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Time: 3:00 hrs

Half Yearly Examination - 2023

MATHEMATICS

PART - I

Reg No.

12158

Max Marks: 50

20 x 1 = 20

19
20

i) Answer all the questions.

ii) Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer.

1. If A is a non-singular matrix and $|A| = 5$ then $|A^{-1}|$ is a) 5 b) $\frac{1}{5}$ c) -5 d) $-\frac{1}{5}$

2. $A = \begin{pmatrix} 1 & \tan \frac{\theta}{2} \\ \tan \frac{\theta}{2} & 1 \end{pmatrix}$ and $AB = I_2$, then B = a) $(\cos^2 \frac{\theta}{2}) A$ b) $(\cos^2 \frac{\theta}{2}) A^{-1}$ c) $(\cos^2 \theta) I$ d) $(\sin^2 \frac{\theta}{2}) A$

3. If $|z| = 2$ the maximum value of $|z + 3 + 4i|$ is a) 5 b) 10 c) 17 d) 3

4. If $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$ and $|9z_1 z_2 + 4z_2 z_3 + z_3 z_1| = 12$ then the value of $|z_1 + z_2 + z_3|$ is a) 1 b) $\sqrt{2}$ c) 3 d) 4

5. Solution of $\sqrt{15} - 2x = x$ is a) 3, -5 b) -3, 5 c) 5 d) 3

6. The number of positive zeros of the polynomial $\sum_{r=0}^n nC_r (-1)^r x^r$ is a) 0 b) n c) $< n$ d) 1

7. The value of $\cot^{-1}(-1)$ is a) $-\frac{\pi}{4}$ b) $\frac{\pi}{4}$ c) $\frac{3\pi}{4}$ d) $\frac{3\pi}{4}$

8. If $\cot^{-1}(\sqrt{\sin u}) + \tan^{-1}(\sqrt{\sin u}) = u$ then $\cos 2u$ is equal to a) $\tan^2 u$ b) 0 c) -1 d) $\tan 2u$

9. Condition that the line $lx + my + n = 0$ to be a tangent to the parabola $y^2 = 4ax$ is a) $am^2 = 4ln$ b) $am^2 = 4ln$ c) $am^2 = 4ln$ d) $an^2 = 4lm$

10. The equation of the normal to the circle $x^2 + y^2 - 2x - 2y + 1 = 0$ which is parallel to $2x + 4y = 3$ is a) $x + 2y = 3$ b) $x + 2y + 3 = 0$ c) $2x + 4y + 3 = 0$ d) $x + 2y + 3 = 0$

11. If the straight line passing through the points A(6, 7, 5), B(8, 10, 6) is perpendicular to the straight line passing through the points C(m, 2, 5) and D(8, 3, -4) then the value of m is a) 2 b) 5 c) 8 d) 1

12. If \vec{a} and \vec{b} are parallel vectors then $|\vec{a} \times \vec{b}|$ is equal to a) 2 b) -1 c) 1 d) 0

13. The value of $\frac{e^{-1} - 1}{e^{-1} + 1}$ is a) e b) e^2 c) $e^{\frac{1}{e}}$ d) $\frac{1}{e^2 + 2}$

14. Angle between $y^2 = x$ and $x^2 = y$ at the origin is a) $\tan^{-1} \frac{3}{4}$ b) $\tan^{-1} \frac{4}{3}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{4}$

15. If $w(x, y) = xy + \frac{e^y}{y^2 + 1}$ then $\frac{\partial^2 w}{\partial x \partial y}$ is a) 1 b) 0 c) $\frac{(y^2 + 1)^2 e^y - e^y}{(y^2 + 1)^2}$ d) $\frac{e^y 2y}{e^y + y^2}$

16. Linear approximation for $g(x) = \cos x$ at $x = \frac{\pi}{2}$ is a) $x + \frac{\pi}{2}$ b) $-x + \frac{\pi}{2}$ c) $x - \frac{\pi}{2}$ d) $-x - \frac{\pi}{2}$

17. The value of $\int_0^{\frac{\pi}{4}} \frac{dx}{4 \sin^2 x + 9 \cos^2 x}$ is a) $\frac{1}{6} \tan^{-1} \left(\frac{2}{3} \right)$ b) $\frac{1}{6} \tan^{-1} \left(\frac{3}{2} \right)$ c) $\tan^{-1} \frac{2}{3}$ d) $\tan^{-1} \left(\frac{3}{2} \right)$

18. If $\int_0^x f(t) dt = x - \int_x^1 f(t) dt$ then the value of $f(1)$ is a) $\frac{1}{2}$ b) 2 c) 1 d) $\frac{3}{4}$

19. Solution of $(1 + x^2) dy = (1 + y^2) dx$ is a) $y - x = c(1 - xy)$ b) $y - x = c(1 + x^2 y^2)$ c) $y - x = c(1 - x^2 y^2)$ d) $y - x = c(1 + xy)$

20. If the solution of the differential equation $\frac{dy}{dx} = \frac{ax + 3}{2y + 1}$ represents a circle, then the value of a is a) 2 b) -2 c) 1 d) -1

PART - II

Note: Answer any seven questions. Question No.30 is compulsory

7 x 2 = 14

5
16

21. If $z = x + iy$ then find $\text{Re}(iz)$ in rectangular form

22. Find the value of $\cot^{-1}(-1) + \tan^{-1}(1)$

23. Find the equation and the length of Latus rectum of $y^2 = -8x$.

24. Find the length of the perpendicular drawn from the origin to the plane $r \cdot (3i - 4j + 12k) = 5$
25. If $v(x, y, z) = xy + yz + zx$, $x, y, z \in \mathbb{R}$ find the differential dv
26. Prove that $\int_0^1 e^{-x} x^n dx = n!$ where n is a positive integer
27. Find the differential equation for the family of all straight lines passing through the origin
28. Suppose $f(x)$ is a differentiable function for all x with $f'(x) \leq 29$ and $f(2) = 17$. What is the maximum value of $f(7)$
29. Find the asymptotes of the curve $f(x) = \frac{x^2}{x^2 - 1}$
30. If A is a non-singular square matrix of order n , then prove that $|\text{adj}(\text{adj}A)| = |A|^{(n-1)^2}$

PART - III

Answer any seven questions. Question No. 40 is compulsory.

7 x 3 = 21

31. Find the inverse by Gauss-Jordan method $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$
32. Simplify $(1+i)^{10}$
33. Find the polynomial equation of minimum degree with rational coefficients having $3 - 2\sqrt{2}$ as a root
34. Show that $\cot(\sin^{-1}x) = \frac{\sqrt{1-x^2}}{x}$, $-1 \leq x \leq 1$ and $x \neq 0$
35. If the normal at the point t_1 on the parabola $y^2 = 4ax$ meets the parabola again at the point t_2 , then prove that $t_1 = -\left(t_2 + \frac{2}{t_2}\right)$
36. Find the co-ordinates of the point where the straight line $r = (2i - j) + 2k + \lambda(3i + 4j) + 2k$ intersects the plane $x - y + z - 5 = 0$
37. Find the value of $x \rightarrow \frac{\pi}{2} (\sin x)^{\cos x}$
38. Evaluate $\int_0^{\pi/2} \frac{dx}{5 + 4 \sin^2 x}$
39. Solve $\frac{dy}{dx} = \frac{x - y + 5}{2(x - y) + 7}$
40. If $U = \sin(xy^2)$ and $x = \log t$, $y = e^t$ then find $\frac{du}{dt}$

PART - IV

Answer all the questions

7 x 5 = 35

41. a) Solve by Gaussian elimination method $2x - 2y + 3z = 2$, $x + 2y - z = 3$, $3x - y + 2z = 1$ (OR) b) If $u = \sin^{-1} \left(\frac{x+y}{\sqrt{x} + \sqrt{y}} \right)$ then prove $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u$
42. a) Find all cube roots of $\sqrt{3} - i$ (OR) b) Find the centre, foci and vertices of the hyperbola $x^2 - 3y^2 + 6x + 6y + 18 = 0$
43. a) Show that the line $5x + 12y = 9$ is a tangent to the hyperbola $x^2 - 9y^2 = 9$ (OR)
b) Prove by vector method that the perpendiculars (altitudes) from the vertices to the opposite sides of a triangle are concurrent.
44. a) Solve $(2x - 1)(x + 3)(x - 2)(2x + 3) + 20 = 0$ (OR)
b) A conical water tank with vertex down of 12 metres height has a radius 5 metres at the top. If water flows into the tank at the rate of 10 cubic m/min how fast is the depth of a water increases when the water is 8m deep?
45. a) Find the number of solutions of the equation $\tan^{-1}(x-1) + \tan^{-1}x + \tan^{-1}(x+1) = \tan^{-1}(3x)$ (OR)
b) A manufacturer wants to design an open box having a square base and a surface area of 108 sq cm. Determine the dimensions of the box for the maximum volume.
46. a) If a plane makes an intercepts 3, 4 and 5 respectively on the co-ordinate axes then find the parametric vector equation and cartesian equation of that plane (OR)
b) Find the area of the region bounded by the parabola $y^2 = x$ and the line $y = x - 2$
47. a) Evaluate $\int \frac{\cos^2 x}{1 + \sin^4 x} dx$ (OR) b) Solve $(1 + x + xy^2) \frac{dy}{dx} + (y + y^2) = 0$