HALF-YEARLY EXAMINATION - 2023

MATHS

STD - XII

TIME : 3.00 Hrs

MARKS : 90

I. Answer all the questions. 20 x 1 = 20

1. If $A=\begin{bmatrix}7 & 3\\ 4 & 2\end{bmatrix}$, then $9I_2 - A =$ a) A^{-1} b) $A^{-1}/2$ c) $3A^{-1}$ d) $2A^{-1}$
2. if $A^T A^{-1}$ is symmetric, then $A^2 =$ a) A^{-1} b) $(A^T)^2$ c) A^T d) $(A^{-1})^2$
3. If $\frac{Z-1}{Z+1}$ is purely imaginary then $|z|$ is a) 1 / 2 b) 1 c) 2 d) 3
4. The Principal argument of $\frac{3}{-1+i}$ is a) $\frac{-5\pi}{6}$ b) $\frac{-2\pi}{3}$ c) $\frac{-3\pi}{4}$ d) $\frac{-\pi}{2}$
5. A zero of x^3+64 is a) 0 b) 4 c) 4i d) -4
6. If α, β and γ are the zeros of x^3+px^2+qx+r, then $\in\frac{1}{\alpha}$ is a) $\frac{-q}{r}$ b) $\frac{-p}{r}$ c) $\frac{q}{r}$ d) $\frac{-q}{p}$
7. The principle value of $\cos^{-1}\frac{\sqrt{3}}{2}$ a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{5\pi}{6}$ d) $\frac{\pi}{6}$
8. Find the general equation of a circle with centre (-3, -4) and radius 3 units.
   a) $x^2+y^2-6x+8y-16=0$ b) $x^2+y^2-6x-8y+16=0$
   c) $x^2+y^2+6x-8y+16=0$ d) $x^2+y^2+6x+8y+16=0$
9. Distance from the origin to the plane $3x-6y+2z+7=0$ is a) 0 b) 1 c) 2 d) 3
10. The position of a particle moving along a horizontal line of any time 't' is given by $s(t)=3t^2-2t-8$. The time at which the particle is at rest is. a) t = 0 b) t = 1 / 3 c) t = 1 d) t = 3
11. If $w(x, y) = x^y, x > 0$ then $\frac{\partial w}{\partial x}$ is equal to a) $x^y \log x$ b) $y \log x$ c) yx^{y-1} d) $x \log y$
12. The area between $y^2 = 4x$ and its latus rectum is a) 2 / 3 b) 4 / 3 c) 8 / 3 d) 5 / 3
13. The value $\int_0^\infty e^{-3x}x^2dx$ a) $\frac{7}{27}$ b) $\frac{5}{27}$ c) $\frac{4}{27}$ d) $\frac{2}{27}$
14. The order and degree of the differential equation $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^{\frac{1}{3}}+x^{\frac{1}{4}}=0$ are respectively
   a) 2, 3 b) 3, 3 c) 2, 6 d) 2, 4
15. The solution of $\frac{dy}{dx}+p(x)y=0$ is a) $y=ce^{\int pdx}$ b) $y=ce^{-\int pdx}$ c) $x=ce^{-\int pdy}$ d) $x=ce^{\int pdy}$
16. Which one of the following is a binary operation on N?
   a) Subtraction b) Multiplication c) Division d) All the above
17. A random variable X has binomial distribution with n = 25 and p = 0.8. Then standard deviation of X is
   a) 6 b) 4 c) 3 d) 2
18. The value of var(4x + 3) a) 7 b) 16 var (x) c) 19 d) 0
19. In the set R of real nos '*' is defined as follows. Which one of the following is not a binary operation on R?
   a) $a * b = \min(a, b)$ b) $a * b = \max(a, b)$ c) $a * b = a$ d) $a * b = a^b$
20. If $u=\frac{1}{\sqrt{x^2+y^2}}$ then $x.\frac{\partial u}{\partial x}+y.\frac{\partial u}{\partial y}=$ a) $\frac{1}{2}u$ b) u c) $\frac{3}{2}u$ d) -u

II. Answer any Seven Questions. Q.No.30 is compulsory 7 x 2 = 14

21. If z = (2 + 3i) (1 - i), Find z^{-1}
22. If $adj\ A=\begin{bmatrix}-1 & 2 & 2\\ 1 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$ find A^{-1}
23. On z, define $\otimes$ by $(m \otimes n) = m^n + n^m \ \forall m, n \in z$ is $\otimes$ binary on z?
24. Find the principal value of $\sin^{-1}[\sin(5\pi/6)]$

25. Write the Maclurin series of e^x

26. Evaluate : $\int_0^1 x^3(1-x)^4 dx$

27. Find a polynomial equation of minimum degree with rational co-efficients, having $2i + 3$ as a root.

28. If $f(x, y) = x^3 - 3x^2 + y^2 + 5x + 6$. Calculate f_x at $(1, -2)$.

29. Find the points on the curve $y = x^3 - 3x^2 + x - 2$ at which the tangent is parallel to the line $y = x$.

30. Find the angle between the lines $\frac{x-4}{2}=\frac{y}{1}=\frac{z+1}{-2}$ and $\frac{x-1}{4}=\frac{y+1}{-4}=\frac{z-2}{2}$

Section - C

III. Write any Seven questions. Q.No. 40 is compulsory **7 x 3 = 21**

31. Find the two positive numbers whose sum is 12 and their product is maximum.

32. A coat of paint of thickness 0.2cm is applied to the faces of a cube whose edge is 10cm. use the differentials to find approximately how many cubic centimeters of paint is used to paint this cube. Also calculate the exact amount of paint used to paint this cube.

33. If $A=\begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, verify that A (adj A) = (adj A) A = $|A| I_2$

34. Find the rank of the matrix $\begin{bmatrix} 3 & 2 & 5 \\ 1 & 1 & 2 \\ 3 & 3 & 6 \end{bmatrix}$

35. Find the square root of $-5 - 12i$

36. Solve the equation $x^4 - 14x^2 + 45 = 0$.

37. Find the vertices, foci for the hyperbola $9x^2 - 16y^2 = 144$.

38. Evaluate : $\int_0^9 \frac{1}{x+\sqrt{x}}\,dx$

39. Solve : $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$

40. Verify (i)closure property (ii)commutative property of the following operation on the given set $a*b = a^b \ \forall\ a,b \in N$.

IV. Answer all the questions. **7 x 5 = 35**

41. a) Solve by cramer's rule the system of equations $x_1 - x_2 = 3$, $2x_1 + 3x_2 + 4x_3 = 17$, $x_2 + 2x_3 = 7$. (OR)

b) If the curve $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$ intersect each other orthogonally then show that $\frac{1}{a}-\frac{1}{b}=\frac{1}{c}-\frac{1}{d}$

42. a) If $z = x + iy$ and $\arg\left(\frac{z-i}{z+2}\right)=\frac{\pi}{4}$ show that $x^2 + y^2 + 3x - 3y + 2 = 0$ (OR)

b) Find the area of the region bounded by the parabola $y2 = x$ and the line $y = x - 2$.

43. a) Solve : $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+4}\right)=\frac{\pi}{4}$

b) The maximum and minimum distances of the Earth from the sun respectively are 152×10^6 km and 94.5×10^6 km. The sun is at one fours of the elliptical orbit. Find the distance from the sun to the other focus.

44. a) Solve : $\frac{dy}{dx}+2y=e^{-x}$ (OR) b) Verify whether the following compound propositions are tautologies or contradictions or contingency $(p \wedge q) \wedge \neg(p \vee q)$

45. a) Prove by vector method $\sin(A - B) = \sin A \cos B - \cos A \sin B$. (OR)

b) Suppose a person deposits 10,000 Indian rupees in a bank account at the rate of 5% per annum compounded continuously. How much money will be in his bank account 18 months later?

46. a) Find the eccentricity, foci, vertices, centre for the ellipse $\frac{(x+1)^2}{100}+\frac{(y-2)^2}{64}=1$ and draw the diagram. (OR)

b) The cumulative distribution function of a discrete random variable is given by

$$F(x)=\begin{cases} 0 & -\infty < x < -2 \\ 0.25 & -2 \le x < -1 \\ 0.60 & -1 \le x < 0 \\ 0.90 & 0 \le x < 1 \\ 1 & 1 \le x < \infty \end{cases}$$

Find i) The probability mass function ii) P (x < 0) and iii) P (x ≥ -1).

47. a) Solve the equation $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$ (OR)

b) For the function $f(x) = 4x^3 + 3x^2 - 6x + 1$. Find the intervals of monotomicity, local extrema, intervals of concavity and points of inflection.