அரியலூர் 12-ஆம் வகுப்பு
அரையாண்டுத் தேர்வு - 2023

I.

1. c	11. c
2. a	12. c
3. c	13. b
4. c	14. d
5. d	15. d
6. c	16. c
7. a	17. a
8. b	18. d
9. b	19. d
10. c	20. c

II.

21. $A^{-1} = \pm \frac{1}{\sqrt{|adj A|}} adj A$

$|adj A| = \begin{vmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{vmatrix}$

$= 2[36-18] = 2[18] = 36$

$\sqrt{|adj A|} = \sqrt{36} = 6$

$A^{-1} = \pm \frac{1}{6} \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$

22. $z = \frac{-2}{1+i\sqrt{3}}$

$\arg z = \arg\left(\frac{-2}{1+i\sqrt{3}}\right)$

$= \arg(-2) - \arg(1+i\sqrt{3})$

$= \left(\pi - \tan^{-1}\frac{0}{2}\right) - \left(\tan^{-1}\frac{\sqrt{3}}{1}\right)$

$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ //

23. $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ ஒரு மூலம், $-\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ மற்றொரு மூலம்.

$\left(x+\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}\right)\left(x-\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}\right) = x^2 - \frac{\sqrt{2}}{\sqrt{3}}$

$\left(x^2 - \frac{\sqrt{2}}{\sqrt{3}}\right)\left(x^2 + \frac{\sqrt{2}}{\sqrt{3}}\right) = x^4 - \frac{2}{3}$

$3x^4 - 2 = 0$

24. $\theta = \sec^{-1}\left(-\frac{2\sqrt{3}}{3}\right)$ எனில்.

$\sec\theta = \frac{-2\sqrt{3}}{3} = \frac{-2}{\sqrt{3}}$

$\cos\theta = -\frac{\sqrt{3}}{2}$

$\theta = \cos^{-1}\left(-\cos\frac{\pi}{6}\right)$

$= \cos^{-1}\left(\cos\left(\pi - \frac{\pi}{6}\right)\right)$

$= \cos^{-1}\left(\cos\frac{5\pi}{6}\right)$

$\theta = \frac{5\pi}{6}$

25. $yt = x + at^2$

$t = 2,\ a = 2$

$y(2) = x + 2(4)$

$2y = x + 8$

$x - 2y + 8 = 0$

26. $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$(x\hat{i} + y\hat{j} + z\hat{k})\cdot(3\hat{i} - 4\hat{j} + 3\hat{k}) = -8$

$(x\hat{i} + y\hat{j} + z\hat{k})(-3\hat{i} + 4\hat{j} - 3\hat{k}) = 8$

$\vec{r}\cdot(-3\hat{i} + 4\hat{j} - 3\hat{k}) = 8$.

27. $f(x) = x^2 - 2x - 3$

$f'(x) = 2x - 2 > 0$

$\forall (2, \infty)$ கூடுமாக உள்ளது.

28. $u(x,y) = \frac{x^2+y^2}{\sqrt{x+y}} = f(x,y)$ என்க.

$f(tx, ty) = \frac{t^2x^2 + t^2y^2}{\sqrt{tx+ty}} = \frac{t^2}{\sqrt{t}} f(x,y)$

$= t^{3/2} f(x,y)$

படி $n = \frac{3}{2}$ உடைய சமபடித்தான சார்பு.

ஆய்லர் தேற்றப்படி,

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$.

29. $\int_0^1 x^m(1-x)^n dx = \frac{m! \times n!}{(m+n+1)!}$

$\int_0^1 x^3(1-x)^4 dx = \frac{3! \times 4!}{(3+4+1)!} = \frac{1}{280}$

30. $y = mx + c$

$y' = m$

$y'' = 0$

III.

31. $A = \begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$; $A^T = \begin{bmatrix} 2 & 1 \\ 9 & 7 \end{bmatrix}$

$|A| = 14 - 9 = 5$; $|A^T| = 14 - 9 = 5$

$A^{-1} = \frac{1}{5}\begin{bmatrix} 7 & -9 \\ -1 & 2 \end{bmatrix}$; $(A^T)^{-1} = \frac{1}{5}\begin{bmatrix} 7 & -1 \\ -9 & 2 \end{bmatrix} \to ①$

$(A^{-1})^T = \frac{1}{5}\begin{bmatrix} 7 & -1 \\ -9 & 2 \end{bmatrix} \to ②$

①, ② லிருந்து $(A^{-1})^T = (A^T)^{-1}$ என நிரூபிக்கப்பட்டது.

32. $z = \cos\theta + i\sin\theta$

$z^n = \cos n\theta + i\sin n\theta$

$\frac{1}{z^n} = \cos n\theta - i\sin n\theta$

$z^n + \frac{1}{z^n} = 2\cos n\theta$

$z^n - \frac{1}{z^n} = 2i\sin n\theta$

33. $lx^2+nx+n=0$

$p+q=\frac{-n}{l}$; $pq=\frac{n}{l}$

L.H.S:

$=\frac{\sqrt{p}}{\sqrt{q}}+\frac{\sqrt{q}}{\sqrt{p}}+\sqrt{\frac{n}{l}}$

$=\frac{p+q}{\sqrt{pq}}+\sqrt{\frac{n}{l}}=\frac{-n}{l}\cdot\frac{\sqrt{l}}{\sqrt{n}}+\sqrt{\frac{n}{l}}$

$=-\sqrt{\frac{n}{l}}+\sqrt{\frac{n}{l}}=0.$

34.

$(\tan^{-1}x+\tan^{-1}y)+\tan^{-1}z=\pi$

$\Rightarrow\tan^{-1}\left(\frac{x+y}{1-xy}\right)+\tan^{-1}z=\pi$

$\Rightarrow\tan^{-1}\left[\frac{\frac{x+y}{1-xy}+z}{1-\frac{xy}{1-xy}\cdot z}\right]=\pi$

$\Rightarrow\tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-xz-yz}\right]=\pi$

$\frac{x+y+z-xyz}{1-xy-xz-yz}=\tan\pi=0$

$x+y+z-xyz=0$

$x+y+z=xyz$ //

35. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ என்ற அதிபரவளையத்தின் செவ்வகலம் S(ae,0) வழியே அமைந்தால்.

$\frac{a^2e^2}{a^2}-\frac{y_1^2}{b^2}=1$

$\frac{-y_1^2}{b^2}=1-e^2$

$y_1^2=b^2(e^2-1)$

$y_1^2=b^2\times\frac{b^2}{a^2}=\frac{b^4}{a^2}$

$y_1=\pm\frac{b^2}{a}$

செவ்வகலத்தின் நீளம் $=\frac{2b^2}{a}$

36. $x+y=12$; $y=12-x$

எண்களின் பெருக்கல் $P=xy$

$P=x(12-x)$

$P=12x-x^2$

$\frac{dP}{dx}=12-2x$

$\frac{dP}{dx}=0$ எனில் $12-2x=0$

$2x=12$

$x=6$

$\frac{d^2P}{dx^2}=-2<0$; $x=6$ ல் பெருமம்.

$x=6$ $\therefore y=6$ //

37. $u=\log(x^3+y^3+z^3)$

$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=\frac{3x^2+3y^2+3z^2}{x^3+y^3+z^3}$.

38. $I=\int_2^3\frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}dx\rightarrow(1)$

$\int_0^a f(x)dx=\int_0^a f(a-x)dx$ எனில்.

$I=\int_2^3\frac{\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}}\rightarrow(2)$

(1)+(2) $2I=\int_2^3\frac{\sqrt{x}+\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}}dx$

$2I=\int_2^3 dx=[x]_2^3=3-2=1$

$I=\frac{1}{2}$

39. $\cos x\frac{dy}{dx}+y\sin x=1$

$\frac{dy}{dx}+y\tan x=\sec x$

$P=\tan x$; $Q=\sec x$

$e^{\int P\,dx}=e^{\int\tan x\,dx}=e^{\log\sec x}=\sec x$

தீர்வு:

$y(I.F)=\int Q(I.F)dx+C$

$y\sec x=\int\sec x.\sec x\,dx+c$

$y\sec x=\tan x+c$

$y=\sin x+c\cos x$. //

40.

$=[\vec a\ \vec b\ \vec c]\begin{vmatrix}1&1&0\\0&1&1\\1&0&1\end{vmatrix}$

$=[\vec a\ \vec b\ \vec c][1(1)-1(-1)+0]$

$=[\vec a\ \vec b\ \vec c][1+1]$

$=2[\vec a\ \vec b\ \vec c]$

IV.

41. (அ) $\frac{dA}{dt}\propto A$; $\frac{dA}{dt}=kA$

$k=5\%\Rightarrow k=0.05$

$\frac{dA}{dt}=0.05A$; $A=ce^{0.05t}$ →(1)

$t=0$ எனில் $A=10,000$;

$c=10,000$

(1) $A=10000e^{0.05t}$

$t=\frac{3}{2}$ எனில் $A=10,000\,e^{0.05\left(\frac{3}{2}\right)}$

$A=10,000\,e^{0.075}$

42. (அ) $2\cos\alpha=x+\frac{1}{x}$ எனில்,

$x=\cos\alpha+i\sin\alpha$

$2\cos\beta=y+\frac{1}{y}$ எனில்

$y=\cos\beta+i\sin\beta$.

$x^m=\cos m\alpha+i\sin m\alpha$

$y^n=\cos n\beta+i\sin n\beta$

மே.நி: $\frac{1}{y^n}=\cos n\beta-i\sin n\beta$

$\frac{x^m}{y^n}=(\cos m\alpha+i\sin m\alpha)(\cos n\beta-i\sin\beta)$

$\frac{x^m}{y^n}=\cos(m\alpha-n\beta)+i\sin(m\alpha-n\beta)$ →(1)

$\frac{y^n}{x^m}=\cos(m\alpha-n\beta)-i\sin(m\alpha-n\beta)$ →(2)

(1)+(2) $\frac{x^m}{y^n}+\frac{y^n}{x^m}=2\cos(m\alpha-n\beta)$

(1)−(2) $\frac{x^m}{y^n}-\frac{y^n}{x^m}=2i\sin(m\alpha-n\beta)$

(ii) $x^my^n=(\cos m\alpha+i\sin n\beta)(\cos n\beta+i\sin n\beta)$

$x^my^n=\cos(m\alpha+n\beta)+i\sin(m\alpha+n\beta)$ →(3)

$\frac{1}{x^my^n}=\cos(m\alpha+n\beta)-i\sin(m\alpha+n\beta)$ →(4)

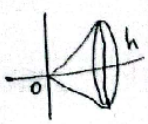
(3)+(4) $x^my^n+\frac{1}{x^my^n}=2\cos(m\alpha+n\beta)$

43. (அ) $y=\frac{x}{h}x$;

$V=\pi\int_a^b y^2dx=\pi\int_0^h\frac{x^2}{h^2}x^2dx$

$=\frac{\pi}{h^2}\left[\frac{x^3}{3}\right]_0^h=\frac{\pi}{3h^2}[h^3]$

$V=\frac{\pi r^2h}{3}$



IV.

42. (ஆ) $[AB]\sim\left[\begin{array}{ccc|c}1&1&3&0\\4&3&\lambda&0\\2&1&2&0\end{array}\right]\begin{matrix}R_1\\R_2\\R_3\end{matrix}$

$\sim\left[\begin{array}{ccc|c}1&1&3&0\\0&-1&\lambda-12&0\\0&-1&-4&0\end{array}\right]\begin{matrix}R_1\\R_2\to R_2-4R_1\\R_3\to R_3-2R_1\end{matrix}$

$\sim\left[\begin{array}{ccc|c}1&1&3&0\\0&-1&\lambda-12&0\\0&0&\lambda-8&0\end{array}\right]R_3\to R_3-R_2$

(i) ஒருங்கியைபுக்கு தீர்வு: $\lambda-8\neq0$, $\lambda\neq8$

(ii) ஒன்றிற்கொன்று சார்புடையதற்கு தீர்வு

$\lambda-8=0$; $\lambda=8$

43. (அ)

$x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$

$x^2\left[\left(x^2+\frac{1}{x^2}\right) - 10\left(x+\frac{1}{x}\right) + 26\right] = 0$

$\left(x^2+\frac{1}{x^2}\right) - 10\left(x+\frac{1}{x}\right) + 26 = 0$

$(y^2-2) - 10y + 26 = 0$

$y^2 - 10y + 24 = 0$

$(y-6)(y-4) = 0$

$y=6$ எனில் $x+\frac{1}{x} = 6$

$x = 3 \pm 2\sqrt{2}$

$y=4$ எனில் $x+\frac{1}{x} = 4$

$x = 2 \pm \sqrt{3}$

43. (ஆ) $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$

$\sin u = \frac{x+y}{\sqrt{x}+\sqrt{y}} = f(x,y)$ என்க.

$f(tx,ty) = \frac{tx+ty}{\sqrt{t}(\sqrt{x}+\sqrt{y})} = t^{1/2} f(x,y)$

இது $n=\frac{1}{2}$ உடைய சமபடித்தான சார்பு. ஆய்லர் தேற்றப்படி,

$x\frac{\partial}{\partial x}\sin u + y\frac{\partial}{\partial y}\sin u = \frac{1}{2}\sin u$

$\cos u\left[x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right] = \frac{1}{2}\sin u$

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$

44. (அ) $x = \cos\alpha,\ y = \cos\beta$

$\alpha + \beta = \pi - \cos^{-1}x$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= xy - \sqrt{1-x^2}\sqrt{1-y^2}$

$\cos(\pi - \cos^{-1}x) = xy - \sqrt{1-x^2}\sqrt{1-y^2}$

$-\cos(\cos^{-1}z) = xy - \sqrt{1-x^2}\sqrt{1-y^2}$

$-z = xy - \sqrt{1-x^2}\sqrt{1-y^2}$

$-xy - z = -\sqrt{1-x^2}\sqrt{1-y^2}$

வர்க்கப்படுத்த

$(xy+z)^2 = (1-x^2)(1-y^2)$

$x^2y^2 + z^2 + 2xyz = 1 - y^2 - x^2 + x^2y^2$

$x^2 + y^2 + z^2 + 2xyz = 1$ //

44. (ஆ)

$f(x) = \log(1+x) \Rightarrow f(0) = 0$

$f'(x) = \frac{1}{1+x}$; $f'(0) = 1$

$f''(x) = \frac{-1}{(1+x)^2}$; $f''(0) = -1$

$f'''(x) = \frac{2}{(1+x)^3}$; $f'''(0) = 2$

$f^{iv}(x) = \frac{-6}{(1+x)^4}$; $f^{iv}(0) = -6$

W.k.t.

$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \cdots$

$f(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$

45. (அ) $x^2 + y^2 + 2gx + 2fy + c = 0 \rightarrow (1)$

(1,0) எனில் $2g + c = -1 \rightarrow (2)$

(-1,0) எனில் $-2g + c = -1 \rightarrow (3)$

(0,1) எனில் $2f + c = -1 \rightarrow (4)$

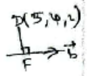
$(2)+(3) \Rightarrow 2c = -2$; $\boxed{c=-1}$

$(2)-(3) \Rightarrow 4g = 0$; $\boxed{g=0}$

$c=-1$ எனில் $(4) \Rightarrow 2f = -1+1 = 0$; $\boxed{f=0}$

∴ வட்டத்தின் சமன்பாடு $x^2 + y^2 = 1$

45. (ஆ) $\vec{a} = -\hat{i} + 3\hat{j} + \hat{k}$



$\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$

கோட்டின் மீதுள்ள புள்ளியின் அமைப்பு

$(2t-1,\ 3t+3,\ -t+1)$

செங்குத்தின் அடிப்புள்ளி F என்க.

$F = (2t-1,\ 3t+3,\ -t+1) \rightarrow (1)$

$\vec{DF} = \vec{OF} - \vec{OD} = (2t-1-5)\hat{i} + (3t+3-4)\hat{j} + (-t+1-2)\hat{k}$

$\vec{DF} = (2t-6)\hat{i} + (3t-1)\hat{j} + (-t-1)\hat{k}$

$\vec{DF}$ ஆனது கொடுக்கப்பட்ட கோட்டிற்கு செங்குத்து என்பதால், $\vec{b}\cdot\vec{DF} = 0$

$2(2t-6) + 3(3t-1) - 1(-t-1) = 0$

$4t - 12 + 9t - 3 + t + 1 = 0$

$14t - 14 = 0$

$14t = 14$

$\boxed{t=1}$

(1) ⇒ ∴ $F = (2(1)-1,\ 3(1)+3,\ -1+1)$

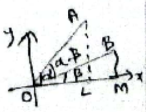
$= (1, 6, 0)$

DF ன் சமன்பாடு

$\frac{x-5}{1-5} = \frac{y-4}{6-4} = \frac{z-2}{0-2}$

$\frac{x-5}{-4} = \frac{y-4}{2} = \frac{z-2}{-2}$ //

46. (ஆ)



$\vec{a} = \cos\alpha\hat{i} + \sin\alpha\hat{j}$

$\vec{b} = \cos\beta\hat{i} + \sin\beta\hat{j}$

$\vec{b}\times\vec{a} = |\vec{b}||\vec{a}|\sin(\alpha-\beta)\hat{k}$

$\vec{b}\times\vec{a} = \sin(\alpha-\beta)\hat{k} \rightarrow (1)$

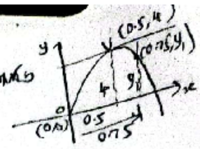
$\vec{b}\times\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos\beta & \sin\beta & 0 \\ \cos\alpha & \sin\alpha & 0 \end{vmatrix}$

$\vec{b}\times\vec{a} = (\sin\alpha\cos\beta - \cos\alpha\sin\beta)\hat{k} \rightarrow (2)$

(1), (2) லிருந்து

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$.

46. (அ)



பரவளையத்தின் சமன்பாடு

$(x-h)^2 = -4a(y-k)$

V(0.5, 4)

$(x-0.5)^2 = -4a(y-4) \rightarrow (1)$

(0,0) வழியே செல்வதால்,

(1) ⇒ $(0-0.5)^2 = -4a(0-4)$

$4a = \frac{0.25}{4}$

(1) ⇒ $(x-0.5)^2 = -\frac{0.25}{4}(y-4) \rightarrow (2)$

$(0.75, y_1)$ புள்ளி வழியே செல்வதால்,

$(0.75-0.5)^2 = \frac{-0.25}{4}(y_1-4)$

$(0.25)^2 \times \frac{4}{0.25} = -(y_1-4)$

$y_1 = 4-1 = 3$

ஆதியிலிருந்து 0.75மீ தொலைவில் உயரம் = 3மீ

47. (அ) $\vec{a} = 3\hat{i} + 6\hat{j} - 2\hat{k}$

$\vec{b} = -\hat{i} - 2\hat{j} + 6\hat{k}$

$\vec{c} = 6\hat{i} - 4\hat{j} - 2\hat{k}$

கார்டீசியன் சமன்பாடு

$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$

$\begin{vmatrix} x-3 & y-6 & z+2 \\ -4 & -8 & 8 \\ 3 & -10 & 0 \end{vmatrix} = 0$

$(x-3)(80) - (y-6)(-24) + (z+2)(64) = 0$

$80x + 24y + 64z - 256 = 0$.

வெக்டர் சமன்பாடு!

$\vec{r} = (1-s-t)\vec{a} + s\vec{b} + t\vec{c}$

$\vec{r} = (1-s-t)(3\hat{i}+6\hat{j}-2\hat{k}) + s(-\hat{i}-2\hat{j}+6\hat{k}) + t(6\hat{i}-4\hat{j}-2\hat{k})$

$\vec{r} = (3\hat{i}+6\hat{j}-2\hat{k}) + s(-4\hat{i}-8\hat{j}+8\hat{k}) + t(3\hat{i}-10\hat{j}+0\hat{k})$.

47. (ஆ)

$I = \int_{\pi/8}^{3\pi/8} \frac{1}{1+\sqrt{\cot x}}\,dx$

$I = \int_{\pi/8}^{3\pi/8} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx \rightarrow (1)$

$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$ ன் படி,

$I = \int_{\pi/8}^{3\pi/8} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx \rightarrow (2)$

(1)+(2) $2I = \int_{\pi/8}^{3\pi/8} dx$

$2I = [x]_{\pi/8}^{3\pi/8} = \frac{3\pi}{8} - \frac{\pi}{8} = \frac{2\pi}{8}$

$2I = \frac{2\pi}{8}$

$I = \frac{\pi}{8}$ //