatrix}$ and $A^{-1} = \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}$ then B^{-1} = a) $\begin{pmatrix} 2 & -5 \\ -3 & 8 \end{pmatrix}$ b) $\begin{pmatrix} 8 & 5 \\ 3 & 2 \end{pmatrix}$ c) $\begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$ d) $\begin{pmatrix} 8 & -5 \\ -3 & 2 \end{pmatrix}$

3. How many roots does the equation $z^2 = \bar{z}$ have? a) 1 b) 2 c) 3 d) 4

4. If α and β are the roots of $x^2 + x + 1 = 0$ then $\alpha^{2020} + \beta^{2020}$ is a) -1 b) -2 c) 1 d) 2

5. The polynomial $x^3 - kx^2 + 9x$ has three real zeros if and only if k satisfies a) $|k| \le 6$ b) $k = 0$ c) $|k| > 6$ d) $|k| \ge 6$

6. For a 5^{th} degree polynomial equations with real coefficients, whcih of the following is possible?
a) it can have 1 real root and 4 purely maginary roots b) it can have 2 real roots and 3 purely imaginary roots
c) it can have 4 real roots and 1 purely imaginary root d) it can have 5 purely imaginary roots

7. $\sin^{-1}\left(\tan\frac{\pi}{4}\right) - \sin^{-1}\left(\sqrt{\frac{3}{x}}\right) = \frac{\pi}{6}$ Then x is a root of the equationa) $x^2 - x - 6 = 0$ b) $x^2 - x - 12 = 0$ c) $x^2 + x - 12 = 0$ d) $x^2 + x - 6 = 0$

8. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$, the value of $x^{2017} + y^{2018} + z^{2019} - \frac{6}{x^{101} + y^{101} + z^{101}}$ is a) 0 b) 1 c) 2 d) 3

9. The locus of a point whose distance from $(-2, 0)$ is $\frac{2}{3}$ times its distance from the line $x = -\frac{9}{2}$ is
a) a circle b) a parabola c) an ellipse d) a hyperbola

10. If P(x, y) by any point on $16x^2 + 25y^2 = 400$ with foci $F_1(3, 0)$ and $F_2(-3, 0)$ then $PF_1 + PF_2$ - is a) 6 b) 8 c) 10 d) 12

11. The distance between the planes $x + 2y + 3z + 7 = 0$ and $2x + 4y + 6z + 7 = 0$ is a) $\frac{\sqrt{7}}{2}$ b) $\frac{7}{2}$ c) $\frac{\sqrt{7}}{2\sqrt{2}}$ d) $\frac{7}{2\sqrt{2}}$

12. The volume of the parallel piped with its edges represented by the vectors $\hat{i} + \hat{j}$, $\hat{i} + 2\hat{j}$, $\hat{i} + \hat{j} + \pi\hat{k}$ a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) π

13. The point of inflection of the curve $y = (x - 1)^3$ is a) (0, 1) b) (1, 0) c) (0, 0) d) (1, 1)

14. Angle between $y^2 = x$ and $x^2 = y$ at the origin is a) $\tan^{-1}\left(\frac{3}{4}\right)$ b) $\tan^{-1}\left(\frac{4}{3}\right)$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{2}$

15. If $g(x, y) = 3x^2 - 5y + 2y^2$, $x(t) = e^t$ and $y(t) = \cos t$ then $\frac{dg}{dt}$ is equal to
a) $6e^{2t} + 5\sin t - 4\cos t \sin t$ b) $6e^{2t} - 5\sin t + 4\cos t \sin t$ c) $3e^{2t} - 5\sin t + 4\cos t \sin t$ d) $3e^{2t} + 5\sin t + 4\cos t \sin t$

16. If $f(x, y) = e^{xy}$, then $\frac{\partial^2 f}{\partial x \partial y}$ is equal to a) xye^{xy} b) $(1 + x)e^{xy}$ c) $(1 + y)e^{xy}$ d) $(1 + xy)e^{xy}$

17. The value of $\int_0^{\pi} \sin^4 x\,dx$ is a) $\frac{3\pi}{2}$ b) $\frac{3\pi}{4}$ c) $\frac{3\pi}{6}$ d) $\frac{3\pi}{8}$

18. If $\int_0^{a} \frac{1}{4 + x^2}dx = \frac{\pi}{8}$ then a - is a) 1 b) 2 c) 3 d) 4

19. The slope at any point of a curve $y = f(x)$ is given by $\frac{dy}{dx} = 3x^2$ and it passes through $(-1, 1)$. Then the equation of the curve is
a) $y = x^3 + 2$ b) $y = x^3 + 5$ c) $y = 3x^2 + 4$ d) $y = 3x^3 + 4$

20. If $\cos x$ is the integrating factor of the linear differential equation $\frac{dy}{dx} + Py = Q$ then P is a) $\tan x$ b) $\cot x$ c) $-\cot x$ d) $-\tan x$

PART - 2

Answer any 7 questions. 30th question must be answered compulsory. **7 x 2 = 14**

21. If $\frac{z + 3}{z - 5i} = \frac{1 + 4i}{2}$, find the complex number z - in rectangular form.

22. Determine the number of positive and negative roots of the equation $x^9 - 5x^8 - 14x^7 = 0$

23. For what value of x, the inequality $\frac{\pi}{2} < \cos^{-1}(3x - 1) < \pi$ holds?

circle ellipse

24. Identify the type of conic section for each of the following equations i) $3x^2 + 3y^2 - 4x + 3y + 10 = 0$ ii) $3x^2 + 2y^2 = 14$

25. Find the magnitude and the direction cosines of the torque about the point $(2, 0, -1)$ of a force $2\hat{i} + \hat{j} - \hat{k}$ whose line of action passes through the origin.

26. Evaluate : $\lim_{x \to 0}\left(\frac{\sin x}{x^2}\right)$

27. Use the linear approximation to find approximate value of $\sqrt[3]{26}$

28. Evaluate the limit $\int_{-\log 2}^{\log 2} e^{-|x|}\, dx$

29. Determine the Order, degree (if exists) of the following differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x \sin\left(\frac{d^2y}{dx^2}\right)$

30. Decrypt the received encoded message (10 1) (6 1) with encryption matrix $\begin{pmatrix} -1 & 1 \\ 2 & 1 \end{pmatrix}$ and the decryption matrix as its inverse, where the system of codes is described by the numbers 1-26 to the letters A-Z respectively and the number 0 to a blank space.

PART - 3

Answer any 7 questions. 40th question must be answered compulsory. **7 x 3 = 21**

31. Solve the following system of equations by Cramer's rule. $\frac{3}{x} + 2y = 12$, $\frac{2}{x} + 3y = 13$

32. Find the value of $\left(\dfrac{1+\sin\frac{\pi}{10}+i\cos\frac{\pi}{10}}{1+\sin\frac{\pi}{10}-i\cos\frac{\pi}{10}}\right)^{10}$

33. Obtain the condition that the roots of $x^3 + px^2 + qx + \gamma = 0$ are in A.P.

34. Find the domain of $\cos^{-1}\left(\frac{2+\sin x}{3}\right)$

35. Find the point of intersection of the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$

36. Write the Maclaurin series expansion for the function $f(x) = \tan^{-1}x, -1 \le x \le 1$

37. If $U(x, y, z) = \log(x^3 + y^3 + z^3)$, find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$

38. Find the area of the region bounded by the y-axis and the parabola $x = 5 - 4y - y^2$

39. Solve $[y(1 - x\tan x) + x^2 \cos x]\, dx - xdy = 0$

40. The Earth is revolving around the Sun in elliptical orbit when Sun is located at one of the focus. If the distance between Sun and the other focus is 575×10^5 km and eccentricity is $\frac{1}{2}$ then find the maximum and minimum distance between the earth and sun in earth's orbit.

PART - 4

Answer all the questions. **7 x 5 = 35**

41. a) Test the consistency of the following system of linear equations and solve it if it is consistent. $x - y + z = -9$, $2x - y + z = 4$, $3x - y + z = 6$, $4x - y + 2z = 7$ (OR) b) If $z = x + iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, show that $x^2 + y^2 + 3x - 3y + 2 = 0$

42. a) Identify the type of conic and find centre, foci, vertices and directrices of the following : $9x^2 - y^2 - 36x - 6y + 18 = 0$
b) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$

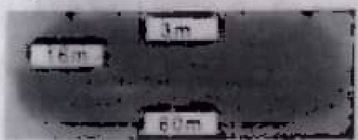
43. a) Find the local extrema of the function $f(x) = 4x^6 - 6x^4$ (OR) b) Solve : $x^4 + 3x^3 - 3x - 1 = 0$

44. a) If $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$, $\vec{b} = 3\hat{i} + 5\hat{j} + 2\hat{k}$, $\vec{c} = -\hat{i} - 2\hat{j} + 3\hat{k}$ then find the following
i) $\vec{a} \times (\vec{b} \times \vec{c})$ ii) $[\vec{a}\ \vec{b}\ \vec{c}]$ iii) $(\vec{a} \cdot \vec{c})\,\vec{b}$ (OR) b) Evaluate : $\int_0^{\pi} \frac{x}{1+\sin x}\, dx$

45. a) A pot of boiling water at 100°C is removed from a stove at time t = 0 and left to cool in the kitchen. After 5 minutes, the water temperature has decreased to 80°C, and another 5 minutes later it has dropped to 65°C. Determine the temperature of the kitchen. (OR)
b) If $a_1, a_2, a_3, \ldots a_n$ is an arithmetic progression with common difference d, prove that
$$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + \ldots \tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right] = \frac{a_n - a_1}{1+a_1a_n}$$

46. a) Find the dimensions of the rectangle with maximum area that can be inscribed in a circle of radius 10cm. (OR)
b) Solve $\left(1+2e^{\frac{x}{y}}\right) dx + 2e^{\frac{x}{y}}\left(1-\frac{x}{y}\right) dy = 0$

47. a) Parabolic cable of a 60m portion of the roadbed of a suspension bridge are positioned as shown below. Vertical cables are to be spaced every 6m along this portion of the roadbed. Calculate the lengths of first two of these vertical cables from the vertex.



(OR) b) Find the vector parametric vector non-parametric and Cartesian form of the equation of the plane passing through the points (-1, 2, 0), (2, 2, -1) and parallel to the straight line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$