HALF YEARLY EXAMINATION - DECEMBER - 2023

Class : XII Maximum Marks : 90

Subject : Mathematics Time Allowed : 3.00 Hours

Part - I

MDU

Note : i) All questions are compulsory $20 \times 1 = 20$

ii) Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer

1. If $A=\begin{pmatrix}2 & 0\\1 & 5\end{pmatrix}$ and $B=\begin{pmatrix}1 & 4\\2 & 0\end{pmatrix}$ then |adj (AB)|=

   a) -40 b) -80 c) -60 d) -20

2. If $(AB)^{-1} = \begin{pmatrix}12 & -17\\-19 & 27\end{pmatrix}$ and $A^{-1} = \begin{pmatrix}1 & -1\\-2 & 3\end{pmatrix}$, then $B^{-1} =$

   a) $\begin{pmatrix}2 & -5\\-3 & 8\end{pmatrix}$ b) $\begin{pmatrix}8 & 5\\3 & 2\end{pmatrix}$ c) $\begin{pmatrix}3 & 1\\2 & 1\end{pmatrix}$ d) $\begin{pmatrix}8 & -5\\-3 & 2\end{pmatrix}$

3. If $|z-2+i| \leq 2$, then the greatest value of $|Z|$ is

   a) $\sqrt{3}-2$ b) $\sqrt{3}+2$ c) $\sqrt{5}-2$ d) $\sqrt{5}+2$

4. The product of all four values of $\left(cos\frac{\pi}{3} + isin\frac{\pi}{3}\right)^{\frac{3}{4}}$ is a) -2 b) -1 c) 1 d) 2

5. A zero of $x^3 + 64$ is a) 0 b) 4 c) $4i$ d) -4

6. If $x^3 + 12x^2 + 10ax + 1999$ definitely has a positive zero ,if and only if

   a) $a \geq 0$ b) $a > 0$ c) $a < 0$ d) $a \leq 0$

7. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is

   a) $[1,2]$ b) $[-1,1]$ c) $[0,1]$ d) $[-1,0]$

8. The radius of the circle passing through the point (6,2) two of whose diameter are $x + y = 6$ and $x + 2y = 4$ is

   a)10 b) $2\sqrt{5}$ c) 6 d) 4

9. The axis of the parabola $y^2 - 2y + 8x - 23 = 0$ is

   a) $y = -1$ b) $x = -3$ c) $x = 3$ d) $y = 1$

10. The volume of the parallelepiped with its edges represented by the vectors $\hat{\imath}+\hat{\jmath}, \hat{\imath}+2\hat{\jmath}, \hat{\imath}+\hat{\jmath}+\pi\hat{k}$ is

    a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) π d) $\frac{\pi}{4}$

11. The equation of the plane passing through (3,4,5) and parallel to the plane $x + 2y - 2z - 9 = 0$ is

    a) $x + 2y - 2z = 4$ b) $x + 2y - 2z = 3$

    c) $x + 2y - 2z = 1$ d) $x + 2y - 2z = 5$

12. The point of inflection of the curve $y = (x-1)^3$ is

a) (0,0) b) (0,1) c) (1,0) d) (1,1)

13. The slope of the line normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$ is

a) 3 b) $\frac{-1}{3}$ c) $\frac{1}{3}$ d) -3

14. The approximate change in the volume V of a cube of side x metres caused by Increasing the side by 1% is

a) $0.3\, x dx\, m^3$ b) $0.03\, x\, m^3$ c) $0.03\, x^2\, m^3$ d) $0.03\, x^3\, m^3$

15. The value of $\int_{-1}^{2} |x|\, dx$ is a) $\frac{1}{2}$ b) $\frac{3}{2}$ c) $\frac{5}{2}$ d) $\frac{7}{2}$

16. The value of $\int_0^{\pi} \sin^4 x dx$ is

a) $\frac{3\pi}{10}$ b) $\frac{3\pi}{8}$ c) $\frac{3\pi}{4}$ d) $\frac{3\pi}{2}$

17. The solution of $\frac{dy}{dx} + p(x)y = 0$ is

a) $y = ce^{\int pdx}$ b) $y = ce^{-\int pdx}$ c) $x = ce^{-\int pdy}$ d) $x = ce^{\int pdy}$

18. If $P(X=0) = 1 - P(X=1)$ .If $E(X) = 3Var(X)$, then $P(X=0)$ is

a) $\frac{2}{3}$ b) $\frac{2}{5}$ c) $\frac{1}{5}$ d) $\frac{1}{3}$

19. $Var(aX + b) = \cdots$ a,b are constant

a) $a^2Var(X)$ b) $aVar(X)$ c) $aVar(X) + b$ d) $a^2Var(X) + b$

20. In the set Q define $a \odot b = a + b + ab$ .For what value of , $3 \odot (y \odot 5) = 7$?

a) $y = \frac{2}{3}$ b) $y = \frac{-2}{3}$ c) $y = \frac{-3}{2}$ d) $y = 4$

Part – II

Note : i) Answer any **Seven** questions 7 × 2 = 14

ii) Question number **30** is compulsory

21. Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal .

22. If $|Z| = 2$ show that $3 \le |Z + 3 + 4i| \le 7$.

23. Find the principal value of $\sin^{-1}(\frac{1}{\sqrt{2}})$.

24. Find the general equation of the circle whose diameter is the line segment joining the points $(-4,-2)$ and $(1,1)$.

25. Find the volume of the parallelepiped whose coterminous edges are given by the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$, $\hat{i} + 2\hat{j} - \hat{k}$ and $3\hat{i} - \hat{j} + 2\hat{k}$.

26. Prove that the function $f(x) = x^2 + 2$ is strictly increasing in the interval (2,7) and strictly decreasing in the interval $(-2,0)$.

27. Find the value of $\int_0^1 x(1-x)^{99} dx$.

28. Let $A = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ be any two Boolean matrices of the same type. Find $A \vee B$ and $A \wedge B$.

29. Determine the order and degree (if exists) of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 = \sqrt{1 + \left(\frac{dy}{dx}\right)}$.

30. Compute $P(X = k)$ for the binomial distribution, Where $B(n, p)$ $n = 10, p = \frac{1}{5}, k = 4$

Part – III

Note : i) Answer any **Seven** questions $7 \times 3 = 21$
ii) Question number **40** is compulsory

31. Solve the following system of linear equations, using matrix inversion method : $5x + 2y = 3, 3x + 2y = 5$.

32. Represent the complex number $-1 - i$ in polar form.

33. Discuss the maximum possible number of positive and negative roots of the polynomial equation $9x^9 - 4x^8 + 4x^7 - 3x^6 + 2x^5 + x^3 + 7x^2 + 7x + 2 = 0$.

34. Find the equation of the parabola with vertex $(-1, -2)$, axis parallel to y-axis and passing through $(3,6)$.

35. Find the angle between the straight line $\frac{x+3}{2} = \frac{y-1}{2} = -z$ with coordinate axes.

36. Establish the equivalence property : $p \to q \equiv \neg p \vee q$

37. Suppose X is the number of tails occurred when three fair coins are tossed once simultaneously. Find the values of the random variable X and number of points in its inverse images.

38. Evaluate : $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}} dx$.

39. Let $f(x, y) = \frac{y^2 - xy}{\sqrt{x} - \sqrt{y}}$ for $(x, y) \neq (0,0)$. Show that $\lim_{(x,y)\to(0,0)} f(x, y) = 0$.

40. Show that the number given by the Rolle's theorem for the function $x^3 - 3x^2, x \in [0,3]$ is 2.

Part – IV

Note : i) Answer all the questions $7 \times 5 = 35$

41. a) Prove by vector method that $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$. (OR)

b) The rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number present. Given that the number triples In 5 hours. Find how many bacteria will be present after 10 hours?

42. a) For the ellipse $4x^2 + y^2 + 24x - 2y + 21 = 0$, find the centre, vertices and foci. Also prove that the length of latus rectum is 2. (OR)

b) Prove that among all the rectangles of the given area square has the least perimeter.

43. a) Let $M = \left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in R - \{0\} \right\}$. Let $*$ be the matrix multiplication. Determine whether M is closed under $*$. If so examine commutative, associative, identity, inverse properties.

(OR)

b) Find the area of the region bounded between the parabolas $y^2 = 4x$ and $x^2 = 4y$.

44. a) A random variable X has the following probability mass function.

x	1	2	3	4	5	6
$f(x)$	k	$2k$	$6k$	$5k$	$6k$	$10k$

Find $i) P(2 < X < 6)$ $ii) P(2 \leq X < 5)$ $iii) P(X \leq 4)$ $iv) P(3 < X)$.

(OR)

b) Investigate for what values of λ and μ the system of linear equations $x + 2y + z = 7, x + y + \lambda z = \mu,$ $x + 3y - 5z = 5$ has i) no solution ii) a unique solution iii) an infinite number of solutions.

45. a) Find the non parametric form of vector equation, and Cartesian equations of the plane passing through the point $(1, -2, 4)$ and perpendicular to the plane $x + 2y - 3z = 11$ and parallel to the line $\frac{x+7}{3} = \frac{y+3}{-1} = \frac{z}{1}$.

(OR)

b) Find the value of : $\cot^{-1}(1) + \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) - \sec^{-1}(-\sqrt{2})$.

46. a) On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4m when it is 6m away from the point of projection. Finally it reaches the ground 12m away from the starting point. Find the angle of projection.

(OR)

b) Find all cube roots of $\sqrt{3} + i$.

47. a) If $v(x, y) = \log\left(\frac{x^2+y^2}{x+y}\right)$, prove that $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = 1$.

(OR)

b) Solve $(x - 4)(x - 7)(x - 2)(x + 1) = 16$.