Class : 12 Register Number

COMMON HALF YEARLY EXAMINATION-2023-24

MATHEMATICS

Time Allowed : 3.00 Hours] [Max. Marks : 90

Part – I

Answer all the Questions : **20X1=20**

1. If $A = \begin{vmatrix} 2 & 0 \\ 1 & 5 \end{vmatrix}$ and $B = \begin{vmatrix} 1 & 4 \\ 2 & 0 \end{vmatrix}$ then $|adjAB| =$

(1) -40 (2) -80 (3) -60 (4) -20

2. If $x^a y^b = e^m$, $x^c y^d = e^n$, $\Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}$, $\Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, then the values of x and y are respectively

(1) $e^{\left(\frac{\Delta_2}{\Delta_1}\right)}, e^{\left(\frac{\Delta_3}{\Delta_1}\right)}$ (2) $\log\left(\frac{\Delta_1}{\Delta_3}\right), \log\left(\frac{\Delta_2}{\Delta_1}\right)$

(3) $\log\left(\frac{\Delta_2}{\Delta_1}\right), \log\left(\frac{\Delta_3}{\Delta_1}\right)$ (4) $e^{\left(\frac{\Delta_1}{\Delta_3}\right)}, e^{\left(\frac{\Delta_2}{\Delta_3}\right)}$

3. If $|z| = 1$, then the value of $\frac{1+z}{1+\bar{z}}$ is

(1) z (2) $\bar{z}$ (3) $\frac{1}{z}$ (4) 1

4. If z is a complex number such that $z \in \mathbb{C}\backslash\mathbb{R}$ and $z + \frac{1}{z} \in \mathbb{R}$ then $|z|$ is

(1) 0 (2) 1 (3) 2 (4) 3

5. A zero of $x^3 + 64i$ is

(1) 0 (2) 4 (3) $4i$ (4) -4

6. If $x^3 + 12x^2 + 10ax + 1999$ definitely has positive zero, if and only if

(1) $a \geq 0$ (2) $a > 0$ (3) $a < 0$ (4) $a \leq 0$

7. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is

(1) $[1, 2]$ (2) $[-1, 1]$ (3) $[0, 1]$ (4) $[-1, 0]$

8. The radius of the circle $3x^2 + by^2 + 4bx - 6by + b^2 = 0$ is

(1) 1 (2) 3 (3) $\sqrt{10}$ (4) $\sqrt{11}$

9. The angle between the lines $\frac{x-2}{3} = \frac{y+1}{-2}, z = 2$ and $\frac{x-1}{1} = \frac{2y+3}{3} = \frac{z+5}{2}$ is

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

10. Distance from the origin to the plane $3x - 6y + 2z + 7 = 0$ is

(1) 0 (2) 1 (3) 2 (4) 3

11. Angle between $y^2 = x$ and $x^2 = y$ at the origin is

(1) $\tan^{-1}\left(\frac{3}{4}\right)$ (2) $\tan^{-1}\left(\frac{4}{3}\right)$ (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{4}$

12. The value of the limit $\lim_{x\to 0}\left(\cot x - \frac{1}{x}\right)$ is

(1) 0 (2) 1 (3) 2 (4) ∞

13. The percentage error of fifth root of 31 is approximately how many times the percentage error in 31?

(1) $\frac{1}{31}$ (2) $\frac{1}{5}$ (3) 5 (4) 31

14. The approximate change in the volume V of a cube of side x metres caused by increasing the side by 1% is

(1) $0.3xdx\ m^3$ (2) $0.03x\ m^3$ (3) $0.03x^2\ m^3$ (4) $0.03x^3\ m^3$

15. The value of $\int_{-1}^{2}|x|dx$ is

(1) $\frac{1}{2}$ (2) $\frac{3}{2}$ (3) $\frac{5}{2}$ (4) $\frac{7}{2}$

16. The differential equation of the family of curves $y = Ae^x + Be^{-x}$, where A and B are arbitary constants is

(1) $\frac{d^2y}{dx^2} + y = 0$ (2) $\frac{d^2y}{dx^2} - y = 0$ (3) $\frac{dy}{dx} + y = 0$ (4) $\frac{dy}{dx} - y = 0$

17. If the function $f(x) = \frac{1}{12}$ for $a < x < b$, represents a probability density function of continuous random variable X, then which of the following cannot be the value of a and b?

(1) 0 and 1 (2) 5 and 17 (3) 7 and 19 (4) 16 and 24

18. Which one of the following is a binary operation on $\mathbb{N}$?

(1)Subtraction (2)Multiplication (3)Division (4)All of the above

19. If cosx is the integrating factor of the linear diferential equation $\frac{dy}{dx} - Py = Q$ then P is

(1) $\log\sin x$ (2) $\cos x$ (3) $\tan x$ (4) $\cot x$

20. The value of $\int_0^{\pi} cos^4xdx$ is

(1) $\frac{3\pi}{10}$ (2) $\frac{3\pi}{8}$ (3) $\frac{3\pi}{4}$ (4) $\frac{3\pi}{2}$

PART -B

Answer any 7 of the following questions. Question no.30 is compulsory: 7X2=14

21. If $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, verify that $A(adjA) = (adjA)A = |A|I$.

22. Show that $(2 + i\sqrt{3})^{10} - (2 - i\sqrt{3})^{10}$ is purely imaginary

23. Find a polynomial equation of minimum degree with rational coefficients, having $2 - \sqrt{3}$ as a root.

24. Find the principal value of $sin^{-1}(2)$, if it exists.

25. For any vector $\vec{a}$, prove that $\hat{i} \times (\vec{a} \times \hat{i}) + \hat{j} \times (\vec{a} \times \hat{j}) + \hat{k} \times (\vec{a} \times \hat{k}) = 2\vec{a}$

26. Evaluate the limit $\lim_{x \to 0} \left(\frac{\sin x}{x^2}\right)$.

27. If $u(x,y) = \frac{x^2+y^2}{\sqrt{x+y}}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$.

28. Determine the order and degree (if exists) of the differential equation

$$dy + (xy - \cos x)dx = 0$$

29. Find the constant C such that the function $f(x) = \begin{cases} Cx^2, 1 < x < 4 \\ 0, \text{ Otherwise} \end{cases}$ is a density function.

30. Solve: $(1+x)\frac{dy}{dx} = 1 + y$

PART - C

Answer any 7 of the following questions. Question no.30 is compulsory: **7X3=21**

31. Find the rank of the matrix $\begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$ by reducing it to an echelon form.

32. Solve the equation $z^3 + 27 = 0$.

33. Solve the equation $x^4 - 9x^2 + 20 = 0$.

34. Find the value of $tan^{-1}(-1) + cos^{-1}\left(\frac{1}{2}\right) + sin^{-1}\left(-\frac{1}{2}\right)$.

35. Prove that the length of the latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{2b^2}{a}$.

36. Find the torque (moment) of the resultant of the three forces represented by $-3\hat{\imath} + 6\hat{\jmath} - 3\hat{k}, 4\hat{\imath} - 10\hat{\jmath} + 12\hat{k}$ and $4\hat{\imath} + 7\hat{\jmath}$ acting at the point with position vector $8\hat{\imath} - 6\hat{\jmath} - 4\hat{k}$, about the point with position vector $18\hat{\imath} + 3\hat{\jmath} - 9\hat{k}$.

37. Prove using mean value theorem that, $|\sin\alpha - \sin\beta| \le |\alpha - \beta|, \alpha, \beta \in \mathbb{R}$.

38. Find the volume of the solid formed by revolving the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$ about the major axis.

39. Using truth table check whether the statements $\neg(p \vee q) \vee (\neg p \wedge q)$ and $\neg p$are logically equivalent.

40. The mean and standard deviation of a binomial variate X are respectively 4 and 1 Find (i) the probability mass function (ii) $P(X = 2)$

PART - D

Answer all the questions.: **7X5=35**

41(a) A boy is walking along the path $y = ax^2 + bx + c$ through the points $(-6,8), (-2,-12)$, and $(3,8)$. He wants to meet his friend at $P(7,60)$. Will he meet his friend? (Use Gaussian elimination method.) (OR)

41(b) If $z = x + iy$ and $arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, show that $x^2 + y^2 + 3x - 3y + 2 = 0$.

42(a) Solve the equation $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$ (OR)

42(b) Let $M = \left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in R - \{0\} \right\}$ and let $*$ be the matrix multiplication. Determine whether M is closed under $*$. If so, examine the closure, commutative, associative, existence of identity and inverse properties.

43(a) Solve : $tan^{-1}\left(\frac{x-1}{x-2}\right) + tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$. (OR)

43(b) Find parametric form of vector equation and Cartesian equations of the plane passing through the points (2,2,1), (1,2,3) and parallel to the straight line passing through the points (2,1,1) and (−1,2,3)

44(a) Prove that $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ (OR)

44(b) A random variable X has the following probability mass function:

X	1	2	3	4	5
$f(x)$	k^2	$2k^2$	$3k^2$	$2k$	$3k$

Find (i) the value of k (ii) $P(2 \le X < 5)$ (iii) $P(3 < X)$

45(a) A ladder 17 metre long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of 5 m/s. When the base of the ladder is 8 metres from the wall. (i)How fast is the top of the ladder moving down the wall? (ii)At what rate, the area of the triangle formed by the ladder, wall, and the floor, is changing? (OR)

45(b) If $u = sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$. Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$.

46(a) Prove that $\int_0^{\frac{\pi}{4}} log(1 + tan\, x)dx = \frac{\pi}{8}log\, 2$. (OR)

46(b)A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain 1,80,000 sq.mtrs in order to provide enough grass for herds. No fencing is needed along the river. What is the length of the minimum needed fencing material?

47(a)The curve $y = (x-2)^2 + 1$ has a minimum point at P. A point Q on the curve is such that the slope of PQ is 2. Find the area bounded by the curve and the chord PQ. (OR)

47(b)Assume that the rate at which radioactive nuclei decay is proportional to the number of such nuclei that are present in a given sample. In a certain sample 10 percentage of the original number of radioactive nuclei have undergone disintegration in a period of 100 years. What radioactive nuclei will remain after 1000 years? (Take the initial amount as A_0)