HSL

HALF YEARLY EXAMINATION - 2023

12 - Std

MATHEMATICS

Time : 3.00 Hrs Marks : 90

PART - A

CHOOSE THE CORRET ANSWER :- 20 X 1 = 20

1. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$, then λ is
   (a) 17 (b) 14 (c) 19 (d) 21

2. Let $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and $4B = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{bmatrix}$. If B is the inverse of A, then the value of x is (a) 2 (b) 4 (c) 3 (d) 1

3. The principal argument of $\frac{3}{-1+i}$ is
   (a) $\frac{-5\pi}{6}$ (b) $\frac{-2\pi}{3}$ (c) $\frac{-3\pi}{4}$ (d) $\frac{-\pi}{2}$

4. According to the rational root theorem, which number is not possible rational root of $4x^7 + 2x^4 - 10x^3 - 5$?
   (a) -1 (b) $\frac{5}{4}$ (c) $\frac{4}{5}$ (d) 5

5. $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$ is equal to
   (a) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ (b) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ (c) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ (d) $\tan^{-1}\left(\frac{1}{2}\right)$

6. The eccentricity of the ellipse $(x-3)^2 + (y-4)^2 = \frac{y^2}{9}$ is
   (a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{3\sqrt{2}}$ (d) $\frac{1}{\sqrt{3}}$

7. The radius of the circle passing through the point (6,2) two of whose diameters are $x + y = 6$ and $x + 2y = 4$ is
   (a) 10 (b) $2\sqrt{5}$ (c) 6 (d) 4

8. If $\vec{a}, \vec{b}, \vec{c}$ are three unit vectors such that $\vec{a}$ is perpendicular to $\vec{b}$, and is parallel to $\vec{c}$ then $\vec{a} \times (\vec{b} \times \vec{c})$ is equal to
   (a) $\vec{a}$ (b) $\vec{b}$ (c) $\vec{c}$ (d) $\vec{0}$

9. If the distance of the point $(1,1,1)$ from the origin is half of its distance from the plane $x + y + z + k = 0$, then the values of k are
   (a) ± 3 (b) ± 6 (c) $-3, 9$ (d) $3, -9$

10. The slope of the line normal to the curve $f(x) = 2\cos 4x$ at $x = \frac{\pi}{12}$ is
    (a) $-4\sqrt{3}$ (b) -4 (c) $\frac{\sqrt{3}}{12}$ (d) $4\sqrt{3}$

11. The maximum value of the function $x^2 e^{-2x}, x > 0$ is

(a) $\frac{1}{e}$ (b) $\frac{1}{2e}$ (c) $\frac{1}{e^2}$ (d) $\frac{4}{e^4}$

12. If $w(x, y) = x^y, x > 0$, then $\frac{\partial w}{\partial x}$ is equal to

(a) $x^y \log x$ (b) $y \log x$ (c) yx^{y-1} (d) $x \log y$

13. The change in the surface area $S = 6x^2$ of a cube when the edge length varies from x_0 to $x_0 + dx$ is

(a) $12x_0 + dx$ (b) $12x_0 dx$ (c) $6x_0 dx$ (d) $6x_0 + dx$

14. The volume of solid of revolution of the region bounded by $y^2 = x(a - x)$ about x-axis is

(a) πa^3 (b) $\frac{\pi a^3}{4}$ (c) $\frac{\pi a^3}{5}$ (d) $\frac{\pi a^3}{6}$

15. The number of arbitrary constants in the general solutions of order n and $n + 1$ are respectively

(a) $n - 1, n$ (b) $n, n + 1$ (c) $n + 1, n + 2$ (d) $n + 1, n$

16. The value of $\int_{-1}^{2} |x^3 - x| \, dx$ is

(a) $\frac{11}{4}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{13}{4}$

17. The degree of the differential equation $\left(\frac{dy}{dx}\right)^2 = sin\left(\frac{dy}{dx}\right)$ is

(a) 1 (b) 2 (c) 1,2 (d) not defined

18. $\tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{8}\right)$ is equal to

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{8}$

19. The interval in which $y = x^2 e^{-x}$ is increasing is

(a) $(-\infty, \infty)$ (b) $(-2, 0)$ (c) $(2, \infty)$ (d) $(0, 2)$

20. Let z_1, z_2, and z_3 are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$. If $z_1 = 1 + i\sqrt{3}$, then z_2 is

(a) 1 (b) -2 (c) -1 (d) 2

PART – B

ANSWER ANY SEVEN QUESTIONS (Q.NO : 30 IS COMPULSORY) :- 7 X 2 = 14

21. Prove that z is real if and only if $z = \bar{z}$

22. Show that the equation $x^9 - 5x^5 + 4x^4 + 2x^2 + 1 = 0$ has atleast 6 imaginary solutions

23. Find the value of $sin^{-1}\left(sin\frac{5\pi}{9}cos\frac{\pi}{9} + cos\frac{5\pi}{9}sin\frac{\pi}{9}\right)$.

24. If $y = 4x + c$ is a tangent to the circle $x^2 + y^2 = 9$ find c.

25. A particle acted on by constant forces $8\hat{i} + 2\hat{j} - 6\hat{k}$ and $6\hat{i} + 2\hat{j} - 2\hat{k}$ is displaced from the point $(1,2,3)$ to the point $(5,4,1)$. Find the total work done by the forces

26. Evaluate : $\lim_{x \to 0}\left(\frac{\sin mx}{x}\right)$.

27. Find the intervals of monotonicity for the function $f(x) = x^{\frac{2}{3}}$.

28. A sphere is made of ice having radius 10 cm. Its radius decreases from 10 cm to 9.8 cm Find approximate change in the surface area

29. Find the differential equation corresponding to the family of curves represented by the equation $y = Ae^{8x} + Be^{-8x}$, where A and B are arbitrary constants.

30. Evaluate : $\int_{-1}^{1} log\left(\frac{5-x}{5+x}\right) dx$

PART – C

ANSWER ANY SEVEN QUESTIONS (Q.NO : 40 IS COMPULSORY) :- 7 X 3 = 21

31. Find the rank of the matrix $\begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$

32. Solve , by Cramer's rule the system of linear equations $3x + 2y + 5z = 6$, $3x + 3y + 6z = 18$, $x + y + 2z = 1$.

33. Show that the equation $z^2 = \bar{z}$ has four solutions.

34. Find the sum of squares of roots of the equation $2x^4 - 8x^3 + 6x^2 - 3 = 0$

35. Find the domain of $cos^{-1}\left(\frac{2+sin\,x}{3}\right)$.

36. A rod of length 1.2m moves with its ends always touching the coordinate axes. The locus of a point P on the rod, which is 0.3m from the end in contact with x-axis is an ellipse. Find the eccentricity.

37. Find the points on the curve $y^2 - 4xy = x^2 + 5$ for which the tangent is horizontal.

38. Evaluate : $\int_0^{\infty} \frac{x^n}{n^x} dx$, where n is a positive integer ≥ 2.

39. Solve : $sin\frac{dy}{dx} = a$, $y(0) = 1$

40. For which values of m , the vectors $\vec{a} = \hat{i} + \hat{j} + m\hat{k}$, $\vec{b} = \hat{i} + \hat{j} + (m+1)\hat{k}$, $\vec{c} = \hat{i} - \hat{j} + m\hat{k}$ are coplanar.

PART – D

ANSWER ALL THE QUESTIONS :- 7 X 5 = 35

41 a. Investigate for what values of λ and μ the system of linear equations $x + 2y + z = 7$, $x + y + \lambda z = \mu$, $x + 3y - 5z = 5$ has (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

(OR)

b. If $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = \hat{i} - \hat{j} - 4\hat{k}$, $\vec{c} = 3\hat{j} - \hat{k}$ and $\vec{d} = 2\hat{i} + 5\hat{j} + \hat{k}$, verify that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}, \vec{b}, \vec{d}]\vec{c} - [\vec{a}, \vec{b}, \vec{c}]\vec{d}$

42 a. If $z = x + iy$ is a complex number such that $Im\left(\frac{2z+1}{iz+1}\right) = 0$, show that $2x^2 + 2y^2 + x - 2y = 0$

(OR)

b. Evaluate : $\int_0^{\pi} x[sin^2(sin\,x) + cos^2(cos\,x)]\,dx$.

43 a. Solve : $6x^5 + x^4 - 43x^3 - 43x^2 + x + 6 = 0$.

(OR)

b. If $v(x, y) = log\left(\frac{x^2+y^2}{x+y}\right)$, prove that $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = 1$

44 a. If $cos^{-1} x + cos^{-1} y + cos^{-1} z = \pi$ and $0 < x, y, z < 1$, then show that $x^2 + y^2 + z^2 + 2xyz = 1$.

(OR)

b. Find the area of the region bounded by the curve $2 + x - x^2 + y = 0$, x-axis, $x = -3$ and $x = 3$.

45a. Find the centre, vertices, foci, and the length of latus rectum of the conics $4x^2 + y^2 + 24x - 2y + 21 = 0$.

(OR)

b. Let $z(x, y) = x\, tan^{-1}(xy)$, $x = t^2$, $y = se^t$, $s, t \in \mathbb{R}$. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ at $s = t = 1$

46a. If the straight lines $\frac{x-1}{2} = \frac{y+1}{\lambda} = \frac{z}{2}$ and $\frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{\lambda}$ are coplanar, find λ and Cartesian equation of the plane containing these two lines.

(OR)

b. A manufacturer wants to design an open box having a square base and a surface area of 108 sq.cm. Determine the dimensions of the box for the maximum volume.

47a. Solve : $\frac{dy}{dx} - 3y\, cotx = sin2x$ given that $y = 2$ when $x = \frac{\pi}{2}$

(OR)

b. A police jeep **A** is travelling from west at 50 km/hr and car **B** is travelling towards north at 60 km/hr. Both are headed for the intersection of the two roads. At what rate are the cars approaching each other when car **A** is 0.3 kilometres and car **B** is 0.4 kilometres from the intersection ?

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