COMMON HALF YEARLY EXAMINATION - 2023

A

Standard XII

MATHEMATICS

Reg No. 1 2 B 0 0 1

Time : 3.00 hrs

Marks : 90

Part - A

20 x 1 = 20

I. Choose the correct answer:

1. If $(AB)^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$, then $B^{-1} =$

a) $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$ b) $\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$ c) $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$

2. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$, then λ is

a) 19 b) 14 c) 21 d) 17

3. The product of all the n^{th} roots of unity is

a) 1 b) 0 c) $(-1)^{n-1}$ d) $(-1)^n$

4. If $\frac{z-1}{z+1}$ is purely imaginary, then $|z|$ is

a) $\frac{1}{2}$ b) 1 c) 2 d) 3

5. If α, β and γ are the zero of $x^3 + px^2 + qx + r$, then $\sum \frac{1}{\alpha}$ is

a) $\frac{-q}{r}$ b) $\frac{-p}{r}$ c) $\frac{q}{r}$ d) $\frac{-q}{p}$

6. The period of $y = \sec x$ is

a) π b) 2π c) 3π d) 4π

7. $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$ is equal to

a) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ b) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ c) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ d) $\tan^{-1}\left(\frac{1}{2}\right)$

8. The locus of the point of intersection of perpendicular tangents to the parabola $y^2 = 16x$ is

a) $x = 4$ b) $x = -4$ c) $y = 4$ d) $x + y = 0$

9. The radius of the circle $3x^2 + by^2 + 4bx - 6by + b^2 = 0$ is

a) 1 b) 3 c) $\sqrt{10}$ d) $\sqrt{11}$

10. If $\vec{a}$ and $\vec{b}$ are parallel vectors, then $[\vec{a}\ \vec{c}\ \vec{b}] =$

a) 2 b) -1 c) 1 d) 0

11. The coordinates of the point where the line $\vec{r} = (6\hat{i} - \hat{j} - 3\hat{k}) + t(-\hat{i} - 4\hat{k})$ meets the plane $\vec{r}.(\hat{i} + \hat{j} - \hat{k}) = 3$ are

a) (2, 1, 0) b) (7, -1, -7) c) (1, 2, -6) d) (5, -1, 1)

12. If the direction cosines of a line are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$, then

a) $c = \pm\sqrt{3}$ b) $c = \pm 3$ c) $c > 0$ d) $0 < c < 1$

13. The function $\sin^4 x + \cos^4 x$ is increasing in the interval

a) $\left[\frac{5\pi}{8}, \frac{3\pi}{4}\right]$ b) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ c) $\left[\frac{\pi}{2}, \frac{5\pi}{8}\right]$ d) $\left[0, \frac{\pi}{4}\right]$

14. The point of inflection of the curve $y = (x-1)^3$ is

a) (0, 0) b) (0, 1) c) (1, 0) d) (1, 1)

15. The minimum value of the function $|3-x| + 9$ is

a) 0 b) 3 c) 6 d) 9

16. Linear approximation for $g(x) = \cos x$ at $x = \pi/2$ is

a) $x + \pi/2$ b) $-x + \pi/2$ c) $x - \pi/2$ d) $-x - \pi/2$

17. The value of $\int_{-\pi/2}^{\pi/2} \sin^2 x \cos x \, dx$ is

a) $3/2$ b) $1/2$ c) 0 d) $2/3$

18. The area between $y^2 = 4x$ and its latest rectum is

a) $2/3$ b) $4/3$ c) $8/3$ d) $5/3$

19. The inverse element of $[3] \in Z_5 - \{[0]\}$ under multiplication modulo 5 is

a) [1] b) [2] c) [3] d) [4]

20. The operation * defined by $a * b = \frac{ab}{7}$ is not a binary operation on

a) Q^+ b) Z c) R d) C

Part - B

II. Answer any 7 questions. (Q.No.30 is compulsory) 7 x 2 = 14

21. If $|z| = 2$, show that $3 \le |z + 3 + 4i| \le 7$

22. Discuss the number of positive and negative real roots of $x^5 - 19x^4 + 2x^3 + 5x^2 + 11 = 0$

23. Find the value of $\sec^{-1}\left(\frac{-2\sqrt{3}}{3}\right)$

24. Examine the position of the point (–4, –3) with respect to the circle $x^2 + y^2 - 5x + 2y - 5 = 0$

25. Find the angle between the line $\vec{r} = (2\hat{i} - \hat{j} + \hat{k}) + t(\hat{i} + 2\hat{j} - 2\hat{k})$ and the plane $\vec{r} \cdot (6\hat{i} + 3\hat{j} + 2\hat{k}) = 8$

26. Write the Maclaurin series for $f(x) = e^x$

27. If $w(x, y, z) = x^2y + y^2z + z^2x$, $x, y, z \in R$, find the differential dw.

28 Evaluate $\int_{-\pi/2}^{\pi/2} (x \cos x + 1)\,dx$

29 Let $A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$. Find $A \vee B$

30 Give your own example for a matrix of rank '1' of order 3×3

Part - C

II. Answer any 7 questions. (Q.No.40 is compulsory) $7 \times 3 = 21$

31 If $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$

32 Show that $|3z + 6 + 12i| = 8$ represent a circle and find its centre and radius

33 Find the condition that the roots of cubic equation $x^3 + ax^2 + bx + c = 0$ are in the ratio $p : q : r$

34 Prove that $\frac{\pi}{2} \le \sin^{-1} x + 3\cos^{-1} x \le \frac{5\pi}{2}$

35 Show that the line $x - y + 4 = 0$ is a tangent to the ellipse $x^2 + 3y^2 = 12$. Also find the coordinates of the point of contact

36 Show that the point of intersection of the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{5} = z$ lies on the plane $2x + 3y - z + 4 = 0$

37 Find the linear approximation for $f(x) = \sqrt{1+x}$, $x \ge -1$ at $x_0 = 3$. Use approximation to estimate f(3.2)

38 Evaluate : $\int_0^1 x^5 (1 - x^2)^5\,dx$

39 Verify whether $(p \wedge q) \wedge \sim(p \vee q)$ is tautology or contradiction or contingency

40 Test the point of inflection of the curve $y = x^4$

Part - D

IV. Answer all the questions. $7 \times 5 = 35$

41 a) Investigate the value of λ and μ the system of linear equations $2x + 3y + 5z = 9$, $7x + 3y - 5z = 8$, $2x + 3y + \lambda z = \mu$, have
(i) no solution (ii) a unique solution (iii) an infinite number of solutions

(OR)

b) Solve $z^3 + 27 = 0$

42 a) Solve $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$

(OR)

b) Find the value of $\tan\left(\sin^{-1}\left(\frac{3}{5}\right) + \cot^{-1}\left(\frac{3}{2}\right)\right)$

43. a) Identify the type of conic $9x^2 - y^2 - 36x - 6y + 18 = 0$ and find the centre, foci, vertices and also draw a rough sketch.

(OR)

b) If $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$, $\vec{b} = 3\hat{i} + 5\hat{j} + 2\hat{k}$, $\vec{c} = -\hat{i} - 2\hat{j} + 3\hat{k}$, verify that

$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$

44. a) Find the non-parametric form of vector equation or parametric form of vector equation and cartesian equation of the plane containing the line $\vec{r} = (\hat{i} - \hat{j} + 3\hat{k}) + t(2\hat{i} - \hat{j} + 4\hat{k})$ and perpendicular to the plane $\vec{r}.(\hat{i} + 2\hat{j} + \hat{k}) = 8$

(OR)

b) A conical water tank with vertex down of 12 meters height has a radius of 5 meters at the top. If water flows into the tank at the rate of 10 cubic m/min, how fast is the depth of the water increases when the water is 8 meters deep?

45. a) On lighting a rocket cracker it gets projected in a parabolic path and reaches maximum height of 4 m when it is 6 cm away from the point of projection. Finally it reaches the ground 12 m away from the starting point. Find the angle of projection.

(OR)

b) Find the area of the region bounded by the parabola $y^2 = x$ and the line $y = x - 2$

46. a) For the function $f(x, y) = \tan^{-1}\left(\frac{x}{y}\right)$, show that $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$

(OR)

b) Prove by vector method $\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$

47. a) Prove that the area of the largest rectangle that can be inscribed in a semi circle of radius r is r^2

(OR)

b) Show that $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

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