ST. ANNE'S ACADEMY

(MATHS TUITION CENTRE)

CLASS – XII – MATHEMATICS

Common Half Yearly Examination – 2023 – 24 (Model Question)

Time Allowed : 3 Hrs Maximum Marks : 90

PART – I

I. Answer ALL questions. **20x1 = 20**

1) If $A=\begin{bmatrix}7 & 3\\ 4 & 2\end{bmatrix}$, then $|A^{-1}|$ =

(1) A^{-1} (2) $\frac{1}{2}$ (3) $\frac{1}{3}$ (4) $2A^{-1}$

2) If $\omega \neq 1$ is a cubic root of unity then, $(1+\omega-\omega^2)^7$ equals

(1) $-\omega^2$ (2) $-128\omega^2$ (3) -128 (4) 128

3) The equation whose roots are opposite in sign to those of $x^2-3x-4=0$ is

(1) $3x^2-3x-4=0$ (2) $x^2+3x-4=0$

(3) $3x^2+3x-4=0$ (4) $x^2+3x+4=0$

4) If $\sin^{-1}x+\sin^{-1}y=\frac{2\pi}{3}$; then $\cos^{-1}x+\cos^{-1}y$ is equal to

(1) $\frac{2\pi}{3}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{6}$ (4) π

5) The equation of the Auxiliary circle to the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ is

(1) $x^2+y^2=16$ (2) $x^2+y^2-6y+7=0$

(3) $x^2+y^2=9$ (4) $x^2+y^2-6y+5=0$

6) If $x+y=k$ is a normal to the parabola $y^2=12x$, then the value of k is

(1) 3 (2) -1 (3) 1 (4) 9

7) If α,β,γ are the angles made by a straight line with the coordinate axes then the value of $\cos 2\alpha+\cos 2\beta+\cos 2\gamma$ equals to.

(1) -2 (2) $\frac{\pi}{3}$ (3) -1 (4) $\frac{\pi}{4}$

8) If $z=x+iy$ is a complex number such that 3x+(3x-y)i =4-6i then the value of z is

(1) $\frac{4}{3}+10i$ (2) 10i (3) 6i (4) 4+6i

9) The polynomial $x^3 - kx^2 + 9x$ has three real zeros if and only if, k satisfies

(1) $|k| \le 6$ (2) $k = 0$ (3) $|k| > 6$ (4) $|k| \ge 6$

10) The minimum value of the function $|3 - x| + 9$ is

(1) 0 (2) 3 (3) 6 (4) 9

11) The point of inflection of the curve $y = (x-1)^3$ is

(1) (0,0) (2) (0,1) (3) (1,0) (4) (1,1)

12) If $f(x,y) = e^{xy}$, then $\frac{\partial^2 f}{\partial x \partial y}$ is equal to

(1) xye^{xy} (2) $(1+xy)e^{xy}$ (3) $(1+y)e^{xy}$ (4) $(1+x)e^{xy}$

13) If $w(x,y,z) = x^2(y-z) + y^2(z-x) + z^2(x-y)$, then $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z}$ is

(1) $xy + yz + zx$ (2) $x(y+z)$ (3) $y(z+x)$ (4) 0

14) The volume of solid of revolution of the region bounded by $y^2 = x(a-x)$ about x-axis is

(1) πa^3 (2) $\frac{\pi a^3}{4}$ (3) $\frac{\pi a^3}{5}$ (4) $\frac{\pi a^3}{6}$

15) If $\frac{\Gamma(n+2)}{\Gamma(n)} = 90$ then n is

(1) 10 (2) 5 (3) 8 (4) 9

16) The value of $\int_{-1}^{2} |x|\, dx$ is

(1) $\frac{1}{2}$ (2) $\frac{3}{2}$ (3) $\frac{5}{2}$ (4) $\frac{7}{2}$

17) The solution of the differential equation $\frac{dy}{dx} + \frac{1}{\sqrt{1-x^2}} = 0$ is

(1) $y + \sin^{-1} x = c$ (2) $x + \sin^{-1} y = 0$ (3) $y^2 + 2\sin^{-1} x = C$ (4) $x^2 + 2\sin^{-1} y = 0$

18) If $\sin x$ is the integrating factor of the linear differential equation $\frac{dy}{dx} + Py = Q$, then P is

(1) $\log \sin x$ (2) $\cos x$ (3) $\tan x$ (4) $\cot x$

19) The domain of the function defined by $f(x)=\sin^{-1} 2x$ is

(1) $[\frac{1}{2}, \frac{1}{2}]$ (2) $[-\frac{1}{2}, \frac{1}{2}]$ (3) $[0, 1]$ (4) $[-1, 0]$

20) The circle passing through $(1,-2)$ and touching the axis of x at $(3,0)$ passing through the point

(1) $(-5,2)$ (2) $(2,-5)$ (3) $(5,-2)$ (4) $(-2,5)$

PART – II

II. Answer any SEVEN questions. [Question 30 is compulsory] 7x2 = 14

21) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 0 & 5 \end{bmatrix}$ by reducing it to a row-echelon form.

22) Find z^{-1}, if $z=(2+3i)(1-i)$.

23) If p and q are the roots of the equation $lx^2+nx+n=0$, show that $\sqrt{\frac{p}{q}}+\sqrt{\frac{q}{p}}+\sqrt{\frac{n}{l}}=0$.

24) Prove that $\tan(\sin^{-1} x)=\frac{x}{\sqrt{1-x^2}}$ for $|x|<1$.

25) A circle of area 9π square units has two of its diameters along the lines $x+y=5$ and $x-y=1$. Find the equation of the circle.

26) Find the angle between the planes $\vec{r}\cdot(\hat{i}+\hat{j}-2\hat{k})=3$ and $2x-2y+z=2$.

27) Let $u(x,y)=e^{-2y}\cos(2x)$ for all $(x,y)\in\mathbb{R}^2$. Prove that u is a harmonic function in $\mathbb{R}^2$.

28) Evaluate $\int_0^1 x\,dx$, as the limit of a sum.

29) For the following differential equation, determine its order, degree (if exists)

$$\left(\frac{d^2y}{dx^2}\right)^3=\sqrt{1+\left(\frac{dy}{dx}\right)}$$

30) Evaluate the following using l'Hôpital Rule :

$$\lim_{x\to\frac{\pi}{2}}\left(\frac{\log\sin x}{(\pi-2x)^2}\right)$$

PART – III

III. Answer any SEVEN questions. [Question 40 is compulsory] **7x3 = 21**

31) Find the equations of the tangent and normal to hyperbola $12x^2 - 9y^2 = 108$ at $\theta = \frac{\pi}{3}$.
(Hint: use parametric form)

32) Find the sum of the squares of the roots of $2x^4 - 8x^3 + 6x^2 - 3 = 0$

33) Find the cube roots of unity.

34) Find the domain of $\sin^{-1}(2-3x^2)$

35) Solve, by Cramer's rule, the system of equations
$x_1 - x_2 = 3, 2x_1 + 3x_2 + 4x_3 = 17, x_2 + 2x_3 = 7.$

36) Assuming $\log_{10} e = 0.4343$, find an approximate value of $\log_{10} 1003$.

37) Evaluate the following

$$\int_0^{\frac{\pi}{2}} \frac{e^{-\tan x}}{\cos^6 x}\, dx$$

38) Show that $y = ae^{-3x} + b$, where a and b are arbitrary constants, is a solution of the differential equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} = 0$.

39) Find the asymptotes of the following curves :

$$f(x) = \frac{x^2+6x-4}{3x-6}$$

40) Find the image of the point whose position vector is $\hat{i} + 2\hat{j} + 3\hat{k}$ in the plane $\vec{r} \cdot (\hat{i} + 2\hat{j} + 4\hat{k}) = 38$. Find also the foot of the perpendicular from the point $\hat{i} + 2\hat{j} + 3\hat{k}$ in the given plane.

PART – IV

IV. Answer ALL questions. **7x5 = 35**

41) a) Suppose z_1, z_2, and z_3 are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$. If $z_1 = 1 + i\sqrt{3}$, then find z_2 and z_3.

OR

b) Evaluate the following using properties of integration :

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\sqrt{\tan x}}\,dx$$

42) a) By using Gaussian elimination method, balance the chemical reaction equation:

$C_2H_6 + O_2 \rightarrow H_2O + CO_2$

OR

b) A hollow cone with base radius a cm and height b cm is placed on a table. Show that the volume of the largest cylinder that can be hidden underneath is $\frac{4}{9}$ times volume of the cone.

43) a) Identify the type of conic and find centre, foci, vertices, and directrices of:

$18x^2 + 12y^2 - 144x + 48y + 120 = 0$

OR

b) If $a_1, a_2, a_3, \ldots a_n$ is an arithmetic progression with common difference d,

prove that $\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+\ldots+\tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right] = \frac{a_n - a_1}{1+a_1a_n}$

44) a) Find the non-parametric form of vector equation and cartesian equation of the plane passing through the point $(1,-2,4)$ and perpendicular to the plane $x+2y-3z=11$ and parallel to the line $\frac{x+7}{3}=\frac{y+3}{-1}=\frac{z}{1}$.

OR

b) Solve : $(2x-1)(x+3)(x-2)(2x+3)+20=0$

45) a) A tank contains 1000 litres of water in which 100 grams of salt is dissolved. Brine runs in a rate of 10 litres per minute, and each litre contains 5grams of dissolved salt. The mixture of the tank is kept uniform by stirring. Brine runs out at 10 litres per minute. Find the amount of salt at any time t.

OR

b) Let $U(x,y)=e^x \sin y$, where $x=st^2,\ y=s^2t,\ \ s,t\in\mathbb{R}$

Find $\frac{\partial U}{\partial s}, \frac{\partial U}{\partial t}$ and evaluate them at $s=t=1$.

46) a) Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent

OR

b) Find the intervals of monotonicity and local extrema of the function

$$f(x) = x\log x + 3x.$$

47) a) Find the area of the region bounded by the parabola $y^2 = x$ and the line $y = x - 2$

OR

b) Find the equation of the circle described on the chord $3x + y + 5 = 0$ of the circle $x^2 + y^2 = 16$ as diameter.