COMMON HALFYEARLY EXAMINATION - 2023 | Reg. No. 1 2 B 0 9

XII - MATHEMATICS

Time Allowed : 3.00 Hrs. Maximum Marks: 90

Part - I

I. Choose the correct answer: **20 x 1 = 20**

1. If $(AB)^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$, then B^{-1}

   a) $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$ b) $\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$ c) $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$

2. If $\omega \neq 1$ is a cubic root of unity and $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2-1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$, then k =

   a) 1 b) -1 c) $\sqrt{3}i$ d) $-\sqrt{3}i$

3. If $x^3 + 12x^2 + 10ax + 1999$ definitely has a positive zero, if and only if

   a) $a \geq 0$ b) $a > 0$ c) $a < 0$ d) $a \leq 0$

4. The value of $\cos^{-1}(\cos x)$, $0 \leq x \leq \pi$ is

   a) $-x$ b) $x - \pi/2$ c) x d) π

5. If $x + y = k$ is a normal to the parabola $y^2 = 12x$, then the value of k is

   a) 3 b) -1 c) 1 d) 9

6. If the planes $\vec{r}.(2\hat{i} - \lambda\hat{j} + \hat{k}) = 3$ and $\vec{r}.(4\hat{i} + \hat{j} - \mu\hat{k}) = 5$ are parallel, then the value of λ and μ are

   a) $\frac{1}{2}, -2$ b) $-\frac{1}{2}, 2$ c) $-\frac{1}{2}, -2$ d) $\frac{1}{2}, 2$

7. The minimum value of the function $|3 - x| + 9$ is

   a) 0 b) 3 c) 6 d) 9

8. The percentage error of fifth root of 31 is approximately how many times the percentage error in 31?

   a) $\frac{1}{31}$ b) $\frac{1}{5}$ c) 5 d) 31

9. The value of $\int_{-1}^{2} |x|\, dx$ is

   a) $\frac{1}{2}$ b) $\frac{3}{2}$ c) $\frac{5}{2}$ d) $\frac{7}{2}$

10. If $P(X = 0) = 1 - P(X = 1)$. If $E[X] = 3Var(X)$, then $P(X = 0)$ is

    a) $\frac{2}{3}$ b) $\frac{2}{5}$ c) $\frac{1}{5}$ d) $\frac{1}{3}$

11. The order and degree of the differential eqn $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/3} + x^{1/4} = 0$ are respectively

    a) 2,3 b) 3,3 c) 2,6 d) 2,4

12. In the last column of the truth table for $\neg(p \vee \neg q)$ the number of final outcomes of the truth value 'F' are

    a) 1 b) 2 c) 3 d) 4

13. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and $A(\text{adj } A) = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, then k =

a) 0 b) $\sin\theta$ b) $\cos\theta$ d) 1

14. Area between the parabola $y^2 = 4x$ and its latus rectum is

a) $\frac{2}{3}$ b) $\frac{4}{3}$ c) $\frac{8}{3}$ d) $\frac{5}{3}$

15. A is a order of non singular matrix then $|adjA|$ =

a) $|A|$ b) $|A|^{n-2}$ c) $|A|^{n-1}$ d) $|A|^{(n-1)^2}$

16. If $x^2 + 1 = 0$ then x = ?

a) ± 1 b) $\pm i$ c) 0 d) ± 2

17. A zero of $x^3 + 216$ is

a) 0 b) 6 c) 6i d) –6

18. If $2\hat{i} - \hat{j} + 3\hat{k}$, $3\hat{i} + 2\hat{j} + \hat{k}$, $\hat{i} + m\hat{j} + 4\hat{k}$ are coplanar, find the value of m.

a) 3 b) –3 c) 2 d) –2

19. $\int_{-\pi/2}^{\pi/2} \tan x \, dx$ =

a) 1 b) –1 c) $\frac{\pi}{2}$ d) 0

20. * is a binary operation then define $a * b = \frac{ab}{7}$ a,b∈Q if a = 7, b = 12, find a * b = ?

a) 10 b) 12 c) 7 d) –12

Part - II

II. Answer any 7 questions. (Q.No.30 is compulsory) **7 x 2 = 14**

21. Solve by matrix inversion method $2x - y = 8$, $3x + 2y = -2$

22. Find the square root of $-5 - 12i$

23. Discuss the maximum possible number of positive and negative roots of the polynomial equation $9x^9 - 4x^8 + 4x^7 - 3x^6 + 2x^5 + x^3 + 7x^2 + 7x + 2 = 0$.

24. Find the value of $2\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)$

25. If $y = 4x + c$ isa tangent to the circle $x^2 + y^2 = 9$, find c.

26. Find the angle between the line $\vec{r} = (2\hat{i} - \hat{j} + \hat{k}) + t(\hat{i} + 2\hat{j} - 2\hat{k})$ and the plane $\vec{r}.(6\hat{i} + 3\hat{j} + 2\hat{k}) = 8$

27. Evaluate the limit $\lim_{x \to 0} \left(\frac{\sin 5x}{x}\right)$

28. Prove that $\int_0^{\infty} e^{-x} x^n \, dx = n!$, where n is a positive integer.

29. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ be any two Boolean matrices of the same type. Find $A \vee B$ and $A \wedge B$.

30. If $u(x,y,z) = \log(e^{2x} + e^{2y} + e^{2z})$, find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$

Part - III

III. Answer any 7 questions. (Q.No.40 is compulsory) **7 x 3 = 21**

31. Find the value of $\left(\dfrac{1 + \sin\frac{\pi}{10} + i\cos\frac{\pi}{10}}{1 + \sin\frac{\pi}{10} - i\cos\frac{\pi}{10}} \right)^{10}$

32. If the equations $x^2 + px + q = 0$ and $x^2 + p'x + q' = 0$ have a common root, show that it must be equal to $\dfrac{pq'-p'q}{q-q'}$ or $\dfrac{q-q'}{p'-p}$

33. Solve: $\sin^{-1}x > \cos^{-1}x$

34. If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{c} = 3\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{a} \times (\vec{b} \times \vec{c}) = l\vec{a} + m\vec{b} + n\vec{c}$, find the values of l, m, n.

35. Find the rank of matrix: $\begin{bmatrix} 4 & 3 & 1 & -2 \\ -3 & -1 & -2 & 4 \\ 6 & 7 & -1 & 2 \end{bmatrix}$

36. Show that $\neg(p \to q) \equiv p \wedge \neg q$

37. Show that the percentage error in the n^{th} root of a number is approximately $1/n$ times the percentage error in the number.

38. Evaluate : $\int_0^{\pi/2} \begin{vmatrix} \cos^4 & x & 7 \\ \sin^5 & x & 3 \end{vmatrix} dx$

39. Suppose a discrete random variable can only take the values 0, 1 and 2. The probability mass function is defined by

$$f(x) = \begin{cases} \frac{x^2+1}{k}, & \text{for } x = 0,1,2 \\ 0, & \text{otherwise} \end{cases}$$, then find the value of k ?

40. Write the Maclaurin series expansion of the following functions: $\tan^{-1}(x)$; $-1 \le x \le 1$

Part - IV

IV. Answer all the questions. **7 x 5 = 35**

41. a) If $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find the products AB and BA and hence solve the system of equations $x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$ ***(OR)***

b) A conical water tank with vertex down of 12 metres height has a radius of 5 metres at the top. If water flows into the tank at a rate 10 cubic m/min, how fast is the depth of the water increases when the water is 8 metres deep?

42. a) If $z = x + iy$ and $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$, show that $x^2 + y^2 = 1$ ***(OR)***

b) Find the area of the region common to the circle $x^2 + y^2 = 16$ and the parabola $y^2 = 6x$.

43. a) A commuter train arrives punctually at a station every half an hour. Everyday in the morning, a student leaves his house to the train station. Let X denote the amount of time, in minutes, that the student waits for the train from the time he reaches the train station. Its known that the pdf of X is

$f(x) = \begin{cases} \frac{1}{30}, & 0 < x < 30 \\ 0, & \text{elsewhere} \end{cases}$. Obtain and interpret the expected value of the random variable X.

(OR)

b) If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0$

44. a) Find parametric form of vector equation and Cartesian equations of the plane passing through the points (2,2,1), (1,–2,3) and parallel to the straight line passing through the points (2,1,–3) and (–1,5,–8).

(OR)

b) Let $M = \left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in R - \{0\}\right\}$ and let * be the matrix multiplication. Determine whether M is closed under *. If so, examine the closure, commutative, associative, existence of identity and inverse properties.

45. a) Prove that $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$

(OR)

b) Prove that $\int_0^{\pi/4}\log(1+\tan x)dx = \frac{\pi}{8}\log 2$

46. a) A tunnel through a mountain for a four lane highway is to have a elliptical opening. The total width of the highway (not the opening) is to be 16m, and the height at the edge of the road must be sufficient for a truck 4m high to clear if the highest point of opening is to be 5m approximately. How wide must the opening be?

(OR)

b) Find the dimensions of the largest rectangle that can be inscribed in a semi circle of radius r cm.

47. a) By vector method, prove that cos(A + B) = cos Acos B – sinAsinB

(OR)

b) The rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number present. Given that the number triples in 5 hours, find how many bacteria will be present after 10 hours?