Class : 12

Register Number

COMMON HALF YEARLY EXAMINATION-2023-24

Time Allowed : 3.00 Hours] **MATHEMATICS** [Max. Marks : 90

PART - I

1. Answer all the questions by choosing the correct answer from the given 4 alternatives **20 x 1 = 20**
2. Write question number, correct option and corresponding answer
3. Each question carries 1 mark

1. If $(AB)^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$, and $A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$, then $B^{-1} =$

(1) $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$ (2) $\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$ (3) $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ (4) $\begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$

2. If $\omega \neq 1$ is a cubic root of unity and $(1+\omega)^7 = A + B\omega$, then (A, B) equals

(1) $(1, 0)$ (2) $(-1, 1)$ (3) $(0, 1)$ (4) $(1, 1)$

3. The polynomial $x^3 - kx^2 + 9x$ has three real zeros if and only if, k satisfies

(1) $|k| \leq 6$ (2) $k = 0$ (3) $|k| > 6$ (4) $|k| \geq 6$

4. If the function $f(x) = \sin^{-1}(x^2 - 3)$, then x belongs to

(1) $[-1, 1]$ (2) $[\sqrt{2}, 2]$ (3) $[-2, -\sqrt{2}] \cup [\sqrt{2}, 2]$ (4) $[-2, -\sqrt{2}]$

5. The length of the diameter of the circle which touches the x-axis at the point $(1,0)$ and passes through the point $(2, 3)$.

(1) $\frac{6}{5}$ (2) $\frac{5}{3}$ (3) $\frac{10}{3}$ (4) $\frac{3}{5}$

6. The radius of the circle passing through the point $(6, 2)$ two of whose diameter are $x + y = 6$ and $x + 2y = 4$ is

(1) 10 (2) $2\sqrt{5}$ (3) 6 (4) 4

7. The volume of the parallelepiped with its edges represented by the vectors $\hat{i} + \hat{j}, \hat{i} + 2\hat{j}, \hat{i} + \hat{j} + \pi\hat{k}$ is

(1) $\pi/2$ (2) $\pi/3$ (3) π (4) $\pi/4$

8. The distance between the planes $x + 2y + 3z + 7 = 0$ and $2x + 4y + 6z + 7 = 0$ is

(1) $\frac{\sqrt{7}}{2\sqrt{2}}$ (2) $\frac{7}{2}$ (3) $\frac{\sqrt{7}}{2}$ (4) $\frac{7}{2\sqrt{2}}$

9. The position of a particle moving along a horizontal line of any time t is given by $s(t) = 3t^2 - 2t - 8$. The time at which the particle is at rest is

(1) $t = 0$ (2) $t = \frac{1}{3}$ (3) $t = 1$ (4) $t = 3$

10. The tangent to the curve $y^2 - xy + 9 = 0$ is vertical when

(1) $y = 0$ (2) $y = \pm\sqrt{3}$ (3) $y = \frac{1}{2}$ (4) $y = \pm 3$

11. If $f(x, y) = e^{xy}$, then $\frac{\partial^2 f}{\partial x \partial y}$ is equal to

(1) xye^{xy} (2) $(1 + xy)e^{xy}$ (3) $(1 + y)e^{xy}$ (4) $(1 + x)e^{xy}$

12. The value of $\int_{-1}^{2} |x|\, dx$ is

(1) $\frac{1}{2}$ (2) $\frac{3}{2}$ (3) $\frac{5}{2}$ (4) $\frac{7}{2}$

13. The integrating factor of the differential equation $\frac{dy}{dx} + P(x)y = Q(x)$ is x . then P(x)

(1) x (2) $\frac{x^2}{2}$ (3) $\frac{1}{x}$ (4) $\frac{1}{x^2}$

14. If $f(x) = \begin{cases} 2x & 0 \le x \le a \\ 0 & \text{otherwise} \end{cases}$ is a probability density function of a random variable, then the value of a is

(1) 1 (2) 2 (3) 3 (4) 4

15. In the last column of the truth table for $\neg(p \vee \neg q)$ the number of final outcomes of the truth value 'F' are

(1) 1 (2) 2 (3) 3 (4) 4

relative

16. If the rank of the matrix $\begin{bmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ -1 & 0 & \lambda \end{bmatrix}$ is 2 then λ is (1) 1 (2) 2 (3) 3 (4) any real number

Eg.1

Creative (Eg.2.16)

17. The number of non zero solutions of the equation $z^2 = \bar{z}$ is

(1) 2 (2) 3 (3) 4 (4) 1

18. $\sin^{-1}(\sin\frac{5\pi}{6}) =$

(1) $\frac{5\pi}{6}$ (2) $\frac{\pi}{6}$ (3) 0 (4) $-\frac{\pi}{6}$

$\frac{dy}{dx} = -mx$

$\int dy = -m\int x\,dx$

19. If $x = r\cos\theta$ and $y = r\sin\theta$ then $\frac{\partial r}{\partial x} =$

(1) $\sin\theta$ (2) $\cos\theta$ (3) $\sec\theta$ (4) $\tan\theta$

20. Solution of $\frac{dy}{dx} + mx = 0$ where m< 0 is

(1) $y = ce^{mx}$ (2) $y = -ce^{mx}$ (3) $x = ce^{my}$ (4) $x = ce^{-my}$

No answer

Crt ans : $\boxed{y = -\frac{mx^2}{2} + C}$

PART – II

1. Answer any 7 questions **7 x 2 = 14**
2. Each question carries 2 marks
3. Question number 30 is compulsory

21. Test for consistency and if possible, solve the following systems of equations by rank method.

$2x + 2y + z = 5$, $x - y + z = 1$, $3x + y + 2z = 4$

Eg.1 22. Simplify $\left(\sin\frac{\pi}{6} + i\cos\frac{\pi}{6}\right)^{18}$.

Eg.1 23. Find solution , if any , of the equation $2\cos^2 x - 9\cos x + 4 = 0$.

24. Find the period and amplitude of $y = 4\sin(-2x)$.

Eg.1 25. Find the general equation of the circle whose diameter is the line segment joining the points (−4,−2) and (1,1).

26. Find the torque of the resultant of the three forces represented by $-3\hat{i} + 6\hat{j} - 3\hat{k}$, $4\hat{i} - 10\hat{j} + 12\hat{k}$, and $4\hat{i} + 7\hat{j}$ acting a the point with position vector $8\hat{i} - 6\hat{j} - 4\hat{k}$, about the point with position vector $18\hat{i} + 3\hat{j} - 9\hat{k}$.

Eg.1 27. Find the local extremum of the function $f(x) = x^4 + 32x$.

28. Evaluate $\lim_{(x,y)\to(0,0)} \cos\left(\frac{x^3+y^2}{x+y+2}\right)$. If the limit exists.

29. Let $A = \{a + \sqrt{5}b : a,b \in Z\}$. Check whether the usual multiplication is a binary operation on A.

30. Solve : $\frac{dy}{dx} + y\cot x = 2\cos x$

PART - III

1. Answer any 7 questions **7 x 3 = 21**
2. Each question carries 3 marks
3. Question number 40 is compulsory

31. A chemist has one solution which is 50% acid and another solution which is 25% acid. How much each should be mixed to make 10 litres of a 40% acid solution ? (Use Cramer's rule to solve the problem).

Eg. 1

32. If z_1, z_2, and z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = |z_1 + z_2 + z_3| = 1$, find the value of $\left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right|$

33. Find the equation of the curve whose slope is $\frac{y-1}{x^2+x}$ and which passes through the point (1,0).

34. Find the equation of the ellipse with length of latus rectum 4, distance between foci $4\sqrt{2}$ and major axis as y - axis.

35. Find the distance of the point $(5, -5, -10)$ from the point of intersection of a straight line passing through the points $A(4,1,2)$ and $B(7,5,4)$ and with the plane $x - y + z = 5$.

36. A beacon makes one revolution every 10 seconds. It is located on a ship which is anchored 5 km from a straight shore line. How fast is the beam moving along the shore line when it makes an angle of 45° with the shore?

37. An urn contains 2 white balls and 3 red balls. A sample of 3 balls are chosen at random from the urn. If X denote the number of red balls chosen, find the values taken by the random variable X and its number of inverse images

Eg. 1

38. Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{4\sin^2 x + 5\cos^2 x}$.

39. Prove that $q \to p \equiv \neg p \to \neg q$

40. Find the value of $\tan\left(2\tan^{-1}\left(\frac{1}{5}\right) - \frac{\pi}{4}\right)$

PART - IV

1. Answer all the questions **7 x 5 = 35**
2. Each question carries 5 marks

Eg. 5

41. a) If the system of equations $px + by + cz = 0$, $ax + qy + cz = 0$, $ax + by + rz = 0$ has a non-trivial solution and $p \neq a, q \neq b, r \neq c$, prove that $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$

(OR)

b) If $z = x + iy$ and $\arg\left(\frac{z-1}{z+2}\right) = \frac{\pi}{4}$, show that $x^2 + y^2 + 3x - 3y + 2 = 0$.

42. a) Solve the equations : $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$

(OR)

b) Find the domain of $\sin^{-1}(2 - 3x^2)$

43. a) On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4m when it is 6m away from the point of projection. Finally it reaches the ground 12m away from the starting point. Find the angle of projection.

(OR)

(OR)

Eg.1 b) Find the vector parametric, vector non-parametric and Cartesian form of the equation of the plane passing through the points $(-1, 2, 0)$, $(2, 2, -1)$ and parallel to the straight line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$

44. a) Find the equation of tangent and normal to the curve given by $x = 7\cos t$ and $y = 2\sin t, t \in R$ at any point on the curve.

(OR)

Eg.1 b) Let $g(x,y) = x^2 - yx + \sin(x+y)$, $x(t) = e^{3t}$, $y(t) = t^2$, $t \in R$. Find $\frac{dg}{dt}$.

45. a) The curve $y = (x-2)^2 + 1$ has a minimum point at P. A point Q on the curve is such that the slope of PQ is 2. Find the area bounded by the curve and the chord PQ.

(OR)

b) A radioactive isotope has an initial mass 200mg, which two years later is 50mg. Find the expression for the amount of the isotope remaining at any time. What is its half-life? (half-life mean the time taken for the radioactivity of a specified isotope to fall to half its original value). Eg.1

46. a) If X is the random variable with probability density function f(x) given by, $f(x) = \begin{cases} x+1, & -1 \le x < 0 \\ -x+1, & 0 \le x < 1 \\ 0, & \text{otherwise} \end{cases}$ then find (i) the distribution function F(x) (ii) $P(-0.5 \le X \le 0.5)$

(OR)

b) Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for the operation $\times_{11}$ on a subset $A = \{1,3,4,5,9\}$ of the set of remainders $\{0,1,2,3,4,5,6,7,8,9,10\}$. Eg. 2.10

47. a) Show that the volume of the largest cone that can be inscribed in a sphere of radius r is $\frac{8}{27}$ volume of sphere

(OR)

b) Find the eccentricity, centre, foci and the vertices of the hyperbola $x^2 - 4y^2 - 8x - 6y - 18 = 0$

Choose

1).

Part II

21). Inconsistant

22). $1+0i$

23). Eg. 3.29

$x = 2n\pi \pm \frac{\pi}{3}$

24). $P = 2\pi$, $A = 4$

25). $x^2+y^2+3x+y-6=0$

$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

26). $\vec{M} = \vec{r} \times \vec{F}$

$= -96\hat{i}+115\hat{j}+15\hat{k}$

27). $(-2,-48)$

28). 1

29). $*$ is binary.

30). $I.F = e^{\int p\,dx}$

$= e^{\int \cot x\,dx}$

$= e^{\log \sin x}$

$\boxed{I.F = \sin x}$

$y(\sin x) = -\frac{\cos 2x}{2} + c$

Part - III

31). 61, 41

32). $|z_1|^2 = 1$

$z_1 \bar{z}_1 = 1$

$\bar{z}_1 = \frac{1}{z_1}$. $\bar{z}_2 = \frac{1}{z_2}$ $\bar{z}_3 = \frac{1}{z_3}$

$\left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right| = 1$

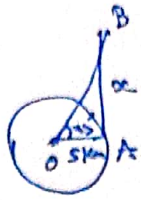
33). $y = \frac{1-x}{x+1}$

34). Given $\frac{2b^2}{a} = 4$ $\frac{x^2}{8}+\frac{y^2}{16}=1$

$2ae = 4\sqrt{2}$

35). 13 units

36). $\frac{dx}{dt} = \frac{\pi}{5}$



$x = 5\tan\theta$

$\boxed{\frac{dx}{dt} = 2\pi}$

37).

x	1	2	3	Tot
Inverse image	3	6	1	10

38). $\frac{\pi}{4\sqrt{5}}$

39).

p	q	$q \to p$	$\sim p$	$\sim q$	$\sim p \to \sim q$
T	T	T	F	F	T
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	T	T

40). $2\tan^{-1}\left[\frac{1}{5}\right] = \tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{5}\right)$

$= \tan^{-1}\left(\frac{5}{12}\right)$

$\tan\left[\tan^{-1}\left(\frac{5}{12}\right) - \tan^{-1}(1)\right]$

$\tan\left[\tan^{-1}\left[\frac{\frac{5}{12}-1}{1+\frac{5}{12}\times 1}\right]\right]$

$= \frac{-7}{17}$

Part IV

41) (a) $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$ (Eg., 1.40)

Expanding $\frac{p}{p-a} + \frac{b}{q-b} + \frac{c}{r-c} = 0$

$\frac{p}{p-a} + \frac{q}{q-b} + \frac{c}{r-c} = 2$

41) (b) $\arg z_1 - \arg z_2 = \arg\left(\frac{z_1}{z_2}\right)$

$\arg[x + i(y-1)] - \arg[x+2+iy] = \frac{\pi}{4}$

$\tan^{-1}\left(\frac{y-1}{x}\right) - \tan^{-1}\left(\frac{y}{x+2}\right) = \frac{\pi}{4}$

H.P.

42 (a). $3, 2, \frac{1}{3}, \frac{1}{2}$

42. (b) Eg 4.4.

$x \in \left[-1, -\frac{1}{\sqrt{3}}\right] \cup \left[\frac{1}{\sqrt{3}}, 1\right]$

43 (a). $x^2 = -4ay$.

$a = \frac{9}{14}$

$\theta = \tan^{-1}\left(\frac{4}{3}\right)$

43 (b).

$\rightarrow \vec{r} = (-\hat{i} + 2\hat{j}) + s(3\hat{i} - \hat{k}) + t(\hat{i} + \hat{j} - \hat{k})$

$\rightarrow [\vec{r} - (-\hat{i} + 2\hat{j})] \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) = 0$

$\rightarrow x + 2y + 3z = 3.$

44 (a)

$x(7\sin t) - y(2\cos t) = 45 \sin t \cos t$

44 (b). ~~sq units~~

$\frac{dg}{dt} = \frac{\partial g}{\partial x}\frac{dx}{dt} + \frac{\partial g}{\partial y}\frac{dy}{dt}$

$= 6e^{6t} - 3t^2 e^{3t} + 3e^{3t}\cos(e^{3t} + t^2)$

$- 2te^{3t} + 2t\cos(e^{3t} + t^2)$

45) (a) $\frac{4}{3}$ sq. units

45) (b) $\boxed{x = ce^{-kt}}$

$k = \frac{1}{2}\log\frac{4}{3}$

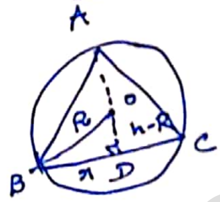
$$\boxed{t_h = \frac{2\log\frac{1}{2}}{\log\frac{3}{4}}}$$

46). (a) (i) $F(x) = \begin{cases} 0 & x \le -1 \\ \frac{x^2}{2} + x + \frac{1}{2} & -1 \le x < 0 \\ -\frac{x^2}{2} + x + \frac{1}{2} & 0 \le x < 1 \\ 1 & 1 \le 0 \end{cases}$

(ii) 0.75

46 (b) Proved.

47 (a). Creative



In $\triangle OBD$,

$R^2 = r^2 + (h-R)^2$

$\boxed{r^2 = 2hR - h^2}$

V of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(2hR - h^2)h$

$= \frac{2\pi h^2 R}{3} - \frac{\pi h^3}{3}$

for max volume

$\frac{dV}{dh} = 0$

$\boxed{h = \frac{4R}{3}}$

$\Rightarrow \quad V = \frac{8}{27} \times \frac{4}{3}\pi R^3$

H.P.

47 (b).

C (4, 2)

e $\frac{\sqrt{3}}{2}$

Focus $(5\sqrt{3}+4, 2)$ $(4-5\sqrt{3}, 2)$

Vertex $(14, 2)$ $(-6, 2)$