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Reg. No.

Time : 3.00 hrs.

Half Yearly Examination - 2023

MATHEMATICS

Max. Marks : 90

PART - I

i) Answer all the questions. 20 x 1 = 20

ii) Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer.

1. If A is a non-singular matrix and $|A| = 5$ then $|A^{-1}|$ is a) 5 b) $\frac{1}{5}$ c) –5 d) $-\frac{1}{5}$

2. $A = \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$ and $AB = I_2$ then B = a) $(\cos^2\frac{\theta}{2})$ A b) $(\cos^2\frac{\theta}{2})$ A^T c) d) $(\cos^2\theta)$ I d) $(\sin^2\frac{\theta}{2})$ A

3. If $|Z| = 2$ the maximum value of $|z + 3 + 4i|$ is a) 5 b) 10 c) 7 d) 3

4. If $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$ and $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 12$ then the value of $|z_1 + z_2 + z_3|$ is a) 1 b) 2 c) 3 d) 4

5. Solution of $\sqrt{15-2x} = x$ is a) 3, –5 b) –3, 5 c) 5 d) 3

6. The number of positive zeros of the polynomial $\sum_{r=0}^{n} nC_r\ (-1)^r\ x^r$ is a) 0 b) n c) < n d) r

7. The value of $\cot^{-1}(-1)$ is a) $-\frac{\pi}{4}$ b) $\frac{\pi}{4}$ c) $\frac{3\pi}{4}$ d) $-\frac{3\pi}{4}$

8. If $\cot^{-1}(\sqrt{\sin\alpha}) + \tan^{-1}(\sqrt{\sin\alpha}) = u$ then cos2u is equal to a) $\tan^2\alpha$ b) 0 c) –1 d) $\tan2\alpha$

9. Condition that the line $\ell x + my + n = 0$ to be a tangent to the parabola $y^2 = 4ax$ is
a) $am^2 = \ell n$ b) $an^2 = \ell m$ c) $am^2 = 4\ell n$ d) $an^2 = 4\ell m$

10. The equation of the normal to the circle $x^2 + y^2 - 2x - 2y + 1 = 0$ which is parallel to $2x + 4y = 3$ is
a) $x + 2y = 3$ b) $x + 2y + 3 = 0$ c) $2x + 4y + 3 = 0$ d) $x - 2y + 3 = 0$

11. If the straight line passing through the points A(6, 7, 5), B(8, 10, 6) is perpendicular to the straight line passing through the points C(m, 2, –5) and D(8, 3, –4) then the value of m is a) 2 b) 5 c) 8 d) 10

12. If $\vec{a}$ and $\vec{b}$ are parallel vectors then $[\vec{a}\ \vec{c}\ \vec{b}]$ is equal to a) 2 b) –1 c) 1 d) 0

13. The value of $\lim_{x\to 1} x^{\frac{1}{x-1}}$ is a) e b) e^2 c) $\frac{1}{e}$ d) $\frac{1}{e^2}$

14. Angle between $y^2 = x$ and $x^2 = y$ at the origin is a) $\tan^{-1}\frac{3}{4}$ b) $\tan^{-1}\frac{4}{3}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{4}$

15. If $w(x, y) = xy + \frac{e^y}{y^2+1}$ then $\frac{\partial^2 w}{\partial x \partial y}$ is a) 1 b) 0 c) $\frac{(y^2+1)^2e^y - e^x}{(y^2+1)^2}$ d) $\frac{-e^y 2y}{e^y + y^2}$

16. Linear approximation for $g(x) = \cos x$ at $x = \frac{\pi}{2}$ is a) $x + \frac{\pi}{2}$ b) $-x + \frac{\pi}{2}$ c) $x - \frac{\pi}{2}$ d) $-x - \frac{\pi}{2}$

17. The value of $\int_0^{\pi/4} \frac{dx}{4\sin^2 x + 9\cos^2 x}$ is a) $\frac{1}{6}\tan^{-1}\left(\frac{2}{3}\right)$ b) $\frac{1}{6}\tan^{-1}\left(\frac{3}{2}\right)$ c) $\tan^{-1}\frac{2}{3}$ d) $\tan^{-1}\left(\frac{3}{2}\right)$

18. If $\int_0^x f(t)dt = x + \int_x^1 tf(t)\,dt$ then the value of f(1) is a) $\frac{1}{2}$ b) 2 c) 1 d) $\frac{3}{4}$

19. Solution of $(1 + x^2)dy = (1 + y^2)\,dx$ is
a) $y - x = c(1 - xy)$ b) $y - x = c(1 + x^2y^2)$ c) $y - x = c(1 - x^2y^2)$ d) $y - x = c(1 + xy)$

20. If the solution of the differential equation $\frac{dy}{dx} = \frac{ax+3}{2y+f}$ represents a circle, then the value of a is a) 2 b) –2 c) 1 d) –1

PART - II

Note : Answer any seven questions. Question No.30 is compulsory. 7 x 2 = 14

21. If $z = x + iy$ then find Re $(i\bar{z})$ in rectangular form.
22. Find the value of $\cot^{-1}(-1) + \tan^{-1}(1)$
23. Find the equation and the length of Latus rectum of $y^2 = -8x$.

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24. Find the length of the perpendicular drawn from the origin to the plane $\vec{r}.(3\hat{i}-4\hat{j}+12\hat{k})=5$.
25. If $v(x, y, z) = xy + yz + zx$, $x, y z \in R$ find the differential dv.
26. Prove that $\int_0^\infty e^{-x} x^n dx = n!$ where n is a positive integer.
27. Find the differential equation for the family of all straight lines passing through the origin.
28. Suppose f(x) is a differentiable function for all x with $f'(x) \leq 29$ and $f(2) = 17$. What is the maximum value of f(7)
29. Find the asymptotes of the curve $f(x) = \frac{x^2}{x^2-1}$
30. If A is a non singular square matrix of order n, then prove that $|adj\ (adjA)| = |A|^{(n-1)^2}$

PART - III

Answer any seven questions. Question No.40 is compulsory. 7 x 3 = 21

31. Find the inverse by Gauss - Jordan method. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$
32. Simplify $(1 + i)^{18}$
33. Find the polynomial equation of minimum degree with rational coefficients having $3 - 2\sqrt{2}$ as a root.
34. Show that $\cot(\sin^{-1}x) = \frac{\sqrt{1-x^2}}{x}$, $-1 \leq x \leq 1$ and $x \neq 0$
35. If the normal at the point 't_1' on the parabola $y^2 = 4ax$ meets the parabola again at the point 't_2' then prove that $t_2 = -\left(t_1 + \frac{2}{t_1}\right)$
36. Find the co-ordinates of the point where the straight line $\vec{r} = (2\hat{i}-\hat{j}+2\hat{k}) + t(3\hat{i}+4\hat{j}+2\hat{k})$ intersects the plane $x - y + z - 5 = 0$
37. Find the value of $\lim_{x \to \frac{\pi}{2}} (\sin x)^{\tan x}$
38. Evaluate $\int_0^{\pi/2} \frac{dx}{5+4\sin^2 x}$
39. Solve $\frac{dy}{dx} = \frac{x-y+5}{2(x-y)+7}$
40. If $U = \sin(xy^2)$ and $x = \log t$, $y = e^t$ then find $\frac{du}{dt}$

PART - IV

Answer all the questions 7 x 5 = 35

41. a) Solve by Gaussian elimination method.

$2x - 2y + 3z = 2$, $x + 2y - z = 3$, $3x - y + 2z = 1$ **(OR)** b) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then prove $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$

42. a) Find all cube roots of $-\sqrt{3} - i$ **(OR)** b) Find the centre, Foci and vertices of the hyperbola $x^2 - 3y^2 + 6x + 6y + 18 = 0$
43. a) Show that the line $5x + 12y = 9$ is a tangent to the hyperbola $x^2 - 9y^2 = 9$ **(OR)**
    b) Prove by vector method that the perpendiculars (altitudes) from the vertices to the opposite sides of a triangle are concurrent.
44. a) Solve $(2x - 1)(x + 3)(x - 2)(2x + 3) + 20 = 0$ **(OR)**
    b) A conical water tank with vertex down of 12 metres height has a radius 5 metres at the top. If water flows into the tank at the rate of 10 cubic m/min how fast is the depth of a water increases when the water is 8m deep?
45. a) Find the number of solutions of the equation $\tan^{-1}(x - 1) + \tan^{-1}x + \tan^{-1}(x + 1) = \tan^{-1}(3x)$ **(OR)**
    b) A manufacturer wants to design an open box having a square base and a surface area of 108 sq.cm. Determine the dimensions of the box for the maximum volume.
46. a) If a plane makes an intercepts 3, 4 and 5 respectively on the co-ordinate axes the find the parametric vector equation and cartesian equation of that plane. **(OR)**
    b) Find the area of the region bounded by the parabola $y^2 = x$ and the line $y = x - 2$.
47. a) Evaluate $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx$ **(OR)** b) Solve $(1 + x + xy^2)\frac{dy}{dx} + (y + y^3) = 0$