



TEACHER'S CARE ACADEMY

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UG TRB MATHEMATICS 2023-2024

$\cos 2x = \cos^2 x - \sin^2 x$

$\frac{\partial z}{\partial x} = 2, \frac{\partial z}{\partial y} = 0 \Rightarrow \vec{n} = (f_x', f_y', f_z')$

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$\sum_{i=1}^n (i^2 - 1)^2$

$1^1 1^1 1^1$
 $1^1 4^2 6^3 4^2 1^1$

$y = \tan x$
 $y = \cot x$

$b = c \cdot \cos A$
 $a^2 = b^2 + c^2 - 2bc \cos A$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\sin 2x = 2 \sin x \cdot \cos x$

$\frac{\sin x}{x} \leq \frac{x}{x^2} - 1$

$A+B+C=180$
 $-2A-7B+2C=193$
 $-18A+5B-3C=16$

$\frac{1}{2}ax^2 + 3bx + 1 = 0$

$x^2 - y^2 + z^2 = 1$
 $x + y + z = 2$
 $x + y + z^2 = 2^2$

$\sin x$
 $\cos x$
 $2 \sin x$
 $\sin 2x$

$\tan x \cdot \cot x = 1$
 $\sin(x+y) = \sin x \cos y + \cos x \sin y$

UNIT I

Algebra & Trigonometry

Your Success is Our Goal....

INDEX

UNIT - I - ALGEBRA AND TRIGONOMETRY

S.No.	Topic	P. No.
1.1	Polynomial Equations	1
1.2	Imaginary and Irrational roots	2
1.3	Relation between roots and coefficients symmetric function of roots in terms of coefficient	9
1.4	Transformation of Equation:	12
1.5	Reciprocal Equation	15
1.6	Increase or Decrease the roots of given equation	23
1.7	Removal of a term	24
1.8	Descartes's rule of signs	27
1.9	Approximate solution of roots of polynomial by Horner's method	29
1.10	Newton's Raphson's Method of cubic polynomial	31
1.11	Summation of series using Binomial	35
1.12	Exponential and Logarithmic series	38
1.13	Logarithmic series	41
1.14	Symmetric	45
1.15	Skew symmetric:	45
1.16	Definition of Hermition	49
1.17	Definition of skew - Hermition	50
1.18	Orthogonal matrix $\Rightarrow AA^T = A^T A = I$	51
1.19	Unitary matrix:	52
1.20	Eigen values	52
1.21	Eigen vectors	55
1.22	Cayley - Hamilton Theorem:	57

1.23	Similar Matrices	59
1.24	Diagonalization of matrices	60
1.25	Diagonalisation of the matrix $\begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$	60
1.26	Prime Number:	63
1.27	Composite Number:	63
1.28	Number of primes	64
1.29	Divisor of a given Number N	64
1.30	Highest Power of prime number P contained in n!	73
1.31	Application of Maxima and Minima	75
1.32	General Fermat's Theorem	76
1.33	Wilson's Theorem:	78
1.34	Expansions of power of six $n x$, $\cos nx$, $\tan nx$	81
1.35	Expansion of $\cos^n \theta$, where n is a positive integer	83
1.36	Expansion of $\sin^n \theta$, in series of multiplies of θ when n is positive integer	86
1.37	Expansion of $\sin^7 \theta$ in series sines of multiples of θ	87
1.38	Summation of C + is method	99
1.39	Expand $\sin 7\theta$ in powers of $\cos \theta$ and $\sin \theta$. Reduce that	102
1.40	Hyper polic function	111
1.41	Relation between circular and Hyperbolic function	116
1.42	Inverse Hyperbolic Function	119
1.43	Logarithm of a complex number:	123
1.44	Principal values and general values	129
1.45	Multiple Choice Question	135

TEACHER'S CARE ACADEMY, KANCHIPURAM**TNPSC-TRB- COMPUTER SCIENCE -TET COACHING CENTER****HEAD OFFICE: NO. 38/23, VAIGUNDA PERUMAL KOIL,****SANNATHI STREET, KANCHIPURAM – 1. CELL: 9566535080****B.Off 2: 65C, Thillai Ngr(West), 4th Cross St, Trichy – 620018****B.Off 3: Vijayaraghavachariar Memorial Hall(Opp to Sundar Lodge), Salem****Trichy: 76399 67359****Salem: 93602 68118****UG TRB – MATHEMATICS – 2023-24****UNIT – I****ALGEBRA & TRIGONOMETRY****1.1. Polynomial Equations:**

$$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_n$$

- Where n is a positive integers and a_0, a_1, \dots, a_n constant is called polynomial in x of nth degree, if $a_0 \neq 0$.

**Fundamental Theorem of Algebra**

- Every polynomial equation of the nth degree has n and only n roots.
- If $f(x) = 0$ is an equation of odd degree then it has atleast one real roots.
Whose sign opposite to that of the last term.
- If $f(x) = 0$ is an even degree another constant terms is negative. The equation has atleast one positive root and atleast one negative root.
- If $f(x) = 0$ has no real root between a and b ($a < b$), then f(a) and f(b) are same sign.

Exercises

- Find the coefficient of x^n in the expansion of e^{a+bx} .

(A) $\frac{e^a \cdot e^b}{n!}$

(B) $\frac{e^a \cdot e^n}{n!}$

(C) $\frac{e^a \cdot e^n}{a!}$

(D) $\frac{e^a \cdot e^n}{b!}$

- The expansion of $\log(1+x)$ is

(A) $\log(1+x) = x - \frac{x^2}{2!} + \dots$

(B) $\log(1+x) = 1 - \frac{x^2}{2!} + \dots$

- (C) $\log(1+x) = x - \frac{x^3}{3!} + \dots$ (D) None of these
3. The number of primes is
 (A) finite (B) prime (C) infinite (D) None of these
4. Every polynomial equation $f(x) = 0$ has atleast one root real or ____
 (A) imaginary (B) real (C) algebraic (D) complex

1.2. Imaginary and Irrational Roots

Solve $x^4 + 4x^3 + 5x^2 + 2x - 2 = 0$, solve $-1+i$ is a root.

Solution:

Given $-1+i$ is a root,

$-1-i$ is also a root.

$$[x - (-1+i)][x - (-1-i)] = ((x+1)-i)((x+1)+i)$$

$$= (x+1)^2 - i^2$$

$$= (x+1)^2 + 1$$

$$= x^2 + 2x + 1 + 1$$

$$= x^2 + 2x + 2$$

- When the polynomial is divided by $x^2 + 2x + 2$. The remainder is zero.
- Equating the co-efficient of x^3 - term of both side

$$\therefore x^4 + 4x^3 + 5x^2 + 2x - 2 = (x^2 + 2x + 2)(x^2 + ax - 1)$$

$$2 + a = 4$$

$$a = 4 - 2$$

$$a = 2$$

$$\therefore f(x) = (x^2 + 2x + 2)(x^2 + 2x - 1)$$

$$\therefore x^2 + 2x - 1 = 0$$

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



$$\begin{aligned}
 a & \frac{-2 \pm \sqrt{4 - 4(-1)}}{2(1)} \\
 & = \frac{-2 \pm \sqrt{8}}{2} \\
 & = \frac{-2 \pm 2\sqrt{2}}{2} \\
 & = \frac{2(-1 \pm \sqrt{2})}{2} \\
 & = -1 \pm \sqrt{2}
 \end{aligned}$$

- The two roots are $(-1 - \sqrt{2}), (-1 + \sqrt{2})$

Solve: $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$ given that $2 + \sqrt{3}$ is a root of the equations.

Solu:

- Since, $2 + \sqrt{3}$ is a roots, $2 - \sqrt{3}$ is also a root.

$$\begin{aligned}
 (x - (2 + \sqrt{3}))[x - (2 - \sqrt{3})] &= [x(-2 - \sqrt{3})][x(-2 + \sqrt{3})] \\
 &= [(x - 2) - \sqrt{3}][(x - 2) + \sqrt{3}] \\
 &= (x - 2)^2 - (\sqrt{3})^2 \\
 &= x^2 - 4x + 4 - 3 \\
 &= x^2 - 4x + 1
 \end{aligned}$$

- When the polynomial is divided by $x^2 - 4x + 1$ the remainder is zero.
- Equality the co-efficient of x^3 term on both side.

$$\therefore x^4 - 10x^3 + 26x^2 - 10x + 1 = (x^2 - 4x + 1)(x^2 - ax + 1)$$

$$-a - 4 = -10$$

$$-a = -10 + 4$$

$$-a = -6$$

$$a = 6$$

$$\therefore f(x) = (x^2 - 4x + 1)(x^2 - 6x + 1)$$

$$x = \frac{-(-6) \pm \sqrt{6^2 - 4 \times 1 \times 1}}{2 \times 1}$$

$$= \frac{6 \pm \sqrt{36 - 4}}{2}$$

$$= \frac{6 \pm \sqrt{32}}{2}$$

$$= \frac{6 \pm 4\sqrt{2}}{2}$$

$$= \frac{2(3 \pm 2\sqrt{2})}{2}$$

$$= 3 \pm 2\sqrt{2}$$

\therefore The two roots are $(3 + 2\sqrt{2})$ and $(3 - 2\sqrt{2})$

Solve: $x^3 - 15x^2 + 71x - 105 = 0$ given that the roots of equation are in A.P.

Soln:

- Let the roots be $\alpha - d, \alpha, \alpha + d$
- Sum of roots = $\alpha - d + d + \alpha + d$

$$= 3\alpha$$

- General formula

$$\alpha - d + \alpha + \alpha + d = p$$

$$3\alpha = +15$$

$$\alpha = \frac{15}{3}$$

$$\alpha = 5$$

- Since $x = 5$ is a root, $x - 5$ is a factor of $f(x)$

$$x^3 - 15x^2 + 71x - 105 = (x - 5)(x^2 + ax + 21)$$

- Equating the coefficient of x^2 term in both side



$$ax - 5 = -15 \Rightarrow a - 5 = 15$$

$$a = -15 + 5$$

$$a = 10$$

$$x^2 - 10x + 21 = (x - 7)(x - 3)$$

$$x - 7 = 0$$

$$x - 3 = 0$$

$$x = 7$$

$$x = 3$$

∴ The roots are (3, 5, 7)

Alter

Product of root = - (-105)

$$(5 - d)(5)(5 + d) = -(-105)$$

$$5(5^2 - d^2) = 105$$

$$25 - d^2 = \frac{105}{5}$$

$$25 - d^2 = 21$$

$$d^2 = 25 - 21$$

$$d^2 = 4$$

$$d = \pm 2$$

∴ The roots are $\alpha - d, \alpha, \alpha + d$ is

$$\Rightarrow (5 - 2, 5, 5 + 2)$$

$$\Rightarrow (3, 5, 7)$$

Solve: $x^3 - 19x^2 + 114x - 216 = 0$ given that the roots are in G.P.

Soln.

- Let the roots be $\left(\frac{\alpha}{r}, \alpha, \alpha r\right)$
- Product of the roots = $\frac{\alpha}{r}, \alpha, \alpha r = -r$

$$\alpha^3 = 216$$

$$\alpha^3 = 6^3$$

$$\alpha = 6$$

- Since $x = 6$ is a root, $(x-6)$ is a factor $x^3 - 19x^2 + 114x - 216 = (x-6)(x^2 + ax + 36)$

$$a - 6 = -19$$

$$a = -19 + 6$$

$$a = -13$$

$$(x^2 - 13x + 36) = (x-9)(x-4)$$

$$(x-9)(x-4) = 0$$

$$x = 9, 4$$

∴ The roots are (6, 4, 9)

Solve: $6x^3 - 11x^2 + 6x - 1 = 0$ given the roots are in H.P.

Soln:

- Put $x = \frac{1}{y}$

$$6x^3 - 11x^2 + 6x - 1 = 0$$

$$6\left(\frac{1}{y}\right)^3 - 11\left(\frac{1}{y}\right)^2 + 6\left(\frac{1}{y}\right) - 1 = 0$$

$$\frac{6}{y^3} - \frac{11}{y^2} + \frac{6}{y} - 1 = 0$$

$$6 - 11y + 6y^2 - y^3 = 0$$

$$y^3 - 6y^2 + 11y - 6 = 0$$

- Sum of the roots

$$\alpha - d + \alpha + \alpha + d = 6$$

$$3\alpha = 6$$

$$\alpha = \frac{6}{3}$$

$$\alpha = 2$$

- Since $y = 2$ is a root, $(y - 2)$ is a factor

$$y^3 - 6y^2 + 11y - 6 = (y - 2)(y^2 + ay + 3)$$

- Equating the co-efficient of y^2 terms in both side

$$a - 2 = -6$$

$$a = -6 + 2$$

$$a = -4$$

$$\therefore y^2 - 4y + 3 = 0$$

$$(y - 1)(y - 3) = 0$$

$$y = 1, 3$$

\therefore The roots are (1, 2, 3)

- The roots of a given equation are

$$\left(1, \frac{1}{2}, \frac{1}{3}\right)$$

Solve: $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$ given that two of its roots are equal in magnitude but opposite in sign.

Soln:

- Let the roots be α, β, γ and δ

$$\alpha + \beta = 0$$

$$\alpha = -\beta$$

$$\therefore x^4 - 2x^3 + 4x^2 + 6x - 21 = (x - \alpha)(x - \beta)(x - \gamma)(x - \delta)$$

$$= [x^2 - (\alpha + \beta)x + \alpha\beta][x^2 - (\gamma + \delta)x + \gamma\delta]$$

$$= (x^2 + \alpha\beta)(x^2 - (\gamma + \delta)x + \gamma\delta)$$

$$= (x^2 - a)(x^2 - 6x + c)$$

$$= x^4 - 6x + cx^2 - ax^2 + 6ax + ac$$

- Equating the co-efficient of x^3, x^2 and x terms are both side

$$-b = -2$$

$$c - a = 4$$

$$ab = 6$$

$$b = 2$$

$$c - 3 = 4$$

$$a \times 2 = 6$$

$$c = 4 + 3$$

$$a = 3$$

$$c = 7$$

$$(x^2 - 3)(x^2 - 2x + 7)$$

$$x^2 - 3 = 0$$

$$x^2 = 3 \Rightarrow x = \pm\sqrt{3}$$

$$x^2 - 2x + 7 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{2 \pm \sqrt{4 - 4 \times 7}}{2}$$

$$x = \frac{2 \pm \sqrt{4 - 28}}{2}$$

$$= \frac{2 \pm \sqrt{-24}}{2}$$

$$= \frac{2 \pm i2\sqrt{6}}{2}$$

$$= \frac{2(1 \pm i\sqrt{6})}{2}$$

$$x = 1 \pm i\sqrt{6}$$

Exercises

1. The series $x - \frac{x^2}{2!} + \frac{x^3}{3!} - \dots \infty$ is

(A) Convergent

(B) Divergent

(C) Infinite

(D) Finite

2. If A and B are symmetric, then AB is symmetric iff A and B are

(A) Symmetric

(B) skew symmetric

(C) commutative

(D) Associative



3. If A and B are Hermitian then $AB+BA$ is Hermitian and $AB-BA$ is
- (A) Hermitian (B) skew symmetric
(C) skew Hermitian (D) Non Hermitian
4. If $A^*A=I$, then a square matrix A is said to be
- (A) unitary (B) orthogonal (C) diagonal (D) None of these
5. The roots of the equation $x + \frac{1}{x} = 1$ are
- (A) 1, -1 (B) $1+i$ and $\frac{1}{2} + \frac{i\sqrt{3}}{2}$
(C) $1+i$ and $1-i$ (D) $\frac{1+i\sqrt{3}}{2}$ and $\frac{1-i\sqrt{3}}{2}$
6. One real root of the equation $x^3 - 7x^2 + 14x - 8 = 0$ is
- (A) -2 (B) $\frac{1}{2}$ (C) $-\frac{1}{2}$ (D) 2

1.3. Relation Between Roots and Coefficients Symmetric Function of Roots In Terms Of Coefficient

If α, β, γ are the roots are the equation $x^3 + px^2 + qx + r = 0$ Find the value of

(i) $\sum \alpha^2\beta$ (ii) $\sum \alpha^2$ (iii) $\sum \alpha^3$

Soln

[i] $\sum \alpha = -p$

[ii] $\sum \alpha\beta = q$

[iii] $\sum \alpha\beta\gamma = -r$

Therefore

$$\begin{aligned} \text{[i]} \quad \sum \alpha^2\beta &= (\sum \alpha\beta)(\sum \alpha) - 3\alpha\beta\gamma \\ &= q(-p) - 3(-r) \\ &= -pq + 3r \\ &= 3r - pq \end{aligned}$$

$$\begin{aligned}
 \text{[ii]} \quad \sum \alpha^2 &= (\sum \alpha)^2 - 2\sum \alpha\beta \\
 &= (-p)^2 - 2(q) \\
 &= p^2 - 2q
 \end{aligned}$$

$$\begin{aligned}
 \text{[iii]} \quad \sum \alpha^3 &= (\sum \alpha)^3 - 3(\sum \alpha)(\sum \alpha\beta) + 3\alpha\beta\gamma \\
 &= -p^3 + (3p)(q) - r \\
 &= -p^3 + 3pq - r
 \end{aligned}$$

Prove that the sum of cubes of the roots $x^3 - 6x^2 + 11x - 6 = 0$ is 36

Soln:

$$\begin{aligned}
 \sum \alpha^3 &= (\sum \alpha)^3 - 3(\sum \alpha)(\sum \alpha\beta) + 3\alpha\beta\gamma \\
 &= (-p)^3 + 3pq + 3r \\
 &= (6)^3 - 3(6)(11) + 3(6) \\
 &= 216 - 198 + 18
 \end{aligned}$$

Here P = 6, q = 11, r = 6

$$\begin{aligned}
 &= 216 - 198 - 18 \\
 &= 234 - 198
 \end{aligned}$$

$$\sum \alpha^3 = 36$$

- Hence, they proved $x^3 - 6x^2 + 11x - 6 = 0$ for cube roots is 36.
- If α, β, γ are the roots $x^3 - x - 1 = 0$ for equation where roots are $\frac{1}{\alpha^3}, \frac{1}{\beta^3}, \frac{1}{\gamma^3}$

Soln:

$$\text{▪ Let } y = \frac{1}{\alpha^3} = \frac{1}{x^3}, y = \frac{1}{x^3}$$

$$\therefore x^3 = \frac{1}{y}$$

Hence, $x^3 - x - 1 = 0$ _____ (1)

$$\frac{1}{y} - x - 1 = 0$$

$$\frac{1}{y} = x + 1$$

$$x = \frac{1-y}{y}$$

- Put $x = \frac{1-y}{y}$ in (1)

$$\left(\frac{1-y}{y}\right)^3 - \left(\frac{1-y}{y}\right) - 1 = 0$$

$$\frac{(1-y)^3}{y^3} - \frac{(1-y)}{y} - 1 = 0$$

$$(1-y)^3 - (1-y)y^2 - y^3 = 0$$

$$1 - 3y + 3y^2 - y^3 - y^2 + y^3 - y^3 = 0$$

$$1 - 3y + 3y^2 - y^3 - y^2 = 0$$

$$1 - 3y + 3y^2 - y^3 - y^2 = 0$$

$$-y^3 + 2y^2 - 3y + 1 = 0$$

$$y^3 - 2y^2 + 3y - 1 = 0$$

∴ This are corresponding equation.

If α, β, γ the roots of $x^3 - 3ax + 6 = 0$ show that $\sum(\alpha - \beta)(\alpha - \gamma) = 9a$

Soln:

- We have $\sum \alpha = 0, \sum \alpha\beta = -3a, \alpha\beta\gamma = -6$

$$\begin{aligned} \sum \alpha^2 &= (\sum \alpha)^2 - 2\sum \alpha\beta \\ &= 0 - 2(-3a) \\ &= 6a \end{aligned}$$

$$\sum(\alpha - \beta)(\alpha - \gamma) = \sum[\alpha^2 - \alpha\gamma - \alpha\beta + \beta\gamma]$$

$$= \sum \alpha^2 - \sum \alpha\gamma + \sum \alpha\beta + \sum \beta\gamma$$

$$= 6a - (-3a) - (-3a) + (-3a)$$

$$= 6a + 3a + 3a - 3a$$

$$= 9a$$

$$\therefore \sum (\alpha - \beta)(\alpha - \gamma) = 9a$$

Exercises

1. Choose the wrong answer from the following choices

Every nth degree equation $f(x) = 0$ has _____.

(A) atleast n roots

(B) atmost n roots

(C) exactly n roots

(D) atleast one real root

2. If the equation $x^3 - 4x^2 + 4x - 16 = 0$ has two roots $2i$ and $-2i$ then the other root is

(A) $1+i$

(B) $1-i$

(C) $2-i$

(D) 4

3. If α, β, γ are the roots of $x^3 + 2x - 6 = 0$ then the value of $\alpha\beta\gamma$ is

(A) 0

(B) 2

(C) 6

(D) -6

4. If the product of the roots of $3x^4 - 4x^3 + 2x^2 + x + a = 0$ is 21 then the value of a is

(A) 7

(B) -7

(C) -63

(D) 63

1.4. Transformation of Equation:

- Transform the equation $x^4 - 8x^3 - x^2 + 68x - 60 = 0$ into 1 which does not contain the terms in x^3 hence the solve the equation.

Soln:

Given: $x^4 - 8x^3 - x^2 + 68x - 60 = 0$ _____ (1)

Take $h = \frac{-a_1}{na_0} = \frac{8}{4} = 2$

$h = 2$

Diminish the root by 2



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UG TRB MATHEMATICS 2023-2024

UNIT II

Differential Calculus, Integral Calculus
& Analytical Geometry

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INDEX

UNIT - II - DIFFERENTIATION

S.N.	Topic	P. N.
2.1	Successive Differentiation n^{th} Derivatives	1
2.2	Standard n^{th} Derivative	11
2.3	Leibnitz Theorem	23
2.4	Partial Differentiation	37
2.5	Maxima And Minima Of Functions Of Two Variables	51
2.6	Method Of Langrange's Multipliers	61
2.7	Curvature And Radius Of Curvature	76
2.8	Radius Of Curvature In Polar Coordinates	94
2.9	Angle Between The Radius Vector And The Tangent	104
2.10	Pedal Equation – P – R Equation	108
2.11	Envelopes And Evolutes	127
2.12	Centre Of Curvature	141
2.13	Asymptotes:	159
2.14	Asymptotes By Inspection:	163
2.15	Double And Triple Integrals	176
2.16	Double Integral In Polar Co-Ordinates	187
2.17	Change Of Order Of Integration	195
2.18	Applications Of Double And Triple Integrals	205
2.19	Surface Area Of Solids	216
2.20	Jacobians	229
2.21	Beta, Gamma Functions	236
2.22	Evaluation Of Integrals Using Beta And Gamma Functions	244
2.23	Pole And Polar Co-Ordinates	264
2.24	Polar Equation Of A Conic	269
2.25.	Multiple Choice Questions	278

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UG TRB – MATHEMATICS – 2023-24

UNIT II : DIFFERENTIATION

2.1. SUCCESSIVE DIFFERENTIATION nth DERIVATIVES

- If y is a function of x , its derivative $\frac{dy}{dx}$ will be some other function of x and the differentiation

of this function with respect to x is called second derivative and is denoted by $\frac{d^2y}{dx^2}$.

$$\text{i.e., } \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d^2y}{dx^2}$$

- Similarly, the third derivative is denoted by $\frac{d^3y}{dx^3}$

$$\text{i.e., } \frac{d}{dx} \left(\frac{d^2y}{dx^2} \right) = \frac{d^3y}{dx^3}$$



- Thus, if we differentiate y twice with respect to x , we get the second derivative. If y is differentiated thrice with respect to x we get the third derivative.

Problem:

1. If $y = \frac{ax+b}{cx+d}$ Find $\frac{d^2y}{dx^2}$.

Solution:

$$y = \frac{ax+b}{cx+d}$$

$$\frac{dy}{dx} = \frac{(cx+d)a - (ax+b)c}{(cx+d)^2}$$

$$= \frac{acx + ad - acx - bc}{(cx+d)^2}$$

$$= \frac{ad - bc}{(cx+d)^2}$$

$$\frac{d^2y}{dx^2} = \frac{0 - (ad - bc)(2)(cx+d)c}{(cx+d)^4}$$

$$= \frac{-2c(ad - bc)}{(cx+d)^3}$$

2. If $x = a(\cos t + t \sin t)$ $y = a(\sin t - t \cos t)$ Find $\frac{d^2y}{dx^2}$.

Solution:

$$y = a(\sin t - t \cos t)$$

$$\frac{dy}{dt} = a(\cos t + t \sin t - \cos t)$$

$$= at \sin t$$

$$x = a(\cos t - t \sin t)$$

$$\frac{dx}{dt} = a(-\sin t + t \cos t + \sin t)$$

$$= at \cos t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{at \sin t}{at \cos t} = \tan t$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d}{dt} \left(\frac{dy}{dx} \right) \frac{dt}{dx}$$

$$= \frac{d}{dt} (\tan t) \cdot \frac{dt}{dx}$$

$$= \sec^2 t \cdot \frac{1}{at \cos t}$$

$$= \frac{\sec^3 t}{at}$$

3. If $y = a \cos 5x + b \sin 5x$ show that $\frac{d^2 y}{dx^2} + 25y = 0$

Solution:

$$y = a \cos 5x + b \sin 5x$$

Differentiating with respect to x ,

$$\frac{dy}{dx} = -5a \sin 5x + 5b \cos 5x$$

$$\frac{d^2 y}{dx^2} = -25a \cos 5x - 25b \sin 5x$$

$$= -25(a \cos 5x + b \sin 5x)$$

$$= -25y$$

$$\frac{d^2 y}{dx^2} + 25y = 0$$

4. If $y = a \cos(\log x) + b \sin(\log x)$ show that $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$

Solution:

$$y = a \cos(\log x) + b \sin(\log x)$$

Differentiating with respect to x ,

$$\frac{dy}{dx} = \frac{-a \sin(\log x)}{x} + \frac{b \cos(\log x)}{x}$$

$$x \frac{dy}{dx} = -a \sin(\log x) + b \cos(\log x)$$

Again, differentiating with respect to x ,



$$x \frac{d^2 y}{dx^2} + \frac{dy}{dx} \cdot 1 = \frac{-a \cos(\log x)}{x} - \frac{b \sin(\log x)}{x}$$

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$$

5. If $(y = x + \sqrt{1+x^2})^m$ show that $(1+x^2)y_2 + xy_1 - m^2 y = 0$

Solution:

$$y = (x + \sqrt{1+x^2})^m$$

Differentiating with respect to x ,

$$\frac{dy}{dx} = m(x + \sqrt{1+x^2})^{m-1} \left[1 + \frac{2x}{2\sqrt{1+x^2}} \right]$$

$$= \frac{m(x + \sqrt{1+x^2})^{m-1} [\sqrt{1+x^2} + x]}{\sqrt{1+x^2}}$$

$$= \frac{m(x + \sqrt{1+x^2})^m}{\sqrt{1+x^2}}$$

$$= \frac{my}{\sqrt{1+x^2}}$$

Cross multiplying and squaring we get,

$$(1+x^2) \left(\frac{dy}{dx} \right)^2 = m^2 y^2$$

$$(1+x^2) 2 \frac{dy}{dx} \cdot \frac{d^2 y}{dx^2} + \left(\frac{dy}{dx} \right)^2 \cdot 2x = m^2 \cdot 2y \cdot \frac{dy}{dx}$$

Cancelling, $2 \frac{dy}{dx}$ we get,

$$(1+x^2) \frac{d^2 y}{dx^2} + \frac{dy}{dx} - m^2 y = 0$$

$$(1+x^2)y_2 + xy_1 - m^2 y = 0$$

6. If $y = e^{a \sin^{-1} x}$ show that $(1+x^2)y_2 + xy_1 - a^2y = 0$.

Solution:

$$y = e^{a \sin^{-1} x}$$

Differentiating with respect to x ,

$$\frac{dy}{dx} = e^{a \sin^{-1} x} \cdot \frac{a}{\sqrt{1-x^2}} = \frac{ay}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2} \frac{dy}{dx} = ay$$

$$(1-x^2) \left(\frac{dy}{dx} \right)^2 = a^2 y^2$$

Differentiating with respect to x ,

$$(1-x^2) \cdot 2 \frac{dy}{dx} \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx} (-2x) = a^2 \cdot 2y \frac{dy}{dx}$$

Cancelling $2 \frac{dy}{dx}$ throughout,

$$(1-x^2)y_2 - xy_1 - a^2y = 0$$

7. If $y = \sin(m \sin^{-1} x)$ show that $(1-x^2)y_2 - xy_1 + m^2y = 0$

Solution:

$$y = \sin(m \sin^{-1} x)$$

$$\sin^{-1} y = m \sin^{-1} x$$

Differentiating with respect to x ,

$$\frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} = \frac{m}{\sqrt{1-x^2}}$$

Squaring and cross multiplying,

$$(1-x^2) \left(\frac{dy}{dx} \right)^2 = m^2 (1-y^2)$$

Differentiating with respect to x we get

$$(1-x^2)2\frac{dy}{dx}\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2(-2x) = m^2\left(-2y\frac{dy}{dx}\right)$$

Cancelling $2\frac{dy}{dx}$,

$$(1-x^2)y_2 - xy_1 + m^2y = 0$$

8. If $2x = y^{\frac{1}{m}} + y^{\frac{-1}{m}}$ prove that $(x^2 - 1)y_2 + xy_1 - m^2y = 0$

Solution:

$$2x = y^{\frac{1}{m}} + y^{\frac{-1}{m}}$$

Differentiating with respect to x , we get,

$$2 = \frac{1}{m} \cdot y^{\frac{1}{m}-1} \cdot y_1 - \frac{1}{m} y^{\frac{-1}{m}-1} \cdot y_1$$

$$= \frac{y_1}{my} \left(y^{\frac{1}{m}} - y^{\frac{-1}{m}} \right)$$

$$2my = y_1 \left(y^{\frac{1}{m}} - y^{\frac{-1}{m}} \right)$$

Squaring,

$$4m^2y^2 = y_1^2 \left(y^{\frac{1}{m}} - y^{\frac{-1}{m}} \right)^2$$

$$4m^2y^2 = y_1^2 \left[\left(y^{\frac{1}{m}} - y^{\frac{-1}{m}} \right)^2 - 4 \right]$$

$$4m^2y^2 = y_1^2(4x^2 - 4)$$

$$m^2y^2 = y_1^2(x^2 - 1)$$

Differentiating with respect to x ,

$$m^2 \cdot 2y \frac{dy}{dx} = y_1^2 2x + (x^2 - 1) 2y_1 \cdot y_2$$

Cancelling $2y_1$, we get, $(x^2 - 1)y_2 + xy_1 - m^2y = 0$

9. If $y = \frac{1}{2}(\sin^{-1} x)^2$ show that $(1-x^2)y_2 - xy_1 = 1$

Solution:

$$y = \frac{1}{2}(\sin^{-1} x)^2$$

Differentiating with respect to x ,

$$y_1 = \frac{1}{2} \cdot 2(\sin^{-1} x) \cdot \frac{1}{\sqrt{1-x^2}}$$

Squaring and cross multiplying we get,

$$(1-x^2)y_1^2 = (\sin^{-1} x)^2$$

$$\text{i.e., } (1-x^2)y_1^2 = 2y$$

Differentiating again with respect to x ,

$$(1-x^2) = 2y_1y_2 + y_1^2(-2x) = 2y_1$$

Cancelling $2y_1$ throughout

$$(1-x^2)y_2 - xy_1 = 1$$

10. If $x = \sin t, y = \sin pt$ obtain $\cos t \frac{dy}{dx} = p \cot p$. Now differentiating both side with

Respect to x deduce $(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + p^2y = 0$.

Solution:

$$x = \sin t, y = \sin pt$$

$$\frac{dx}{dt} = \cos t, \frac{dy}{dt} = p \cos pt$$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{p \cos pt}{\cos t}$$

$$\cos t \frac{dy}{dx} = p \cos pt$$

Differentiating both sides with respect to x ,

$$\cos t \frac{d^2 y}{dx^2} + \frac{dy}{dx} (-\sin t) \frac{dt}{dx} = p^2 (-\sin pt) \frac{dt}{dx}$$

$$\therefore \cos t \frac{dx}{dt} \frac{d^2 y}{dx^2} - \sin t \frac{dy}{dx} + p^2 \sin pt = 0$$

$$(1 - \sin^2 t) \frac{d^2 y}{dx^2} - \sin t \frac{dy}{dx} + p^2 \sin pt = 0 \quad \left(\text{since } \frac{dx}{dt} = \cos t\right)$$

$$(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + p^2 y = 0$$

11. If $y = (\tan^{-1} x)^2$ show that $(1 + x^2)^2 y_2 + 2x(1 + x^2) y_1 = 2$

Solution:

$$y = (\tan^{-1} x)^2$$

Differentiating with respect to x ,

$$y_1 = \frac{2 \tan^{-1} x}{1 + x^2}$$

$$(1 + x^2) y_1 = 2 \tan^{-1} x$$

Again differentiating $(1 + x^2) y_2 + y_1 2x = \frac{2}{1 + x^2}$

$$= (1 + x^2)^2 y_2 + 2x(1 + x^2) y_1 = 2$$

12. If $y = a \sin^m x$ prove that $\sin^2 x \cdot \frac{d^2 y}{dx^2} = (m^2 \cos^2 x - m)y$

Solution:

$$y = \sin^m x$$

Differentiating with respect to x ,

$$\frac{dy}{dx} = m \sin^{m-1} x \cos x$$

$$\frac{d^2y}{dx^2} = m(m-1)\sin^{m-2}x \cdot \cos^2x - m\sin^m x$$

Multiplying both sides by $\sin^2 x$,

$$\begin{aligned} \therefore \sin^2 x \frac{d^2y}{dx^2} &= m(m-1)\sin^m x \cos^2 x - m\sin^m x \cdot \sin^2 x \\ &= m(m-1)y \cos^2 x - my \sin^2 x \\ &= m^2 y \cos^2 x - my \cos^2 x - my \sin^2 x \\ &= m^2 y \cos^2 x - my(\cos^2 x + \sin^2 x) \\ \therefore \sin^2 x \frac{d^2y}{dx^2} &= m^2 y \cos^2 x - my \\ &= (m \cos^2 x - m)y \end{aligned}$$

13. If $y = -x^3 \log x$ prove that $x \frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + 3x^2 = 0$

Solution:

$$y = -x^3 \log x$$

$$\frac{dy}{dx} = -\frac{x^3}{3} - 3x^2 \log x$$

$$= -x^2 - 3x^2 \log x$$

$$\frac{d^2y}{dx^2} = -2x - \frac{3x^2}{x} - 6x \log x$$

$$x \frac{d^2y}{dx^2} = -2x^2 - 3x^2 - 6x^2 \log x$$

$$= -3x^2 - 2(x^2 + 3x^2 \log x)$$

$$\therefore x \frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + 3x^2 = 0$$

EXERCISES

1. If $y = a \cos 5x + b \sin 5x$ then $\frac{d^2 y}{dx^2}$ is _____.
- (A) 25 (B) -25 (C) -25 y (D) y
2. If $x = \sin t, y = \sin pt$ then $\frac{dy}{dt} =$ _____.
- (A) $\sin pt$ (B) $p \sin pt$ (C) $p \cos pt$ (D) $\cos pt$
3. If $y = (\tan^{-1} x)^2$ then $y_1 =$ _____.
- (A) $\frac{2 \tan^{-1} x}{1+x^2}$ (B) $\tan^{-1} x$ (C) $\frac{\tan^{-1} x}{1+x^2}$ (D) $\frac{2}{1+x^2}$
4. If $x = at^2, y = 2at$ then $y_2 =$ _____.
- (A) $\frac{1}{2at^3}$ (B) $\frac{-1}{2at^3}$ (C) $\frac{1}{at^3}$ (D) $\frac{-1}{t^3}$
5. If $y = \frac{1}{2}(\sin^{-1} x)^2$ then _____.
- (A) $(1-x^2)y_1^2 = 2y$ (B) $y_1^2 = 2y$ (C) $(1-x^2)y_1^2 = y$ (D) none
6. The n^{th} derivative of e^{ax} is
- (A) $y_n = a^n$ (B) $y_n = a^n e^{ax}$ (C) $y_n = e^{ax}$ (D) $y_n = ne^{ax}$
7. The n^{th} derivative of $\sin(ax+b)$ is
- (A) $a^n \sin(ax+b)$ (B) $a^n \sin\left(ax+b+\frac{\pi}{2}\right)$
- (C) $\sin\left(ax+b+\frac{n\pi}{2}\right)$ (D) $n \sin\left(ax+b+\frac{n\pi}{2}\right)$
8. If $y = \tan^{-1}\left(\frac{x}{a}\right)$ then $y_1 =$ _____.
- (A) $\frac{a}{x^2+a^2}$ (B) $\frac{1}{x^2+a^2}$ (C) $\frac{1}{x^2-a^2}$ (D) $\frac{a}{x^2-a^2}$



9. If n^{th} derivative of $\frac{1}{(2x+3)^2}$ is _____.

(A) $\frac{(-1)^n (n+1)!}{(2x+3)^{n+2}}$ (B) $\frac{(-1)^n 2^n (n+1)!}{(2x+3)^{n+2}}$ (C) $\frac{(n+1)!}{(2x+3)^{n+2}}$ (D) $\frac{(-1)2^n}{(2x+3)^{n+2}}$

10. The n^{th} derivative of $\sin 2x$ is _____.

(A) $2^n \sin\left(2x + \frac{n\pi}{2}\right)$ (B) $2^n \sin 2x$ (C) $\sin\left(2x + \frac{n\pi}{2}\right)$ (D) none

2.2. STANDARD n^{th} DERIVATIVE

1. n^{th} derivative of e^{ax} .

Solution:

$$y = e^{ax}$$

$$y_1 = e^{ax} \cdot a$$

$$y_2 = e^{ax} \cdot a^2$$

$$y_3 = e^{ax} \cdot a^3$$

$$\therefore y_n = a^n e^{ax}$$

2. n^{th} derivative of $\frac{1}{ax+b}$

Solution:

$$y = \frac{1}{ax+b} = (ax+b)^{-1}$$

$$y_1 = -1(ax+b)^{-2} \cdot a$$

$$y_2 = (-1)(-2)(ax+b)^{-3} \cdot a^2$$

$$y_3 = (-1)(-2)(-3)(ax+b)^{-4} \cdot a^3$$

$$\therefore y_n = (-1)(-2)(-3)\dots(-n)(ax+b)^{-(n+1)} \cdot a^n$$

$$= \frac{(-1)^n n! a^n}{(ax+b)^{n+1}}$$

3. n^{th} derivative of $\frac{1}{(ax+b)^2}$

$$y = (ax+b)^{-2}$$

$$y_1 = (-2)(ax+b)^{-3} \cdot a$$

$$y_2 = (-2)(-3)(ax+b)^{-4} \cdot a^2$$

$$y_3 = (-2)(-3)(-4)(ax+b)^{-5} \cdot a^3$$

.

.

.

$$y_n = (-2)(-3)(-4)\dots(-n+1)(ax+b)^{-(n+2)} \cdot a^n$$

$$= \frac{(-1)^n (n+1)! a^n}{(ax+b)^{n+2}}$$

4. n^{th} derivative of $\log(ax+b)$

$$y = \log(ax+b)$$

$$y_1 = \frac{1}{ax+b} \cdot a$$

$$\therefore y_n = \frac{(-1)^{n-1} (n-1)! a^n}{(ax+b)^n}$$

5. n^{th} derivative of $\sin(ax+b)$

$$y = \sin(ax+b)$$



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UG TRB MATHEMATICS 2023-2024

UNIT III

Differential Equations & Laplace Transformations

Your Success is Our Goal....

INDEX - UNIT - III - DIFFERENTIAL EQUATIONS

S.N.	Topic	P. N.
3.1.	Ordinary Differential Equations	1
3.2.	Homogeneous Differential Equations	3
3.3.	Exact Differential Equation	5
3.4.	Integrating Factors	8
3.5.	Linear Equations	11
3.6.	Reduction of Order	13
3.7.	Second Order Linear Differential Equations	16
3.8.	Second Order Differential Equations with Constant Coefficients	17
3.9.	Method of undetermined Co-Efficients	35
3.10.	Variation of Parameter	49
3.11.	System of First Order Equations	56
3.12.	Simultaneous Linear Equations with Constant Co-Efficients	60
3.13.	Partial Differential Equation	63
	3.13.1. Eliminating arbitrary constants	64
	3.13.2. Elimination of Arbitrary Functions	69
3.14.	Complete Solution (or) Complete Integral	79
3.15.	Chapits Method	100
3.16.	Lagrange's Linear Partial Differential Equations	103
3.17.	Special Types of First Order Equations	109
3.18.	Laplace Transform	117
3.19.	Laplace Transform of Elementary Functions	118
3.20.	Properties of Laplace Transform	121
	3.20.1. Laplace Transform of Derivatives	123
	3.20.2. Laplace Transform of Integrals	124
3.21.	Evaluation of integrals using L.T, Initial Theorems, Final Theorems	128
3.22.	Laplace Transform of Periodic Functions	138
3.23.	Inverse Laplace Transform	140
3.24.	Problems of Convolution Theorem	152
3.25.	Application of Laplace Transforms	157
3.26.	Simultaneous Differential Equations	170
3.27.	Multiple Choice Questions	174

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UG TRB - MATHEMATICS – 2023-24

UNIT – III

DIFFERENTIAL EQUATIONS AND LAPLACE TRANSFORMATION

3.1. Ordinary Differential Equations:

- An ordinary differential equation is an equation which is defined for one or more functions of one independent variable and its derivations. It is abbreviated as ODE. Example $\frac{dy}{dx} = x + 3$
- When the function involved in the equation depends on only a single variable, its derivatives are ordinary derivatives and the differential equation is classed as an ordinary differential equation.
- On the other hand, if the function depends on several independent variables the differential equation is classed as a partial differential equation.

Order and Degree of Ordinary Differential Equations:

- The order of differential equation is the highest derivative in the equation is the **highest derivative** in the equation. The degree of the diffi. equation of the highest power to which the derivative is raised.

Problems:

1. Solve: $9yy' + 4x = 0$

Solution:

$$9y \frac{dy}{dx} = -4x \Rightarrow 9ydy = -4x dx$$



Integrating we get,

$$\frac{9y^2}{2} = \frac{-4x^2}{2} + c$$

$$\Rightarrow \frac{y^2}{4} = \frac{-x^2}{9} + c \Rightarrow \frac{x^2}{9} + \frac{y^2}{4} = c$$

2. Solve: $\frac{dy}{dx} = \frac{2x + y - 1}{4x + 2y - 4}$

Solution:

Let $V = \frac{2x + y}{2}$,

The D.E. becomes

$$\therefore \frac{dy}{dx} = 1 + \frac{1}{2} \left(\frac{2v - 1}{4v - 4} \right)$$

$$\Rightarrow \frac{8v - 8}{10v - 9} v = dx \Rightarrow \left[\frac{8v - \frac{36}{5} - \frac{4}{5}}{10v - 9} \right] dv = dx$$

$$\Rightarrow \left[\frac{4}{5} - \frac{4}{5} \left[\frac{1}{10v - 9} \right] \right] dv = dx$$

Int. we get

$$\frac{4v}{5} - \frac{2}{25} \log(10v - 9) + c = x$$

$$\Rightarrow \frac{2}{5}(2x + y) - \frac{2}{25} \log(10x + 5y - 9) + c = x$$

$$\Rightarrow \frac{x}{5} + \frac{2y}{5} - \frac{2}{25} \log(10x + 5y - 9) + c = 0$$

Exercises

1. An ordinary differential equation is an equation which is defined for one or more functions of _____ independent variables.

(A) several

(B) one

(C) two

(D) more than one

2. The order of this equation is _____ $\left[\frac{d^2y}{dx^2} + 2y \right]^3 + \frac{d^3y}{dx^3} + y = 0$
- (A) 2 (B) 1 (C) 3 (D) none
3. A general solution of the equation $y' = \cos x$ is _____.
- (A) $y = \sin x + c$ (B) $y = \operatorname{cosec} x + c$ (C) $y = \cos x + c$ (D) $y = \sec x + c$

3.2. Homogeneous Differential Equations:

Homogeneous Function:

- A function $f(x, y)$ in x and y is said to be a homogenous function if the degree of each term in the function is constant. In general, a homo function $f(x, y)$ of degree n is expressible as

$$f(x, y) = \lambda^n f\left(\frac{y}{x}\right)$$

Homogeneous Differential Equation

- A differential Equation in which all the functions are of the same degree is called a homogenous differential equation

Example:

$$\frac{dy}{dx} = \frac{x^2 - y^2}{xy} \text{ is a homogeneous differential equation.}$$

- Homogenous differential equations are differential equations with homogeneous functions. They are equations containing a differentiation operator, a function and a set of variable. The general form of the homogenous differential equation is $f(x, y)dy + g(x, y)dx = 0$, where $f(x, y)$ and $g(x, y)$ is a homo. function.
- Homo. functions are defined as functions in which the total power of all the terms of the function is constant.
- Homo. function and homogenous differential equation are represented in the below form.

$$\text{Homo. function: } f(x, y) = \lambda^n f\left(\frac{y}{x}\right)$$

$$\text{Homo. Differential equation: } \frac{dy}{dx} = f(x, y)$$

Exercises

4. The solution of the differential equation $xy^2dy - (x^3 + y^3)dx = 0$ is _____.

(A) $y^3 = 3x^3 + c$

(B) $y^3 = 3x^3 \log(cx)$

(C) $y^3 = 3x^3 + \log(cx)$

(D) none

5. The solution of differential equation $\cos(x+y)dy = dx$ is _____.

(A) $y = x \sec\left(\frac{y}{x}\right) + c$

(B) $y + \cos^{-1}\left(\frac{y}{x}\right) = c$

(C) $y = \tan\left(\frac{x+y}{2}\right) + c$

(D) $y = \cot\left(\frac{x+y}{2}\right) + c$

3.3. Exact Differential Equation:

- A differential equation is said to be exact if it can be derived directly from its primitive without any further operation of elimination or reduction. Thus the differential equation

$$M(x, y)dx + N(x, y)dy = 0 \quad \text{_____ (1)}$$

it exact if it can be derived by equating the differential of some function $V(x, y)$ to zero.

Let $v(x, y) = c$ be the solution

Differentiating this we get

$$\frac{\partial u}{\partial x} dx + \frac{\partial v}{\partial y} dy = 0 \quad \text{_____ (2)}$$

(1) and (2) are identical

$$M = \frac{\partial u}{\partial x}, N = \frac{\partial u}{\partial y}$$

If we eliminate v between there by means of the equivalence of the relation

$$\frac{\partial}{\partial x} \left(\frac{\partial u}{\partial y} \right) = \frac{\partial^2 u}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial u}{\partial x} \right) \text{ we get}$$

Thus, the condition for $Mdx + Ndy = 0$ to be an exact equation is

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Rule for solving $Mdx + N dy = 0$ when it is exact

- (i) First integrate M w.r.to x regarding y as a constant.
- (ii) Then integrate w.r.to y those terms in N which do not contain x .
- (iii) The sum of the expressions obtained in (i) and (ii), when equated to an arbitrary constant, will be the solution.

Problems:

1. Solve $(\sin x \cos y + e^{2x})dx + (\cos x \sin y + \tan y)dy = 0$

Solution:

Here $M = \sin x \cos y + e^{2x}$

$N = \cos x \sin y + \tan y$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, the equation is exact, integrating M w.r.to x regarding y as a constant we get

$$\left[-\cos x \cos y + \frac{1}{2} e^{2x} \right]$$

In N , the term not involving x namely $\tan y$ is integrated w.r.to y giving $\log \sec y$

\therefore the solution is

$$-\cos x \cos y + \frac{e^{2x}}{2} + \log \sec y = c$$

2. Solve $(ye^{xy} - 2y^3)dx + (xe^{xy} - 6xy^2 - 2y)dy = 0$

Solution:

$M = ye^{xy} - 2y^3, \frac{\partial M}{\partial y} = e^{xy} + xye^{xy} - 6y^2$

$N = xe^{xy} - 6xy^2 - 2y, \frac{\partial N}{\partial x} = e^{xy} + xye^{xy} - 6y^2$

Since $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, the equation is exact

$$\int Mdx = \int (ye^{xy} - 2y^3)dx$$

$$= y \frac{e^{xy}}{y} - 2xy^3 = e^{xy} - 2xy^3$$

Integrating those terms in N which do not contain x, with respect to y, we get

$$\int N dy = \int -2y dy = -y^2, \text{ omitting terms involving } x \text{ in N.}$$

$$\therefore \text{The solution is } e^{xy} - 2xy^3 = y^2 = c$$

3. Solve $y(2x^2y + e^x)dx - (e^x + y^3)dy = 0$

Solution:

$$M = 2x^2y^2 + ye^x : \frac{\partial M}{\partial y} = 4x^2y + e^x$$

$$N = -(e^x + y^3) : \frac{\partial N}{\partial x} = -e^x$$

As $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$, the equation is not exact, However, we can rearrange the equation as

$$ye^x dx - e^x dy + (2x^2 dx - y dy)y^2 = 0$$

Now dividing by y^2 , we have

$$\frac{ye^x dx - e^x dy}{y^2} + 2x^2 dx - y dy = 0$$

$$d\left(\frac{e^x}{y}\right) + 2x^2 dx - y dy = 0$$

Integrating, we find the solution as $\frac{e^x}{y} + \frac{2x^3}{3} - \frac{y^2}{2} = c$

4. Solve $(\log x + y)dx - xdy = 0$

Solution:

Observing that the equation is not proper we arrange it as

$$\log x dx + y dx - x dy = 0$$

Dividing by x^2 (an integrating factor) we get

$$\frac{1}{x^2} \log x dx + \left(\frac{y dx - x dy}{x^2} \right) = 0$$

$$\int \frac{1}{x^2} \log x dx + \int d \left(\frac{-y}{x} \right) = 0$$

$$\int \log x d \left(\frac{-1}{x} \right) - \frac{y}{x} = c$$

$$\frac{-\log x}{x} + \int x^{-2} dx - \frac{y}{x} = c$$

$$\frac{-\log x}{x} - \frac{1}{x} - \frac{y}{x} = c$$

or $cx + y + \log x + 1 = 0$ is the solution.

Exercises

6. A differential equation of the form $M(x, y)dx + N(x, y)dy = 0$ is said to be _____ if it can be directly obtained from its primitive by differentiation.
- (A) Linear equation (B) Separable equation
(C) Exact equation (D) Lagrange's equation
7. The solution of $[\sec x \tan x \tan y - e^x] dx + [\sec x \sec^2 y] dy = 0$ is _____.
- (A) $\tan y - e^x = c$ (B) $\sec x \tan y - e^x = c$
(C) $\tan x \sec y - e^x = c$ (D) $\sec x \tan y = c$
8. The exact condition value of $(x^3 + 3xy^2) dx + (3x^2 y + y^3) dy = 0$ is _____.
- (A) $6xy$ (B) $3xy$ (C) $2xy$ (D) $12xy$
9. The diff. equation $2y dx - (3y - 2x) dy = 0$ is
- (A) exact and homogenous but not linear
(B) exact, homogenous and linear
(C) exact and linear but not homogeneous
(D) homogenous and linear but not exact



3.4. Integrating Factors:

Rule 1:

- When $Mx + Ny \neq 0$, and the equation is a homogenous one, $\frac{1}{Mx + Ny}$ is an integrating factor.

Problems:

1. Solve $x^2 y dx - (x^3 + y^3) dy = 0$

Solution:

The equation is not exact and $Mx + Ny = y^4 \neq 0$. Hence $-\frac{1}{y^4}$ can be used as an I.F. then

$$-\frac{x^2}{y^3} dx + \left(\frac{x^3 + y^3}{y^4} \right) dy = 0$$

$$\frac{\partial M}{\partial y} = \frac{3x^2}{y^4}, \quad \frac{\partial N}{\partial x} = \frac{3x^2}{y^4}$$

Hence the equation has become exact

$$\int M dx = \int \frac{x^2}{y^3} dx = -\frac{x^3}{3y^3}$$

In N, integrating the term not containing x , namely $\frac{1}{y}$ w.r.to y we get $\log y$

$$\therefore \text{the solution is } -\frac{x^3}{3y^3} + \log y = c$$

Rule 2:

If the equation is of the form $f_1(xy) dx + x f_2(xy) dy = 0$ and $Mx - Ny \neq 0$, then $\frac{1}{Mx - Ny}$

is an I.F.

2. Solve $y(x^2 y^2 + xy + 1) dx + x(x^2 y^2 - xy + 1) dy = 0$

Solution:

The equation is not exact since $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

$$Mx - Ny = x^3y^3 + x^2y^2 + xy - x^3y^3 + x^2y^2 - xy = 2x^2y^2 \neq 0$$

Using $\frac{1}{Mx - Ny} = \frac{1}{2x^2y^2}$ as an I.F. we get

$$\left(\frac{x^2y^2 + xy + 1}{2x^2y} \right) dx + \left(\frac{x^2y^2 - xy + 1}{2xy^2} \right) dy = 0$$

$$\left(y + \frac{1}{x} + \frac{1}{x^2y} \right) dx + \left(x - \frac{1}{y} + \frac{1}{xy^2} \right) dy = 0$$

Now $\frac{\partial M}{\partial y} = 1 - \frac{1}{x^2y^2}$ and $\frac{\partial N}{\partial x} = 1 - \frac{1}{x^2y^2}$

∴ The equation is exact and the solution is

$$\int \left(y + \frac{1}{x} + \frac{1}{x^2y} \right) dx + \int -\frac{1}{y} dy = c$$

$$xy + \log x - \frac{1}{xy} - \log y = c$$

Rule 3:

(i) If $\frac{1}{N} \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right)$ is a function of x alone, say $f(x)$, then $e^{\int f(x) dx}$ is an integration factor.

(ii) If $\frac{1}{M} \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right)$ is a function of y alone, say $g(y)$, then $e^{\int g(y) dy}$ is an integrating factor.

3. Solve $(xy^3 + y)dx + 2(x^2y^2 + x + y + y^4)dy = 0$

Solution:

The equation is not exact and $\frac{\partial M}{\partial y} = 3xy^2 + 1$, $\frac{\partial N}{\partial x} = 4xy^2 + 2$

$$\frac{1}{M} \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) = \frac{1}{y} = g(y)$$

$e^{\int g(y) dy} = e^{\log y} = y$ is an integrating factor multiplying by y we get the equation

$$(xy^4 + y^2)dx + 2(x^2y^3 + xy + y^5)dy = 0$$

Now the equation is exact and the solution is

$$\int (xy^4 + y^2)dx + 2\int y^5 dy = c$$

$$3y^4x^2 + 6xy^2 + 2y^6 = c$$

Rule 4:

If the equation $Mdx + Ndy = 0$ is of the form

$$x^a y^b (mydx + nxdy) + x^r y^s (pydx + qxdy) = 0$$

where a, b, m, n, r, s, p, q are constants, then

$x^h y^k$, is an integrating factor, where h and k are determined using the condition that after multiplication by $x^h y^k$, the equation becomes exact.

4. Solve $(y^3 - 2yx^3)dx + (2xy^2 - x^3)dy = 0$

Solution:

The equation is not an exact one and it can be rewritten as

$$y(y^2 - 2x^2)dx + x(2y^2 - x^2)dy = 0$$

$$y^2(ydx + 2xdy) + x^2(-2ydx - xdy) = 0$$

So that is of the form mentioned in rule IV above. Multiplying the equation by $x^h y^k$ we get

$$(x^h y^{3+k} - 2x^{h+2} y^{k+1})dx + (2x^{h+1} x^{h+3} y^k)dy = 0$$

Now $\frac{\partial M}{\partial y} = (3+k)x^h y^{k+2} - 2(k+1)x^{h+2} y^k$ and $\frac{\partial N}{\partial x} = 2(h+1)x^h y^{k+2} - (h+3)x^{h+2} y^k$

Using the condition $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ and equating the coefficients of like lowered terms on both

sides, we get

$$3+k = 2(h+1)$$

$$2k+2 = h+3$$

Solving them we get $k = 1$, $h = 1$ so that the integrating factor is xy

The equation (1) for these values of h and k becomes

$$(xy^4 - 2x^3y^2) + (2x^2y^3 - x^4y)dy = 0$$

At this equation is exact, the solution is

$$\int (xy^4 - 2x^3y^2) dx = c, \frac{x^2y^4}{2} - \frac{2x^4y^2}{4} = c$$

$$x^2y^4 - x^4y^2 = k$$

Exercises

10. If the equation is of the form $f_1(x, y)dx + f_2(x, y)dy = 0$, when $Mx + Ny \neq 0$ then the integrating factor is _____.

- (A) $\frac{1}{Mx + Ny}$ (B) $Mx + Ny$ (C) $\frac{1}{Mx - Ny}$ (D) $Mx - Ny$

11. For the equation $(xy^3 + y)dx + 2(x^2y^2 + x + y + y^4)dy = 0$ the integrating factor is _____.

- (A) x (B) $2x$ (C) y (D) $-y$

12. If $\frac{1}{M} \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right)$ is a function of y alone, say $g(y)$ then _____ is an integrating factor.

- (A) $e^{\int f(x)dx}$ (B) $e^{\int g(y)dy}$ (C) $e^{\int g(y)dx}$ (D) none

3.5. Linear Equations:

- A differential equation of the form $\frac{dy}{dx} + Py = Q$ where P and Q are function of x , is said to be a linear equation in y .

Multiplying both sides by $Qe^{\int p dx}$ we get

$$e^{\int p dx} \left(\frac{dy}{dx} + Py \right) = Qe^{\int p dx}$$

$$\frac{d}{dx} \left(ye^{\int p dx} \right) = Qe^{\int p dx}$$

Integrating we get the solution as $ye^{\int p dx} = \int Qe^{\int p dx} dx + c$

Problems

1. Solve $\frac{dy}{dx} + y \cot x = 4x \operatorname{cosec} x$ given that $y = 0$ when $x = \frac{\pi}{2}$.

Solution:

Comparing with $\frac{dy}{dx} + Py = Q$ we find that

$$P = \cot x, Q = 4x \operatorname{cosec} x$$

$$\int P dx = \int \cot x dx = \log \sin x$$

$$e^{\int P dx} = e^{\log \sin x}$$

Solution is $y \sin x = \int 4x \operatorname{cosec} x \sin x dx$

$$= \int 4x dx = 2x^2 + c$$

$$y = 0 \text{ when } x = \frac{\pi}{2} \text{ gives } c = \frac{\pi^2}{2}$$

\therefore The solution is

$$y \sin x = 2x^2 - \frac{\pi^2}{2}$$

2. Solve $(1 + y^2) dx = (\tan^{-1} y - x) dy$

Solution:

$$\frac{dx}{dy} + \frac{1}{1+y^2} x = \frac{\tan^{-1} y}{1+y^2}$$

This is an equation of the type

$$\frac{dx}{dy} + Px = Q, \text{ which is linear in } x,$$

$$P = \frac{1}{1+y^2}; Q = \frac{\tan^{-1} y}{1+y^2}$$

$$\int P dy = \int \frac{dy}{1+y^2} = \tan^{-1} y$$

$$e^{\int P dy} = e^{\tan^{-1} y}$$

∴ the solution is $xe^{\int P dy} = \int Qe^{\int P dy} dy + c$

$$xe^{\tan^{-1} y} = \int e^{\tan^{-1} y} \frac{\tan^{-1} y}{1+y^2} dy + c$$

Putting $t = \tan^{-1} y$ on the R.H.S, we get

$$\begin{aligned} xe^{\tan^{-1} y} &= \int te' dt + c \\ &= te^t - e^t + c \end{aligned}$$

∴ Solution is $xe^{\tan^{-1} y} = e^{\tan^{-1} y} (\tan^{-1} y - 1) + c$ or $x = \tan^{-1} y - 1 + ce^{-\tan^{-1} y}$

Exercises

13. Solving the differential equation $\frac{dy}{dx} + \frac{y}{x} = 4x^2$ we get the solution _____.

(A) $x^2 + c$

(B) $x^3 + \frac{c}{x}$

(C) $x^2 + \frac{c}{x}$

(D) $x^3 + c$

14. The solution of the differential equation $x \frac{dy}{dx} - y = 3$ represents a family of _____.

(A) straight line

(B) circle

(C) ellipse

(D) parabola

15. A differential equation of the form $\frac{dy}{dx} + Py = Q$ has the solution as _____.

(A) $ye^{\int P dx} = \int Q dx + c$

(B) $ye^{\int P dx} = \int Qe^{\int P dx} dx + c$

(C) $y = \int Qe^{\int P dx} dx + c$

(D) $ye^{\int P dx} = \int e^{\int P dx} dx + c$

3.6. Reduction of Order:

Equations Reducible to Linear Form

Consider the equation $\frac{dy}{dx} + Py = Qy^n$

Where P and Q are functions of x



Dividing by y^n we get

$$y^{-n} \frac{dy}{dx} + y^{1-n} P = Q$$

Putting $V = y^{1-n}$, $\frac{dv}{dx} = (1-n)y^{-n} \frac{dy}{dx}$

Using the above equation

$$\frac{dv}{dx} + (1-n)vP = (1-n)Q$$

which is a linear equation in v and hence can be solved by the previous method.

Problems

1. Solve $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$

Solution:

Dividing by $\cos^2 y$ we get

$$\sec^2 y \frac{dy}{dx} + 2x \tan y = x^3$$

Let $v = \tan y$ then $\frac{dv}{dx} = \sec^2 y \frac{dy}{dx}$ _____ (1)

\therefore (1) becomes $\frac{dv}{dx} + 2vx = x^3$

$$P = 2x; Q = x^3$$

$$\int P dx = \int 2x dx = x^2 \text{ and } e^{\int P dx} = e^{x^2}$$

Solution is $ve^{\int P dx} = \int Qe^{\int P dx} dx + c$

$$\therefore ve^{x^2} = \int x^3 e^{x^2} dx + c = \int xx^2 e^{x^2} dx + c$$

Put $t = x^2$; $dt = 2x dx$

$$\therefore ve^{x^2} = \frac{1}{2} \int te^t dt$$

$$ve^{x^2} = \frac{1}{2}(te^t - e^t) + c = \frac{1}{2}(x^2e^{x^2} - e^{x^2}) + c$$

$$v = \frac{1}{2}(x^2 - 1) + ce^{-x^2}$$

The solution is

$$\tan y = \frac{1}{2}(x^2 - 1) + ce^{-x^2}$$

2. Solve $\cos x \frac{dy}{dx} - y \sin x = y^3 \cos^2 x$

Solution:

Dividing by y^3 , we get $\frac{1}{y^3} \frac{dy}{dx} \cos x - \frac{1}{y^2} \sin x = \cos^2 x$

Dividing by $\cos x$ we get $\frac{dy}{dx} \frac{1}{y^3} - \frac{1}{y^2} \tan x = \cos x$

Substituting $v = \frac{1}{y^2}$ gives $\frac{dv}{dx} = \frac{-2}{y^3} \frac{dy}{dx}$

Now the above equation becomes $-\frac{1}{2} \frac{dv}{dx} - v \tan x = \cos x$

or $\frac{dv}{dx} + 2v \tan x = -2 \cos x$

$P = 2 \tan x, Q = -2 \cos x$

$$\int P dx = 2 \int \tan x dx = 2 \log(\sec x)$$

$$e^{\int P dx} = e^{2(\log \sec x)} = \sec^2 x$$

$$v \sec^2 x = -\int 2 \cos x \sec^2 x dx$$

$$= -2 \int \sec x dx$$

$$= -2 \log(\sec x + \tan x) + c$$

$$\frac{\sec^2 x}{y^2} = c - 2 \log(\sec x + \tan x) \text{ is the solution}$$



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UG TRB MATHEMATICS 2023-2024

UNIT IV

Vector Calculus

& Fourier Series, Fourier Transforms

Your Success is Our Goal....

INDEX

UNIT - II - DIFFERENTIATION

S.N.	TOPIC	P. N.
4.1	Vector Differentiation	1
4.2	Vectocity of a Particle	1
4.3	Vector Valued Function and Scalar Potential	2
4.4	Gradient of a Scalar Point Function	2
4.5	Divergence and Curl of a Vector Point Function	2
4.6	Directional Derivative of a Scalar Point Function	4
4.7	Unit Normal	4
4.8	Laplacian Operator	4
4.9	Harmonic Function	4
4.10	Line Integration	41
	4.10.1.Conservative field	
	4..10.2. Scalar point function	
4.11	Surface Integral	42
4.12	Volume Integral	42
4.13	Integral Theorem	42
	4.13.1. Green's theorem in the plane	
	4.13.2. Gauss divergence theorem	
	4.13.3. Stoke's theorem	
4.14	Fourier Series	100
	4.14.1. Fourier Series function defined	
	4.14.2. Fourier series for even and odd functions defined in $-\pi \leq x \leq \pi$	
	4.14.3. Half range series	
4.15	Fourier Transform	147
	4.15.1. Fourier cosine formula	
	4.15.2. Fourier inverse cosine transform	
4.16	Simple Properties of Fourier Transforms	148
4.17	Convolution Theorem	149
4.18	Parsavel's Identity	149
4.19	Multiple Choice Question	189

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UG TRB – MATHEMATICS – 2023-24

UNIT - IV - VECTOR

4.1. VECTOR DIFFERENTIATION

- Vector function: If for each value of scalar variable u there corresponds a vector f , then f is said to be a vector function of the scalar variable u . It is written as $f(u)$

Constant Function:

- A vector whose magnitude is constant and whose direction is in a fixed direction is a constant vector.

Note:

- A scalar function has only a magnitude while a vector function has both magnitude and direction.

Derivative of a Vector Function

- It is denoted by Δf , then

$$\frac{d\vec{f}}{du} = \lim_{\Delta u \rightarrow 0} \frac{\Delta \vec{f}}{\Delta u}$$



4.2. VELOCITY OF A PARTICLE

- The displacement of the particle in time interval is Δt . So the rate of displacement of the particle at P is

$$\lim_{\Delta t \rightarrow 0} \frac{\Delta r}{\Delta t} \text{ (or) } \frac{dr}{dt}$$

- But the rate of displacement is the velocity of the particle. It is denoted by v .

$$\text{(i.e.,)} \quad \vec{v} = \frac{d\vec{r}}{dt}$$

4.3. VECTOR VALUED FUNCTION AND SCALAR POTENTIAL

- Vector point Function: Suppose, in a physical situation for every point (x, y, z) , there corresponds a vector.
- $f(x, y, z)\vec{i} + g(x, y, z)\vec{j} + h(x, y, z)\vec{k}$, then this vector function is said to be a vector point function.

Scalar Point Function:

- In a physical situation, for every point (x, y, z) , there corresponds a scalar $\phi(x, y, z)$. Then $\phi(x, y, z)$ is said to be a scalar point function.

Level Surfaces:

- If $\phi(x, y, z)$ is a scalar, then the equation $\phi(x, y, z) = c$, where c is a varying constant, represents surface called level surfaces. Thus, the value ϕ is a constant.

4.4. GRADIENT OF A SCALAR POINT FUNCTION

- If ϕ is a scalar point function, then $\vec{i} \frac{\partial \phi}{\partial x} + \vec{j} \frac{\partial \phi}{\partial y} + \vec{k} \frac{\partial \phi}{\partial z}$ is called the gradient of ϕ at (x, y, z)

Notation:

- Gradient of ϕ is denoted by $\text{grad } \phi$ or $\nabla \phi$ where ∇ is the operator which stands for

$$\vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z}$$

- Thus ϕ is a scalar but $\nabla \phi$ is a vector

4.5. DIVERGENCE AND CURL OF A VECTOR POINT FUNCTION

Divergence:

- The scalar point functions

$$\frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial v_3}{\partial z}$$

- is called the divergence of the vector point function $V_1i + V_2j + V_3k$

Notation:

- Divergence of V or $div V$ or $\nabla \cdot \vec{V}$

$$\begin{aligned} \nabla \cdot V &= \left(i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z} \right) \cdot (V_1i + V_2j + V_3k) \\ &= \frac{\partial V_1}{\partial x} + \frac{\partial V_2}{\partial y} + \frac{\partial V_3}{\partial z} \end{aligned}$$

Curl:

- The vector point function

$$i \left(\frac{\partial V_3}{\partial y} - \frac{\partial V_2}{\partial z} \right) + j \left(\frac{\partial V_1}{\partial z} - \frac{\partial V_3}{\partial x} \right) + k \left(\frac{\partial V_2}{\partial x} - \frac{\partial V_1}{\partial y} \right)$$

- is called the curl of the vector point function $v_1\vec{i} + v_2\vec{j} + v_3\vec{k}$

Notation:

- Curl of V is $\text{curl } V$ (or) $\nabla \times \vec{V}$

$$\nabla \times \vec{V} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_1 & V_2 & V_3 \end{vmatrix}$$

Particular cases of $\nabla \times \vec{V}$ and $\nabla \cdot \vec{V}$

Solenoidal Vector:

- If $\nabla \cdot \vec{V} = 0$, then V said to be solenoidal.

Irrotational Vector:

- If $\nabla \times \vec{V} = 0$, then V is said to be irrotational.



4.6. DIRECTIONAL DERIVATIVE OF A SCALAR POINT FUNCTION

- Suppose $\phi(x, y, z)$ is a scalar point function and $\phi(p)$ is the value of ϕ at P. If P' is any point, then $\lim_{p' \rightarrow p} \frac{\phi(p') - \phi(p)}{pp'}$.
- is called the directional derivative of ϕ . The directional derivative is a scalar. Actually it is the rate of change of ϕ in the given direction.

4.7. UNIT NORMAL

- This directional derivative of ϕ in the direction specified by the unit vector \hat{e} having direction cosines l, m, n is $(\nabla\phi) \cdot \hat{e}$.
- The unit vector normal to the surface $\phi(x, y, z) = c$ at any point (x, y, z) is

$$\hat{n} = \frac{\nabla\phi}{|\nabla\phi|}$$

4.8. LAPLACIAN OPERATOR

- The operator ∇^2 defined by

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

- is called Laplacian differential operator, when it operator on a scalar pint function, it results in a scalar. When it operates on a vector point function, it results in a vector.

4.9. HARMONIC FUNCTION:

- For every scalar point function, having continuous second partials, $\nabla \times (\nabla\phi) = 0$.
- In words curl of a gradient vanishes.
- For every vector point function \vec{A} , having continuous second partials,

$$\nabla \cdot (\nabla \times \vec{A}) = 0 \text{ . In words}$$

- Divergence of a curl vanishes.

VECTOR CALCULUS AND FOURIER SERIES, FOURIER TRANSFORMS

4.1 to 4.9 – EXAMPLES

PROBLEMS

1. A particle moves along the curve $x = e^{-t}$, $y = 2 \cos 3t$, $z = 2 \sin 3t$. Determine the velocity and acceleration at any time t and their magnitudes at $t = 0$

Soln:

$$\begin{aligned} r &= xi + yj + zk \\ &= e^{-t}i + 2 \cos 3t j + 2 \sin 3t k \end{aligned}$$

$$\frac{dr}{dt} = -e^{-t}i - 6 \sin 3t j + 6 \cos 3t k$$

$$\frac{dr}{dt}_{(t=0)} = -i + 6k \quad (\text{velocity at time})$$

$$\text{Magnitude of the velocity} = \sqrt{1+36} = \sqrt{37}$$

$$\bar{a} = \frac{d^2\bar{r}}{dt^2} = e^{-t}\bar{i} - 18 \cos 3t \bar{j} - 18 \sin 3t \bar{k}$$

$$\frac{d^2\bar{r}}{dt^2} = i - 18j = \text{acceleration at time } t = 0$$

$$|\bar{a}| = \sqrt{1+324} = \sqrt{325} = 5\sqrt{13}$$

2. A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 5$ where t is the time. Find the components of its velocity and acceleration at $t = 1$ in the direction $i + j + 3k$

Soln:

$$\begin{aligned} r &= xi + yj + zk \\ &= (t^3 + 1)i + t^2j + (2t + 5)k \end{aligned}$$

$$\dot{v} = \frac{dr}{dt} = 3t^2i + 2tj + 2k$$

$$\text{Velocity at } t = 1 \text{ is } V = 3i + 2j + 2k$$

$$a = \frac{dr^2}{dt^2} = 6ti + 2j$$

Acceleration at $t = 1$ is $\vec{a} = 6\vec{i} + 2\vec{j}$

$$b = i + j + 3k$$

$$= \frac{\vec{v} \cdot \vec{b}}{|\vec{b}|}$$

$$= (3\vec{i} + 2\vec{j} + 2\vec{k}) \frac{\vec{i} + \vec{j} + 3\vec{k}}{\sqrt{11}}$$

$$= \frac{3+2+6}{\sqrt{11}} = \frac{11}{\sqrt{11}} = \sqrt{11}$$

Acceleration component in the direction of b at $t = 1$

$$= \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

$$= (6\vec{i} + 2\vec{j}) \frac{(\vec{i} + \vec{j} + 3\vec{k})}{\sqrt{11}}$$

$$= \frac{6+2}{\sqrt{11}}$$

$$= \frac{8}{\sqrt{11}}$$

3. If $\phi(x, y, z) = x^2y + y^2x + z^2$ find $\nabla\phi$ at the point $(1, 1, 1)$

Soln:

$$\phi(x, y, z) = x^2y + y^2x + z^2$$

$$\frac{\partial\phi}{\partial x} = 2xy + y^2$$

$$\frac{\partial\phi}{\partial y} = x^2 + 2xy$$

$$\frac{\partial\phi}{\partial z} = 2z$$

$$\begin{aligned}\nabla\phi &= \bar{i} \frac{\partial\phi}{\partial x} + \bar{j} \frac{\partial\phi}{\partial y} + \bar{k} \frac{\partial\phi}{\partial z} \\ &= (2xy + y^2)\bar{i} + (x^2 + 2xy)\bar{j} + 2z\bar{k}\end{aligned}$$

$$\nabla\phi_{(1,1,1)} = 3\bar{i} + 3\bar{j} + 2\bar{k}$$

4. If $r = xi + yj + zk$ and $r = |\vec{r}|$ prove that

$$(i) \nabla_r = \frac{1}{r} \vec{r} \quad (ii) \nabla\left(\frac{1}{r}\right) = \frac{-\vec{r}}{r^3}$$

$$(iii) \nabla r^n = nr^{n-2} \vec{r} \quad (iv) \nabla f(r) = f'(r) \frac{\vec{r}}{r} = f'(r) \nabla_r$$

$$(v) \nabla(\log r) = \frac{\vec{r}}{r^2} \quad (vi) \nabla f(r) \times \vec{r} = 0$$

Soln:

$$(i) r = x\hat{i} + y\hat{j} + z\hat{k} \quad \therefore |\vec{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$r^2 = x^2 + y^2 + z^2$$

Differentiating partially with respect to x, we get

$$2r = \frac{\partial r}{\partial x} = 2x \quad \therefore \frac{\partial r}{\partial x} = \frac{x}{r}$$

$$\frac{\partial r}{\partial y} = \frac{y}{r} \quad \text{and} \quad \frac{\partial r}{\partial z} = \frac{z}{r}$$

$$\nabla_r = \bar{i} \frac{\partial r}{\partial x} + \bar{j} \frac{\partial r}{\partial y} + \bar{k} \frac{\partial r}{\partial z}$$

$$= \frac{x\bar{i} + y\bar{j} + z\bar{k}}{r}$$

$$= \frac{\vec{r}}{r}$$

$$(ii) \nabla\left(\frac{1}{r}\right) = \bar{i} \frac{\partial}{\partial x}\left(\frac{1}{r}\right) + \bar{j} \frac{\partial}{\partial y}\left(\frac{1}{r}\right) + \bar{k} \frac{\partial}{\partial z}\left(\frac{1}{r}\right)$$

$$= \frac{-1}{r^2} \left[\bar{i} \frac{\partial r}{\partial x} + \bar{j} \frac{\partial r}{\partial y} + \bar{k} \frac{\partial r}{\partial z} \right]$$

$$= \frac{-1}{r^2} \left(i \frac{x}{r} + j \frac{y}{r} + k \frac{z}{r} \right)$$

$$= \frac{-1}{r^2} \left(\frac{\dot{r}}{r} \right) = \frac{-\dot{r}}{r^3}$$

$$\text{(iii)} \quad \nabla r^n = \bar{i} \frac{\partial}{\partial x} (r^n) + \bar{j} \frac{\partial}{\partial y} (r^n) + \bar{k} \frac{\partial}{\partial z} (r^n)$$

$$= nr^{n-1} \left[i \frac{\partial r}{\partial x} + j \frac{\partial r}{\partial y} + k \frac{\partial r}{\partial z} \right]$$

$$= nr^{n-1} \left[\frac{x\bar{i} + y\bar{j} + z\bar{k}}{r} \right]$$

$$= nr^{n-2} \dot{r}$$

$$\text{(iv)} \quad \nabla f(r) = \bar{i} \frac{\partial}{\partial x} f(r) + \bar{j} \frac{\partial}{\partial y} f(r) + \bar{k} \frac{\partial}{\partial z} f(r)$$

$$= f'(r) \left[i \frac{\partial r}{\partial x} + j \frac{\partial r}{\partial y} + k \frac{\partial r}{\partial z} \right]$$

$$= f'(r) \frac{(xi + yj + zk)}{r}$$

$$= f'(r) \frac{\dot{r}}{r}$$

$$= f'(r) \nabla(r)$$

$$\text{(v)} \quad \nabla (\log r) = \bar{i} \frac{\partial}{\partial x} (\log r) + \bar{j} \frac{\partial}{\partial y} (\log r) + \bar{k} \frac{\partial}{\partial z} (\log r)$$

$$= \frac{1}{r} \left[i \frac{\partial r}{\partial x} + j \frac{\partial r}{\partial y} + k \frac{\partial r}{\partial z} \right]$$

$$= \frac{1}{r} \frac{x\hat{i} + y\hat{j} + z\hat{k}}{r}$$

$$= \frac{\dot{r}}{r^2}$$

$$\text{(vi)} \quad \nabla f(r) \times \dot{r}$$

$$\nabla f(r) = \frac{f'(r)}{r} \vec{r}$$

$$\nabla f(r) \times \vec{r} = \frac{f'(r)}{r} \vec{r} \times \vec{r} = 0 \text{ since } r \times r = 0$$

5. If $u = x + y + z$

$$v = x^2 + y^2 + z^2$$

$$w = yz + zx + xy \text{ prove that } \text{grad } u \times \text{grad } v \times \text{grad } w = 0$$

Soln:

$$\text{grad } u = \nabla u = \vec{i} \frac{\partial u}{\partial x} + \vec{j} \frac{\partial u}{\partial y} + \vec{k} \frac{\partial u}{\partial z}$$

$$\text{grad } v = \nabla v = \vec{i} \frac{\partial v}{\partial x} + \vec{j} \frac{\partial v}{\partial y} + \vec{k} \frac{\partial v}{\partial z}$$

$$= 2(xi + yj + zk)$$

$$\text{grad } w = \vec{i} \frac{\partial w}{\partial x} + \vec{j} \frac{\partial w}{\partial y} + \vec{k} \frac{\partial w}{\partial z}$$

$$= (y+z)i + (z+x)j + (x+y)k$$

$$(\text{grad } u)(\text{grad } v \times \text{grad } w) = \begin{vmatrix} 1 & 1 & 1 \\ 2x & 2y & 2z \\ y+z & z+x & x+y \end{vmatrix}$$

$$= 2 \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ y+z & z+x & x+y \end{vmatrix}$$

$$= 2 \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x+y+z & x+y+z & x+y+z \end{vmatrix}$$

$$= 2(x+y+z) \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix}$$

$$= 0 \text{ since two rows are identical}$$

6. Find the directional derivative of $xyz - xy^2z^3$ at the point $(1, 2, -1)$ in the direction of the vector $i - j - 3k$.

Soln:

$$\phi = xyz - xy^2z^3$$

$$\nabla \phi = \vec{i} \frac{\partial \phi}{\partial x} + \vec{j} \frac{\partial \phi}{\partial y} + \vec{k} \frac{\partial \phi}{\partial z}$$

$$= (yz - y^2z^3)\vec{i} + (xz - 2xyz^3)\vec{j} + (xy - 3xy^2z^2)\vec{k}$$

$$\vec{n} = \frac{\vec{i} - \vec{j} - 3\vec{k}}{\sqrt{11}}$$

$$\frac{d\phi}{dn} = \nabla \phi \cdot \vec{n} = \text{directional derivative of } \phi \text{ in the direction of the vector } \vec{i} - \vec{j} - 3\vec{k}$$

$$= \frac{[(yz - y^2z^3) - (xz - 2xyz^3) - 3(xy - 3xy^2z^2)]}{\sqrt{11}}$$

$$\nabla \phi \vec{n}_{(1,2,-1)} = \frac{(-2+4) - (-1+4) - 3(2-12)}{\sqrt{11}} = \frac{29}{\sqrt{11}}$$

7. Show that (i) $\text{grad} (\vec{r} \cdot \vec{a}) = \vec{a}$ (ii) $\text{grad} [\vec{r} \cdot \vec{a}, \vec{b}] = \vec{a} \times \vec{b}$ where \vec{a} and \vec{b} are constant vectors and $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$

Soln:

$$\text{Let } \vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$$

$$\vec{b} = b_1\vec{i} + b_2\vec{j} + b_3\vec{k}$$

$$(i) \vec{a} \cdot \vec{r} = a_1x + a_2y + a_3z$$

$$\text{grad} (\vec{a} \cdot \vec{r}) = \left[\vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right] (a_1x + a_2y + a_3z)$$

$$= a_1\vec{i} + a_2\vec{j} + a_3\vec{k} = \vec{a} \quad \text{--- (1)}$$

$$(ii) \text{grad} [\vec{r} \cdot \vec{a}, \vec{b}] = \text{grad} (\vec{r} \cdot \vec{a} \times \vec{b})$$

$$= \vec{a} \times \vec{b} \text{ using (1)}$$

8. Find the unit vector normal to the surface $x^2 + 3y^2 + 2z^2 = 6$ at the point $(2, 0, 1)$

Soln:

$$\phi = x^2 + 3y^2 + 2z^2$$

$$\nabla\phi = 2xi + 6yj + 4zk$$

$$\nabla\phi_{(2,0,1)} = 4\hat{i} + 4\hat{k}$$

$$\hat{n} = \frac{\nabla\phi}{|\nabla\phi|} = \frac{4\hat{i} + 4\hat{k}}{4\sqrt{2}} = \frac{\hat{i} + \hat{k}}{\sqrt{2}}$$

the unit normal vector at the point $(2, 0, 1)$ to the given surface

$$= \frac{1}{\sqrt{2}}(\hat{i} + \hat{k})$$

9. Find the maximum value of the directional derivative of the function

$\phi = 2x^2 + 3y^2 + 5z^2$ at the point $(1, 1, -4)$

Soln:

$$\phi = 2x^2 + 3y^2 + 5z^2$$

$$\nabla\phi = \hat{i} \frac{\partial\phi}{\partial x} + \hat{j} \frac{\partial\phi}{\partial y} + \hat{k} \frac{\partial\phi}{\partial z}$$

$$= 4xi + 6yj + 10zk$$

$$\nabla\phi_{(1,1,-4)} = 4\hat{i} + 6\hat{j} - 40\hat{k}$$

Maximum value of the directional derivative at the point $(1, 1, -4)$

$$= \sqrt{16 + 36 + 1600} = \sqrt{1652}$$

10. Find the angle between the normal to the surface $xy - z^2 = 0$ at the point $(1, 4, -2)$ and $(-3, -3, 3)$

Soln:

$$\phi = xy - z^2$$

$$\nabla \phi = \hat{i} \frac{\partial \phi}{\partial x} + \hat{j} \frac{\partial \phi}{\partial y} + \hat{k} \frac{\partial \phi}{\partial z}$$

$$= yi + xj - 2zk$$

$$\nabla \phi_{(1,4,-2)} = 4\hat{i} + \hat{j} + 4\hat{k}$$

$$\nabla x_{(-3,-3,3)} = -3\hat{i} - 3\hat{j} - 6\hat{k}$$

Unit normal vector to the surface at the point (1, 4, -2) is

$$\hat{n}_1 = \frac{\nabla \phi}{|\nabla \phi|} = \frac{4\hat{i} + \hat{j} + 4\hat{k}}{\sqrt{33}}$$

Unit normal vector at the point (-3, -3, 3) is

$$\hat{n}_2 = \frac{-3\hat{i} - 3\hat{j} - 3\hat{k}}{\sqrt{9+9+36}} = \frac{-3\hat{i} - 3\hat{j} - 3\hat{k}}{\sqrt{54}}$$

If θ is the angle between the normal then

$$\cos \theta = \hat{n}_1 \cdot \hat{n}_2 = \frac{-12 - 3 - 24}{\sqrt{33}\sqrt{54}} = \frac{-39}{9\sqrt{22}} = \frac{-3}{3\sqrt{22}}$$

$$\therefore \theta = \cos^{-1}\left(\frac{-13}{3\sqrt{22}}\right)$$

11. Show that the surface $5x^2 - 2yz - 9x = 0$ and $4x^2y + z^3 - 4 = 0$ are orthogonal at (1, -1, -2)

Soln:

$$\text{Let } \phi_1 = 5x^2 - 2yz - 9x$$

$$\phi_2 = 4x^2y + z^3 - 4$$

$$\nabla \phi_1 (10x - 9) = \hat{i} - 2z\hat{j} - 2y\hat{k}$$

$$\nabla \phi_1 (1, -1, 2) = \hat{i} - 4\hat{j} + 2\hat{k}$$

$$\nabla \phi_2 = 8xy\hat{i} + 4x^2\hat{j} + 3z^2\hat{k}$$

$$\nabla \phi_2 (1, -1, 2) = -8\hat{i} + 4\hat{j} + 12\hat{k}$$

$$\nabla \phi_1 \cdot \nabla \phi_2 = -8 - 16 + 24 = 0$$

∴ The surface are orthogonal at the point (1, -1, 2)

12. Find ϕ if $\nabla \phi = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$

Soln:

$$\nabla \phi = \vec{i} \frac{\partial \phi}{\partial x} + \vec{j} \frac{\partial \phi}{\partial y} + \vec{k} \frac{\partial \phi}{\partial z} \quad \text{--- (1)}$$

$$\text{Also } \nabla \phi = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k \quad \text{--- (2)}$$

Comparing (1) and (2), we get

$$\frac{\partial \phi}{\partial x} = 6xy + z^3 \quad \text{--- (1)}$$

$$\frac{\partial \phi}{\partial y} = 3x^2 - z \quad \text{--- (2)}$$

$$\frac{\partial \phi}{\partial z} = 3xz^2 - y \quad \text{--- (3)}$$

Integrating (1), (2) and (3), w.r.t. x, y, z respectively we get,

$$\phi = 3x^2y + xz^3 + f_1(y, z) \quad \text{--- (4)}$$

$$\phi = 3x^2y - yz + f_2(x, z) \quad \text{--- (5)}$$

$$\phi = xz^3 - yz + f_3(x, y) \quad \text{--- (6)}$$

From (4), (5) and (6) $\phi = 3x^2y + xz^3 - yz + c$ where c is an arbitrary constant.

13. Find ϕ if $\nabla \phi = (y + \sin z)i + xj + x \cos z k$

Soln:

$$\nabla \phi = \vec{i} \frac{\partial \phi}{\partial x} + \vec{j} \frac{\partial \phi}{\partial y} + \vec{k} \frac{\partial \phi}{\partial z} \quad \text{--- (1)}$$

$$= (y + \sin z)i + xj + x \cos z k \quad \text{--- (2)}$$

Comparing (1) and (2) we get,



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UG TRB MATHEMATICS 2023-2024



UNIT V Algebraic Structures

Your Success is Our Goal....

UG TRB 2022-23 MATHEMATICS
UNIT - V – ALGEBRAIC STRUCTURE
INDEX

S.NO	CHAPTER NAME	P.NO
5.1	GROUPS	1
	5.1.1. Binary Operations	1
	5.1.2. Group Under “.”:	2
	5.1.3. Group Under “+”:	2
	5.1.4. Abelian Group:	3
	5.1.5. Non-Abelian Group	3
	5.1.6. Types of Functions	3
	5.1.7. Order of A Group:	4
	5.1.8. Finite Group:	4
5.2.	SUBGROUPS	23
5.3	CYCLIC GROUP	25
	5.3.1. Cyclic sub-Group:	26
	5.3.2. Congruent:	36
5.4.	LAGRANGE’S THEOREM	38
5.5.	COUNTING PRINCIPLES	47
5.6.	NORMAL SUBGROUP	50
5.7.	QUOTIENT GROUP	54
5.8.	HOMOMORPHISM	57
	5.8.1. Properties Of Homomorphism:	57
	5.8.2. Kernal Of Homomorphism:	62

	5.8.3. Types Of Homomorphism:	62
	5.8.4. Canonical Homorphism:	62
	5.8.5. (1 st Isomorphic Theorem):	63
	5.8.6. Cyclic Group Of Homomorphism:	65
	5.8.7. SECOND ISOMORPHIC THEOREM:	68
	5.8.8. Third Isomorphk Theorem:	70
5.9.	AUTOMORPHISM	72
	5.9.1. Inner Automorphism:	74
	5.9.2. Centre Of A Group:	76
5.10.	CAYLEY'S THEOREM	76
5.11.	PERMUTATION GROUPS	78
5.12.	RINGS	80
	5.12.1. Ring with Unity:	80
	5.12.2. Commutativ Ring:	80
	5.12.3. Commutativ Ring with Unity:	80
	5.12.4. Properties of a Ring:	82
5.13	SOME SPECIAL CLASSES OF RINGS	85
	5.13.1. Zero Divisor	85
	5.13.2. Unit element in Ring:	85
	5.13.3. Division Ring:	85
	5.13.4. Quotient Ring:	85
5.14.	INTEGRAL DOMAIN	89
5.15.	HOMOMORPHISM OF RING	94

5.16.	IDEAL AND QUOTIENT RINGS	97
	5.16.1. Ideals:	97
	5.16.2. Principal Ideal:	98
	5.16.3. Maximum Ideal:	98
5.17.	PRIME IDEAL & MAXIMUM IDEAL	101
	5.17. 1. Prime Ideal:	101
	5.17.2. Maximum Ideal:	102
5.18.	THE FIELD OF QUOTIENT OF AN INTEGRAL DOMAIN	103
5.19.	EUCLIDEAN RINGS	109
5.20.	ALGEBRA OF LINEAR TRANSFORMATION	112
	5.20.1. Linear Transformation:	112
	5.20.2. Kernel of a Linear Transformation:	112
	5.20.3. Algebra:	112
	5.20.4. Algebra with Unit Element:	112
	5.20.5. Minimal Polynomial:	113
	5.20.6. Range of T:	114
5.21.	CHARACTERISTIC ROOTS	116
	5.21.1. Characteristic Vector:	117
5.22.	MATRICES	117
5.23.	CANONICAL FORM	117
	5.23.1. Quotient Space:	117
5.24.	TRIANGULAR FROM	118
5.25.	PROBLEMS OF CONVERTING LINEAR TRANSFORMATION TO MATRICES AND VICE VERSA	121

5.26.	VECTOR SPACES	122
5.27.	DEFINITION AND EXAMPLES	124
	5.27.1. Homomorphism:	124
	5.27.2. Kernal of T:	124
	5.27.3. Quotient set:	124
	5.27.4. Quotient Space:	125
	5.27.5. Internal Direct Sum:	126
	5.27.6. External Direct Sum:	126
5.28.	LINEAR DEPENDENCE AND INDEPENDENCE	128
	5.28.1. Linear combination:	128
	5.28.2. Linear Span:	128
	5.28.3. Linearly Independent:	128
	5.28.4. Linearly Dependent:	129
5.29.	SUB- SPACE	130
5.30.	DUAL SPACE	130
	5.30.1. Second Dual Space	131
5.31.	INNER PRODUCT SPACE	132
	5.31.1. Orthogonal:	132
	5.31.2. Orthogonal Complement:	133
	5.31.3. Orthonormal Set:	133
	5.31.4. Schwartz Inequality:	133
5.32.	IMPORTANT QUESTIONS (MCQ)	136

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UG TRB – MATHS – 2022-23

UNIT - V

ALGEBRAIC STRUCTURE

5.1 GROUPS

5.1.1. BINARY OPERATIONS:

Binary operation means “way of putting two things together.

Eg. The set of all natural number under addition



Closure Property Under “.”:

Let A be a set with binary operation “.”. Thus operation is said to be closure if
 $a, b \in A \Rightarrow a \cdot b \in A$

Associative Property Under “.”:

Let A be a set with binary operation “.”. Thus operation is said to be associative

$$\text{if } a, b, c \in A \Rightarrow (a \cdot b) \cdot c = a \cdot (b \cdot c)$$

Identity Element Under “.”:

Let a be a set with binary operation “.”. An element e is said to be identify element if
 $a \cdot e = e \cdot a = a \quad \forall a \in A$

Inverse Element Under “.”:

Let A be a set with binary operation ‘.’. Suppose that A contains an identity element e.

$$\text{If } a \in A \text{ and if } a^{-1} \in A \ni a \cdot a^{-1} = a^{-1} \cdot a = e$$

Where a^{-1} is called inverse element of A.

5.1.2. GROUP UNDER “.”:

A non-empty set G with binary operation “.” is called a group if it satisfies the following conditions.

(i) Closure:

$$\text{If } a, b, \in G \Rightarrow a \cdot b \in G \quad \forall a, b \in G$$

(ii) Associative:

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c \quad \forall a, b, c \in G$$

(iii) Identify:

$$\text{If an element } e \in G \ni a \cdot e = e \cdot a = a \quad \forall a \in G$$

(iv) Inverse:

$$\forall a \in G \text{ if an element } a^{-1} \in G$$

$$\Rightarrow a \cdot a^{-1} = e = a^{-1} \cdot a$$

Where a^{-1} is the inverse element of G .

5.1.3. GROUP UNDER “+”:

A non- empty set G with binary operations ‘+’ is called a group if it satisfies the following conditions.

(i) closure:

$$\text{If } a, b \in G \Rightarrow a + b, \in G \quad \forall a, b \in G$$

(ii) Associative:

$$a + (b + c) = (a + b) + c \quad \forall a, b, c \in G$$

(iii) Identify:

$$\text{If an element } e \quad (a + e) = e + a = a \quad \forall a \in G$$

(iv) Inverse

$$\forall a \in G, \text{ if an element } a^{-1} \in G$$

$$a + a^{-1} = e = a^{-1} + a$$

Where a^{-1} is the inverse element of G of G



Commutative Property:

Let A be a set with binary operation “.” If $a.b = b.a \forall a, b \in A$, then A satisfies commutative property.

5.1.4. ABELIAN GROUP:

If $(G, .)$ is a group than $(G, .)$ is abelian, if the group of the operation “.” is commutative.

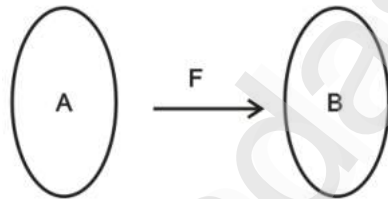
$$(i.e.,) a . b = b . a \quad \forall a, b \in G$$

5.1.5. NON-ABELIAN GROUP:

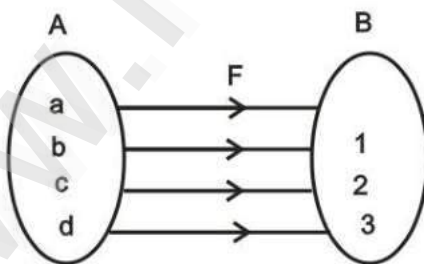
A group which is not abelian is called non-abelian group.

5.1.6. TYPES OF FUNCTIONS**One - To - One Function:**

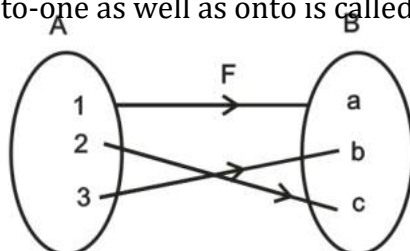
A function $f : A \rightarrow B$ is said to be a one-to-one function if distinct element of A have distinct image of B .

**Onto function:**

A function $f : A \rightarrow B$ is said to be onto if every element of B has atleast one - preimage in A .

**Bijjective Function:**

A function which is one-to-one as well as onto is called bijective function.



5.1.7. ORDER OF A GROUP:

The number of elements in a group G is called order of a group, it is denoted by $O(G)$.

Eg. $G = \{1, -1, i, -i\}$

$$O(G) = 4$$

5.1.8. FINITE GROUP:

A group G is called finite if it consists of only finite number of elements and we say that the group is of finite order.

PROBLEMS:

1. Prove that (S, \cdot) is a group where S is the set of all 4th roots of unity.

Solution:

$$\text{Let } S = \{1, -1, i, -i\}$$

\cdot	1	-1	i	-i
1	1	-1	i	-i
-1	-1	1	-i	i
i	i	-i	-1	1
-i	-i	i	1	-1

Closure:

$$\text{Let } 1, i \in S$$

$$\Rightarrow 1 \cdot i \in S$$

$\therefore (S, \cdot)$ satisfies closure property.

Associative:

$$\text{Let } 1, -1, i \in S$$

$$1 \cdot (-1, i) = (1 \cdot (-1)) \cdot i$$

$$1 \cdot (-i) = (-1) \cdot i$$

$$-i = -i$$

$\therefore (S, \cdot)$ satisfies associative property.

Identity:

$$a \cdot e = e \cdot a = a$$

$$1 \cdot i = i \cdot 1 = i$$

$$1 \cdot (-i) = (-i) \cdot 1 = -i$$

$$1 \cdot 1 = 1 \cdot 1 = 1$$

$$1 \cdot (-1) = (-1) \cdot 1 = -1$$

$\therefore 1$ is the identity element of S .

Inverse Law:

$$a \cdot a^{-1} = a^{-1} \cdot a = e$$

Inverse of $1 = 1$

Inverse of $i = -i$

Inverse of $-1 = 1$

Inverse of $-i = i$

\therefore Inverse exists

$\therefore (S, \cdot)$ is a group.

2. Find the residue group of integers under addition modulo 5.**Solution:**

$$\text{Let } z_5 = \{[0], [1], [2], [3], [4]\}$$

\oplus_5	[0]	[1]	[2]	[3]	[4]
[0]	[0]	[1]	[2]	[3]	[4]
[1]	[1]	[2]	[3]	[4]	[0]
[2]	[2]	[3]	[4]	[0]	[1]
[3]	[3]	[4]	[1]	[1]	[2]
[4]	[4]	[0]	[1]	[2]	[3]

(i) Closure

$$\text{Let } [0], [1] \in z_5$$

$$\Rightarrow [0] \oplus_5 [1] = [1] \in z_5$$

$\therefore (z_5, \oplus_5)$ satisfies closure property.

(ii) Associative

Let $[2] [3] [4] \in z_5$

$$[2] \oplus_5 ([3] + [4]) = ([2] \oplus_5 [3]) \oplus_5 [4]$$

$$[2] \oplus_5 [2] = [0] \oplus_5 [4]$$

$$[4] = [4]$$

$\therefore (z_5, \oplus_5)$ satisfies property.

Identity Law:

$$a \oplus e = e \oplus a = a$$

$$[0] \oplus_5 [0] = [0]$$

$$[1] \oplus_5 [0] = [1]$$

$$[2] \oplus_5 [0] = [2]$$

$$[3] \oplus_5 [0] = [3]$$

$$[4] \oplus_5 [0] = [4]$$

$\therefore [0]$ is the identity element.

Inverse Law:

$$a \oplus a^{-1} = a^{-1} \oplus a = e$$

$$[0] \oplus_5 [0] = [0]$$

$$[1] \oplus_5 [4] = [0]$$

$$[2] \oplus_5 [3] = [0]$$

$$[3] \oplus_5 [2] = [0]$$

$$[4] \oplus_5 [1] = [0]$$

\therefore Inverse exists

$\therefore (z_5, \oplus_5)$ is a group.

Associate property:

For any $a, b \in \Rightarrow a*(b*c)*c$

Here, for any $a, b \in \Rightarrow Z_5 \Rightarrow a*(b*c) = (a*c) = (a*b)*c$

Let us take $[1], [3], [4] \in Z_5$

Consider

$$[1] \oplus_5 ([3] \oplus_5 [4]) = [1] \oplus_5 [2] = [3]$$

Consider

$$([1] \oplus_5 [3]) \oplus_5 [4] = [4] \oplus_5 [4] = [3]$$

$\therefore (Z, \oplus_5)$ satisfies associative property

Identify Property:

In the table, $[0]$ is an identity element in Z_5

$$[1] \oplus_5 [0] = [0]$$

$$[1] \oplus_5 [0] = [1]$$

$$[2] \oplus_5 [0] = [2]$$

$$[3] \oplus_5 [0] = [3]$$

$$[4] \oplus_5 [0] = [4]$$

Inverse Property:

$$[1] \oplus_5 [0] = [0]$$

$$[1] \oplus_5 [4] = [0]$$

$$[2] \oplus_5 [3] = [0]$$

$$[3] \oplus_5 [2] = [0]$$

$$[4] \oplus_5 [1] = [0]$$

Inverse element is exist.

Hence, (Z, \oplus_5) is a group.

Problem - 3:

Find the residue class of integers under addition modulo 7 and prove that it is a group.

Solution:

Let $Z_7 = \{0, 1, 2, 3, 4, 5, 6\}$ be the set of all residue class of integer for Z_7 under addition.

To prove: (Z, \oplus_7) is a group.

Closure property:

\oplus_7	[0]	[1]	[2]	[3]	[4]	[5]	[6]
[0]	[0]	[1]	[2]	[3]	[4]	[5]	[6]
[1]	[1]	[2]	[3]	[4]	[5]	[6]	[0]
[2]	[2]	[3]	[4]	[5]	[6]	[0]	[1]
[3]	[3]	[4]	[5]	[6]	[0]	[1]	[2]
[4]	[4]	[5]	[6]	[0]	[1]	[2]	[3]
[5]	[5]	[6]	[0]	[1]	[2]	[3]	[4]
[6]	[6]	[0]	[1]	[2]	[3]	[4]	[5]

for any $a, b \in G \Rightarrow a * b \in G$

In the above table, any two elements in Z_7 , their addition is in Z_7 .

Associative Property:

for any $a, b \in G \Rightarrow a * b \in G$

Here, for any $a, b \in Z_7 \Rightarrow a * (b * c) = (a * c) * c$

Let us take $[1], [3], [4] \in Z_7$

Consider

$$[1] \oplus_7 ([3] \oplus_7 [4]) = [1] \oplus_7 [0] = [1]$$

Consider

$$([1] \oplus_7 [3]) \oplus_7 [4] = [4] \oplus_7 [4] = [1]$$

Identity Property:

In the table, [0] is an identity element in Z_7

$$[0] \oplus_7 [0] = [0]$$

$$[1] \oplus_7 [0] = [1]$$

$$[2] \oplus_7 [0] = [2]$$

$$[3] \oplus_7 [0] = [3]$$

$$[4] \oplus_7 [0] = [4]$$

$$[5] \oplus_7 [0] = [5]$$

$$[6] \oplus_7 [0] = [6]$$

Inverse Property:

$$[0] \oplus_7 [0] = [0]$$

$$[1] \oplus_7 [6] = [0]$$

$$[2] \oplus_7 [5] = [0]$$

$$[3] \oplus_7 [4] = [0]$$

$$[4] \oplus_7 [3] = [0]$$

$$[5] \oplus_7 [2] = [0]$$

$$[6] \oplus_7 [1] = [0]$$

Inverse element is exist for each element of Z_7 and in Z_7

Hence (Z, \oplus_7) is a group.

Problem - 4:

Find the residue class of integers under multiplication modulo 7 and prove that it is a group.

Solution:

Let $Z_7 = \{1, 2, 3, 4, 5, 6\}$ be the set of all residue class of integer for Z_7 under addition

To prove: $Z_7 = (Z, \oplus_7)$ is a group.

\oplus_7	[1]	[2]	[3]	[4]	[5]	[6]
[1]	[1]	[2]	[3]	[4]	[5]	[6]
[2]	[2]	[4]	[6]	[1]	[3]	[5]
[3]	[3]	[6]	[2]	[5]	[1]	[4]
[4]	[4]	[1]	[5]	[2]	[6]	[3]
[5]	[5]	[3]	[1]	[6]	[4]	[2]
[6]	[6]	[5]	[4]	[3]	[2]	[1]

Closure Property:

For any $a, b \in G \Rightarrow a * b \in G$

In the above table, any two elements in Z_7 their addition is in Z_7 .

Associative Property:

For any $a, b, c \in G \Rightarrow a * (b * c) = (a * b) * c$

Here, for any $a, b, c \in Z_7 \Rightarrow a * (b * c) * c$

Let us take, $[1], [3], [4] \in Z_7$

Consider

$$[1] \oplus_7 ([3] \oplus_7 [4]) = [1] \times_7 [5] = [5]$$

Consider $([1] \oplus_7 [3]) \oplus_7 [4] = [3] \oplus_7 [4] = [5]$

$\therefore (Z, \oplus_7)$ satisfies associative property

Identity property:

In the table, $[1]$ is an identity element in Z_7

$$[1] \oplus_7 [1] = [1] \quad [2] \oplus_7 [1] = [2]$$

$$[3] \oplus_7 [3] = [3] \quad [4] \oplus_7 [1] = [4]$$

$$[5] \oplus_7 [5] = [5] \quad [6] \oplus_7 [1] = [6]$$

Inverse property:

$$[1] \oplus_7 [1] = [1] \quad [2] \oplus_7 [4] = [1]$$

$$[3] \oplus_7 [5] = [1] \quad [4] \oplus_7 [2] = [1]$$

$$[5] \oplus_7 [3] = [1] \quad [6] \oplus_7 [6] = [1]$$

Inverse element is exist for each element of Z_7 and in Z_7

Hence (Z, \oplus_7) is a group.

Problem - 5:

Prove that (S, \cdot) where S is the set of all fourth roots of unity is a group

Solutions:

Let S be set of all fourth root of unity

(i.e.,) $S = \{1, -1, i, -i\}$

	1	-1	i	-i
1	1	-1	i	-i
-1	-1	1	-i	i
i	i	-i	-1	1
-i	-i	i	1	-1

Closure property:

for any $a, b \in G \Rightarrow a * b \in G$

In the above table, any two elements in S their addition is in S .

Associative property:

for any $a, b \in G \Rightarrow a * b \in G$

Let us take, $1 \cdot (-1 \cdot i) \in S$

Consider,

$$1 \cdot (-1 \cdot i) = 1 \cdot (-i) = -i$$

Consider,

$\therefore (S_1, \cdot)$ satisfies associative property.

Identity Property:

Here, 1 is in identity element

$$\begin{array}{l} 1 \cdot 1 = 1 \\ -1 \cdot 1 = -1 \\ i \cdot 1 = i \\ -i \cdot 1 = -i \end{array}$$

Inverse Property:

$$1 \cdot 1 = 1$$

$$-1 \cdot -1 = 1$$

$$i \cdot -i = 1$$

$$i \cdot -i = 1$$

$$-i \cdot i = 1$$

Inverse element is exist for each element of S and in S

Hence (S, \cdot) is a group.

Problem - 6:

Show that the set of all rational numbers except 1 is a group under the binary operation * defined as $a * b = a + b - ab$ is group.

Solution:

$$\text{Let } Q - \{1\} = \left\{ \frac{p}{q} \mid p, q \in N \text{ \& } p, q \neq 0, 1 \right\}$$

Closure property:

$$\text{For any } a, b \in Q - \{1\}$$

$$\Rightarrow a * b = a + b - ab \in Q - \{1\}$$

$$\therefore a * b \in Q - \{1\}$$

Associative Property:

$$\text{For any } a, b, c \in Q - \{1\}$$

$$\Rightarrow a * (b * c) = (a * b) * c \text{ consider,}$$

$$\Rightarrow a * (b * c) = a * (b + c - bc)$$

$$= a + b + c - bc - a(b + c - bc)$$

$$= a + b + c - bc - ab - ac + abc$$

Consider

$$\begin{aligned}(a * b) * c &= (a + b - ab) * c \\ &= a + b + c - ab - (a + b - ab)c \\ &= a + b + c - ab - ac - bc + abc\end{aligned}$$

$\therefore Q - \{1\}$ satisfies associative property.

Identity Property:

For any $a \in G, \exists e \in G$ such that $a * e = e * a = a$

$$a * e = a$$

$$a + e - ae = a$$

$$e - ae = 0$$

$$e(1 - a) = 0$$

$$e = 0$$

$$\therefore 0 \in Q - \{1\}$$

Inverse Property:

for each $a \in G, \exists a^{-1} \in G$ such that $a * a^{-1} = a^{-1} * a = e$

Consider,

$$a * a^{-1} = 0$$

$$a + a^{-1} - aa^{-1} = 0$$

$$a^{-1}(1 - a) = -a$$

$$a^{-1} = \frac{-a}{1 - a}$$

$$a^{-1} = \frac{a}{a - 1}$$

Inverse element exists in $Q - \{1\}$ for each a

Hence set of all rational numbers except 1 is a group under the binary operations * defined as

$a * b = a + b - ab$ is group.

Problem - 7:

Prove that $(Q, *)$ is group with respect to $*$ as defined as $a * b = \frac{ab}{2} \forall a, b \in Q$

Closure property:

For any $a, b \in Q$

$$a * b = \frac{ab}{2} \in Q$$

$$\therefore a * b \in Q$$

Associative property:

For any $a, b, c \in Q$

$$a * (b * c) = (a * b) * c$$

Consider,

$$\begin{aligned} a * (b * c) &= a * \left(\frac{bc}{2} \right) \\ &= \frac{abc}{4} \end{aligned}$$

Consider,

$$\begin{aligned} (a * b) * c &= \left(\frac{ab}{2} \right) * c \\ &= \frac{abc}{4} \end{aligned}$$

Hence associative property is satisfied

Identity Property:

For any $a \in G, \exists e \in G$ such that $a * e = e * a = a$

Consider,

$$a * e = a$$

$$\frac{ae}{2} = a$$

$$ae = 2a$$

$$e = 2$$

$$\therefore 2 \in Q$$

Hence identity elements in Q

Inverse Property:

for each $a \in G, \exists a^{-1} \in G$ such that $a * a^{-1} = a^{-1} * a = e$

Consider,

$$a * a^{-1} = 2$$

$$\frac{aa^{-1}}{2} = 2$$

$$\frac{aa^{-1}}{2} = 4$$

$$a^{-1} = \frac{4}{a}$$

Inverse element exist in Q for each a.

Hence $(Q, *)$ is group with respect to *

Problem - 8:

Prove that $(Z, *)$ is group with respect to * as defined as $a * b = a + b + 1 \forall a, b \in Z$

Solution

Closure property:

For any $a, b \in Z$

$$a * b = a + b + 1 \in Z$$

$$\therefore a * b \in Z$$

Associative Property:

for any $a, b, c \in Z \Rightarrow a * (b * c) = (a * b) * c$

5.32. ALGEBRA STRUCTURE - MCQ

1. A group G is said to be _____ if for every $a, b \in G$, $a \cdot b = b \cdot a$
- A) semigroup
B) abelian
C) monoid
D) quasi group
2. Let $G = \{a^i, i = 0, 1, 2, \dots, n-1\}$ where $a^0 = a^n = e$, $a^{i+j} = \begin{cases} a^{i+j} & \text{if } i+j < n \\ a^{i+j-n} & \text{if } i+j \geq n \end{cases}$, the G is a
- A) cyclic group of order $n-1$
B) cyclic group of order $2n$
C) cyclic group of order n
D) cyclic group of order $n+1$
3. Every subgroup of _____ is normal.
- A) cyclic group
B) Abelian group
C) Cyclic or abelian
D) cyclic and abelian
4. Let G be the set of all 2×2 matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where $a, b, c, d \in R$ such that $ad - bc = 1$, then G is _____.
- A) finite abelian group
B) finite non-abelian group
C) infinite abelian group
D) infinite non-abelian group
5. Which of the following is incorrect?
- A) The identity G is unique
B) Every $a \in G$ has a unique inverse in G
C) For every $a \in G$, $(a^{-1})^{-1} = a$
D) for all $a, b \in G$ $(a \cdot b)^{-1} = a^{-1}b^{-1}$
6. G is a finite group of order 4 and $a \in G$, then $a^4 =$
- A) 4
B) 2
C) e
D) 1
7. If G has a element $a \neq e$ such that $a^2 = e$, then G is a group of
- A) odd order
B) even order
C) finite order
D) infinite order
8. For any _____ construct a non-abelian group of order $2n$
- A) $n > 1$
B) $n \geq 2$
C) $n \geq 1$
D) $n > 2$



9. Let G be the set of all 2×2 matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where a, b, c, d are integers modulo 2, such that $ad - bc = 1$ is a group under multiplication, then $|G| =$
- A) 6 B) 48 C) 4 D) 3
10. Let G be the set of all 2×2 matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where $ad - bc = 1$, a, b, c, d are integers mod 3, forms group under multiplication then $|G| =$
- A) 48 B) 6 C) 4 D) 9
11. A non-empty subset H of a group G is a subgroup of G if
- A) $a, b \in H \Rightarrow ab \in H$ B) $a \in H \Rightarrow a^{-1} \in H$
 C) $a, b \in H \Rightarrow ab^{-1} \in H$ D) all A, B, C
12. If H is a non-empty _____ of a group G and H is closed under multiplication, then H is a subgroup of G
- A) infinite subset B) finite subset
 C) proper subset D) improper subset
13. Let $G = (z, +)$ Let H be a subset consisting of all multiples of m ($H = m\mathbb{Z}$) then H is _____ of G .
- A) subgroup B) not subgroup
 C) may be subgroup D) none of these
14. If H is a subgroup of G , then index of H is no. of _____ of H in G .
- A) all right cosets of G B) distinct right cosets
 C) distinct left cosets D) both c and b
15. If G is a finite group and $a \in G$, then $a^{|G|} =$
- A) $0A$ B) $0(G)$
 C) e D) 0
16. If n is a +ve integer and a is relatively prime onto n , then $a^{\phi(n)} \equiv 1 \pmod{n}$ is
- A) Euler theorem B) Fermat theorem
 C) Sylow's theorem D) Cayley's theorem



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UG TRB MATHEMATICS 2023-2024



UNIT VI Real Analysis

Your Success is Our Goal....

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UNIT - VI – REAL ANALYSIS

INDEX

S.No.	CHAPTER NAME	P.No.
6.1	SETS	1
	6.1.1. Order of Sets	
	6.1.2. Types of Sets	
	6.1.3. Operations of Set	
	6.1.4. Properties of Sets	
6.2.	SUBGROUPS	11
6.3	COUNTABLE AND UNCOUNTABLE SETS	25
	6.3.1. Real Number	
6.4.	FUNCTION	26
	6.4.1. Image and Range	
	6.4.2. Inverse Image	
6.5.	REAL – VALUED FUNCTION, EQUIVALENCE AND COUNTABILITY	31
	6.5.1. Real valued functions:	
	6.5.2. Operational on Real - valued functions:	
	6.5.3. Composition of function	
6.6.	EQUIVALENT FUNCTION	32
6.7.	COUNTABILITY	33
6.8.	INFIMUM AND SUPREMUM OF A SUBSET OF R	34
	6.8.1. Infimum of a subset of R	7
	6.8.2. Supremum of a subset of R	
	6.8.3. Bounded above - upper bound	
	6.8.4. Bounded below	

	6.8.5. Bounded sets	
	6.8.6. Greatest lower bound (G.L.B)	
	6.8.7. Least upper bound (L.U.B)	
	6.8.8. Least upper bound axiom	
6.9.	BOLZANO – WAITRESS THEOREM	37
6.10.	SEQUENCE OF REAL NUMBER	39
	6.10.1. Subsequence of positive integer:	
	6.10.2. Limit of a sequence.	
6.11.	CONVERGENT AND DIVERGENT SEQUENCES	42
6.12.	DIVERGENT SEQUENCE	45
6.13	MONOTONE SEQUENCE	47
6.14	CAUCHY SEQUENCE	61
6.15	LIMIT SUPERIOR AND LIMIT INFERIOR OF A SEQUENCE	63
	6.15.1. Limit superior	
	6.15.2. limit inferior	
6.16	SUB SEQUENCE	66
	6.16.1. Subsequence of positive integers	
6.17	INFINITE SERIES	68
6.18	ALTERNATING SERIES	71
	6.18.1. Theorem: Leibnitz theorem	
6.19	CONDITIONAL CONVERGENCES AND ABSOLUTE CONVERGENCES	74
	6.19.1. Definition of Conditional Convergences	
	6.19.2. Positive terms of a series	
	6.19.3. Negative terms of a series	

6.20	TEST OF ABSOLUTE	78
	6.20.1. Theorem Comparison Test	
	6.20.2. Theorem ratio test	
	6.20.3 Theorem: Root test.	
6.21	CONTINUITY AND UNIFORM CONTINUITY OF A REAL VALUED FUNCTION OF A REAL VARIABLE	82
	6.21.1. Continuity function of a real variable	
	6.21.2. Uniform continuity of a real valued function	
6.22	LIMIT OF A FUNCTION AT A POINT	84
6.23	CONTINUITY AND DIFFERENTIABILITY OF REAL VALUED FUNCTION	86
	6.23.1. Continuity of real valued function	
	6.23.2. Continuity Function	
	6.23.3. Differentiability of real valued function	
6.24	ROLLE'S THEOREM	87
6.25	MEAN VALUE THEOREM	89
	6.25.1. Theorem: Generalized law of mean	
6.26	INVERSE FUNCTION THEOREM, TAYLOR'S THEOREM WITH REMAINDER FORM	92
	6.26.1. Inverse function theorem	
	6.26.2. Maclaurin series	
6.27	POWER SERIES EXPANSION	97
6.28	RIEMANN INTEGRABILITY	100
6.29	SEQUENCE AND SERIES OF FUNCTION	102
	6.29.1. Point wise convergence of sequence of function	
	6.29.2. Uniform convergence of sequence of function	
	6.29.3. Theorem (Dini's theorem for sequence of function)	

	6.29.4. Uniform convergence of series of function	
	6.29.5. Weierstrass M –test	
6.30	METRIC SPACES	105
6.31	LIMITS OF A FUNCTION AT A POINT IN METRIC SPACES	107
6.32	FUNCTION CONTINUOUS ON A METRIC SPACES	109
6.33	VARIOUS REFORMULATIONS OF CONTINUITY OF A FUNCTION IN A METRIC SPACE.	111
6.34	OPEN SET	113
6.35	CLOSED SETS	114
	6.35.1. Closed subset.	
	6.35.2. Homeomorphism	
	6.35.3. Dense	
6.36	DISCONTINUOUS FUNCTIONS ON THE REAL LINE	117
	6.36.1. Oscillation of f over J .	
	6.36.2. Nowhere Dense	
6.37	IMPORTANT MCQ - REAL ANALYSIS	120

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UNIT - VI

REAL ANALYSIS

6.1. SETS



DEFINITION OF SETS:

❖ A set is a collection of objects chosen from some universe

➤ Example: $\{1,2,3,4\}$ is a set of numbers

6.1.1. Order of Sets:

❖ The order of a set defines the number of elements a set is having. It describes the size of a set. The order of a sets is also known as the cardinality.

6.1.2. Types of Sets:

(i) Empty set - A set which doesn't contain any element. It is denoted by $\{ \}$ or ϕ

(ii) Singleton set - A set which contains a single element.

(iii) Finite set - A set which consists of a definite number of elements.

(iv) Infinite set - A set which is not finite.

(v) Equivalent set - If the number of elements is the same for two different sets, then they are called equivalent sets.

- (vi) Equal sets - The two sets A and B are said to be equal if they have exactly the same elements, the order of elements do not matter.
- (vii) Disjoint sets - Two sets are said to be disjoint if the sets does not contain any common element.
- (viii) Subsets - A sets 'A' is said to be a sub sets of B if every element of A is also an element of B, denoted as $A \subseteq B$.
- (ix) proper subset - If $A \subseteq B$ and $A \neq B$, then A is called the proper subset of B and it can be written as $A \subset B$.
- (x) superset - Sets A is said to be the suspect of B if all the elements of sets B are the elements of set A. it is represented as $A \supset B$
- (xi) universal set - A set which contains all the sets relevant to a certain condition is called the universal set. It is the set of all possible values.

6.1.3. Operations of Set:

(i) Union Sets:

If set A and set B are two sets, then A union B is the set that contains all the elements of a set A and set B. It is denoted as $A \cup B$.

➤ **Example:**

$$A = \{1, 2, 3\} \text{ and } B = \{4, 5, 6\}$$

$$A \cup B = \{1, 2, 3, 4, 5, 6\}$$

(ii) Intersection of Sets:

If sets A and set B are two sets, then A intersection B is the set that contains only the common elements between set A and set B . If denoted as $A \cap B$

➤ **Example:**

$$A = \{1, 2, 3\} \text{ and } B = \{4, 5, 6\}$$

$$A \cap B = \{ \} \text{ or } \phi$$

(iii) Complement of Sets:

The complement of sets of any set, say p is the set of all elements in the universal set that are not in set P. If is denoted by 'p'

➤ Properties of complements sets

a) $P \cup P' = \cup$

b) $P \cap P' = \phi$

c) $(P')' = P$

d) $\phi' = \cup$ and $\cup' = \phi$

(iv) Cartesian product of sets:

If set A and set B are two sets then the Cartesian product of set A and set B is a set of all ordered pairs (a, b) such that a is an element of A and b is an element of B. It is denoted by $A \times B$

$$A \times B = \{(a, b); a \in A \text{ and } b \in B\}$$

(v) Difference of sets:

If set A and set B are two, then set A different set B is a set which has element of A but no elements of B. It denoted as $A - B$

➤ **Example:**

$$A = \{1, 2, 3\} \text{ and } B = \{3, 2, 4\}$$

$$A - B = \{1\}$$

6.1.4. Properties of Sets:

(i) commutative property : $A \cup B = B \cup A$ and $A \cap B = B \cap A$

(ii) Associative property : $A \cup (B \cap C) = (A \cup B) \cap C$

$$A \cap (B \cup C) = (A \cap B) \cup C$$

(iii) Distributive property : $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

(iv) De Morgan's law: Law of union : $(A \cup B)' = A' \cap B'$

Law of intersection : $(A \cap B)' = A' \cup B'$

(v) complement law : $A \cup A' = A' \cup A = \cup$ and $A \cap A' = \phi$

(vi) Idempotent law and law of null and universal set for any finite set A,

(a) $A \cup A = A$

(b) $A \cap A = A$

(c) $\phi' = \cup$

(d) $\phi = \cup'$



Ex:

- The set $f = \{ \langle x, x^2 \rangle \mid -\infty < x < \infty \}$ is the function defined by

➤ $f(x) = x^2 \quad (-\infty < x < \infty)$

➤ $f(1) = 1 \quad f(-1) = 1$

➤ $f(2) = 4 \quad f(-2) = 4$

Define: Image and Range:

- Let 'f' be a function from X to Y for any $x \in X, f(x) = y \in Y$ here $f(x) = y$ is called an image of 'x' under f. Let 'f' be a function from X to Y define, $f(x) = \{ y \mid y = f(x); f \text{ or some } x \in X \}$ is called a range of 'f'.

Define: Inverse Image:

- Let 'f' is a function $f : X \rightarrow Y$ such that $f(x) = y \Rightarrow x = f^{-1}(y)$, here $f(x)$ is called an image of y under 'f'. $f^{-1}(y)$ is called an inverse image of x under 'f'.

Let B be a subset of Y. i.e., $B \subset Y$

$$f^{-1}(B) = \{ x \mid f(x) = y; \text{ for } y \in B \}$$

Define: One-One function (or) Injective:

- A function $f : X \rightarrow Y$ is said to be a one-one function if for any $x_1, x_2 \in X$. Such that $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$ (OR) $x_1 = x_2 \Rightarrow f(x_1) = f(x_2)$
- i.e., The distinct elements in X has distinct image in Y.

Define: Onto function (or) Surjective:

- A function $f : X \rightarrow Y$ is said to be a onto function, if the range of 'f' is equal to Y. i.e.,

$$f(x) = y$$

$$f : R \rightarrow R$$

Let $f_1 : R \rightarrow (0, \infty)$

$$f_1(x) = x^2$$

$$f_1(-2) = 4$$

$$f_1(-1) = 1$$

$$f_1(0) = 0$$

$$f_1(1) = 1$$

$$f_1(2) = 4$$



Range of $f_1(0, \infty) \subset R$. It is a onto function but not into

Let $f_2 : R \rightarrow R$

$$f_2(x) = x$$

Range of $f_2(-\infty, \infty) = R$

Define 1 - 1 Correspondence (or) Bijective:

- If the function f is both one-one and onto then we say that the function f is 1 - 1 Correspondance (or) Bijective.

Define: Constant function:

- The function f is said to be constant function, if all the images are same. i.e., $f(x) = k$ for all x in domain

Define: Inverse function:

- Let 'f' be a function from X to Y, such that f is one-one and onto function.

\therefore The function $f^{-1} : Y \rightarrow X$ is called a inverse function of 'f'.

Define Characteristic function:

- If $A \subset S$ then the characteristic function ψ_A is defined as,

$$\psi_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \in A' \end{cases}$$

Theorem - 1:

- If $f : A \rightarrow B$ and $X \subset B, Y \subset B$ Then $f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y)$ (or) The inverse image of the union of two sets is the union of the inverse images.

Proof:

- Given that $f : A \rightarrow B$ and $X \subset B, Y \subset B$

To prove: $f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y)$

Let $b \in X \cup Y$

Since $f : A \rightarrow B$

$\therefore f(a) = b$ such that $a \in A, b \in B$ and hence $X \subset B, Y \subset B$

For some $a \in A$,

$$f(a) \in X \cup Y \rightarrow (1)$$

$$\therefore f(a) \text{ (or) } f(a) \in Y$$

$$a \in f^{-1}(X) \text{ (or) } a \in f^{-1}(Y)$$

$$\Rightarrow a \in f^{-1}(X) \cup f^{-1}(Y)$$

From (1), $f(a) \in X \cup Y$

$$a \in f^{-1}(X \cup Y)$$

$$\Rightarrow f^{-1}(X \cup Y) \subseteq f^{-1}(X) \cup f^{-1}(Y) \rightarrow (*)$$

Now, let $a \in f^{-1}(X) \cup f^{-1}(Y)$

$$a \in f^{-1}(X) \text{ (or) } a \in f^{-1}(Y)$$

$$f(a) \in X \quad (\text{or}) \quad f(a) \in Y$$

$$f(a) \in X \cup Y$$

$$a \in f^{-1}(X \cup Y)$$

$$\therefore f^{-1}(X) \cup f^{-1}(Y) \subseteq f^{-1}(X \cup Y) \rightarrow (**)$$

From (*) and (**)

$$f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y)$$

Hence proved

Theorem - 2:

If $f : A \rightarrow B, X \in A, Y \in A$ then $f(X \cup Y) = f(X) \cup f(Y)$

Proof:

Given that $f : A \rightarrow B, X \in A, Y \in A$

To prove:

$$f(X \cup Y) = f(X) \cup f(Y)$$

Suppose $b \in f(X \cup Y)$

Since f is a function from A to B

$$\therefore b = f(a), \text{ for some } a \in X \cup Y$$

$$\Rightarrow a \in X \quad (\text{or}) \quad a \in Y$$

$$\Rightarrow f(a) \in f(X) \quad (\text{or}) \quad \Rightarrow f(a) \in f(Y)$$

$$\Rightarrow f(a) \in f(X) \cup f(Y)$$

$$\Rightarrow b \in f(X) \cup f(Y)$$

$$\therefore f(X \cup Y) \subseteq f(X) \cup f(Y) \quad \text{--- (*)}$$

Since f is a function from A to B

$$\therefore v = f(a); \text{ for some } a \in X \cup Y$$

Suppose, $b \in f(X) \cup f(Y)$

$b \in f(X)$ (or) $f(Y)$

From (*) and (**)

$$f^{-1}(X \cap Y) = f^{-1}(X) \cap f^{-1}(Y)$$

Hence proved.

Define: Real Valued Function

- If $f : X \rightarrow R$ then f is called a Real valued function. If $x \in X$ then $f(x)$ is also called the value of f at x .

Ex.

1. $f(x) = x^2$ or $(-\infty < x < \infty)$ it is a real valued function.

2. $f : Z \rightarrow C$

$$f(x) = ix$$

It is not a real valued function but it is a complex valued function.

Note:

1. If $A \subset B$ then every element of A is an element of B .
2. If A is a proper subset of B then $A \subset B$ and $A \neq B$.
3. If A is an improper subset of B then $A \subset B$ and $A = B$.
4. If $A \subseteq B$ and $B \subseteq A \Rightarrow A = B$
5. If $a \in A$ and $a \in B$ here a is an arbitrary then $A \subseteq B$

Operations on real valued function:

Let $f : A \rightarrow T, g : B \rightarrow R$

We define, $f + g$ as the function whose value at $x \in A$ is equal to $f(x) + g(x)$

$$\text{i.e., } (f + g)(x) = f(x) + g(x), (x \in A)$$

$$\text{Similarly, } (f - g)(x) = f(x) - g(x), (x \in A)$$

$$(fg)(x) = f(x)g(x), (x \in A)$$

$$(cf)(x) = cf(x), (x \in A) \text{ and } c - \text{constant}$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, (x \in A)$$

$$|f|(x) = |f(x)|, (x \in A)$$

$$\text{Max}(f, g)(x) = \text{Max}((f(x), g(x))), (x \in A)$$

$$\text{Min}(f, g)(x) = \text{Min}((f(x), g(x))), (x \in A)$$

Define: Composition of function:

- Let X, Y, Z are three non-empty sets. Let us define function, $f: X \rightarrow Y$ and $g: Y \rightarrow Z$. The function f composition of g is denoted by $g \circ f: X \rightarrow Y \rightarrow Z$

$$\Rightarrow g \circ f: X \rightarrow Z$$

- It is defined by, for any $x \in X$ such that $(g \circ f)(x) = g[f(x)]$. The composition function if possible only if, the co-domain of f is equal to the domain of g.

➤ Ex.

$$\text{Let } f(x) = 1 + \sin x \text{ on } (-\infty < x < \infty)$$

$$g(x) = x^2 \text{ on } (-\infty < x < \infty)$$

The find $(g \circ f)(x)$

Solution:

By the definition of composition function

$$\begin{aligned} g \circ f(x) &= g[f(x)] \\ &= g[1 + \sin x] \\ &= (1 + \sin x)^2 \\ &= 1 + \sin^2 x + 2\sin x \text{ on } (-\infty < x < \infty) \end{aligned}$$



Define: Equivalent set

- If there exist a 1 - 1 corresponds between the sets A and B then we say that A and B are equivalence sets of equivalent sets.

Note:

1. Any two sets containing exactly same number of elements are equivalent
2. Every set A is equivalent to itself.
3. If A and B are equivalent. Then B and A are equivalent
4. If A and B are equivalent and B and C are equivalent then A and C also equivalent

Define: Equivalent function:

- Two sets A and B are said to be equivalent sets if there exist a one-one and onto functions from A to B.

➤ Ex.

$$f : Z \rightarrow 2Z \cup \{0\}$$

$$f(z) = 2x$$

Here f is one-one on to function therefore Z and $2Z \cup \{0\}$ are equivalent set.

Exceise Questions:

1. How many elements are there in the complement of set A?

A) 0	B) 1
C) All the elements of A	D) None of these
2. Empty set is a _____.

A) Infinite set	B) Finite set
C) unknown set	D) universal set
3. Order of the power set P(A) of a set A of order n is equal to

A) n	B) $2n$	C) 2^n	D) n^2
------	---------	----------	----------
4. The cardinality of the power set of $\{x : x \in N, x \leq 10\}$ is _____.

A) 1024	B) 1023	C) 2048	D) 2043
---------	---------	---------	---------
5. The range of the function $f(x) = 3x - 2$, is:

A) $(-\infty, \infty)$	B) $R - \{3\}$	C) $(-\infty, 0)$	D) $(0, -\infty)$
------------------------	----------------	-------------------	-------------------



6.37. REAL ANALYSIS - IMPORTANT MCQ**Choose the Correct Answer:**

- The cardinal number of empty is
 (A) $n(\phi) = \infty$ (B) $n(\phi) = 1$ (C) $n(\phi) = 0$ (D) $n(\phi) = -\infty$
- which one is countable set
 (A) Algebraic number (B) Transcendental number
 (C) Cantor set (D) irrational number
- The element of a_{41} is
 (A) 4 (B) 5 (C) 3 (D) 2
- Every bounded and infinite set has a
 (A) Interior point (B) limit point
 (C) Derived set (D) Neighborhoods points
- Which one is an closed set
 (A) ϕ (B) ϕ' (C) \mathbb{N} (D) (a, b)
- The set of all real number is
 (A) uncountable (B) countable (C) finite (D) none of these
- The interval $[0, 1]$ is
 (A) uncountable (B) countable
 (C) finite (D) at most countable
- The cardinality of the set $x = \{a, e, i, o, u\}$ if _____
 (A) $n(x) = 5$ (B) $n(x) = \infty$
 (C) $n(x) = 2$ (D) $n(x) = 4$



9. The extended real line $\bar{R} =$ _____
- (A) R (B) \bar{R} (C) $R \cup \{-\infty, \infty\}$ (D) $R \cap \{-\infty, \infty\}$
10. If $S = [0, 1)$ then exterior of $S =$ _____
- (A) $(0, 1)$ (B) $(-\infty, 0) \cup (1, \infty)$
 (C) $(-\infty, 0)$ (D) $(1, \infty)$
11. If S is such that $S \cap S^1 = \emptyset$, then
- (A) S is uncountable (B) S is countable
 (C) S is compact (D) S is not closed
12. The Lévesque measure of cantor set C is
- (A) 1 (B) 0 (C) 4 (D) prime no
13. The continuity on a set A implies uniform continuity if A is
- (A) complete (B) compact (C) open (D) closed
14. Compact implies
- (A) bounded only (B) closed only
 (C) closed and bounded (D) none of these
15. If $\lim_n x_n = l$, then $\lim_n \frac{x_1 + x_2 + \dots + x_n}{n} =$ _____
- (A) l (B) $l + n$ (C) $\frac{l}{n}$ (D) $l - n$
16. The series $\sum_{n=1}^{\infty} ar^{n-1}$ _____
- (A) converges if $|r| < 1$ (B) diverges to if $r \geq 1$
 (C) oscillates if $r < -1$ (D) all are true





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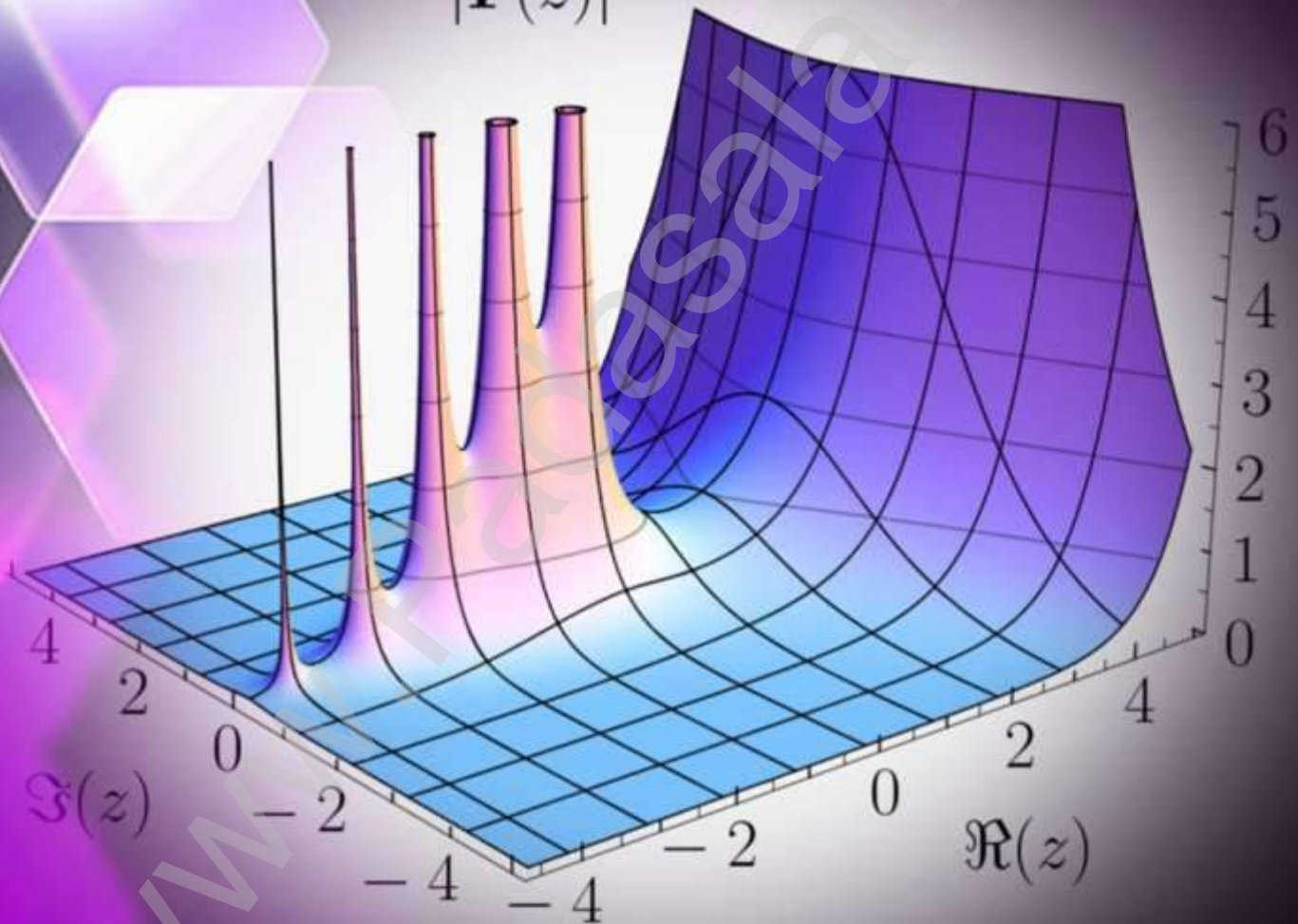
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$$|\Gamma(z)|$$



UNIT VII Complex Analysis

Your Success is Our Goal....

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S.NO	CHAPTER NAME	P.NO
7.1	Function of Complex Variables	1
7.2	Limits	4
7.3	Mappings	7
7.4	Theorems on Limit	14
7.5	Continuity	36
7.6	Differentiability	37
7.7	Cauchy Riemann Equations	39
7.8	Analytic Functions:	45
7.9	Harmonic Function:	65
7.10	Conformal Mapping	68
7.11	Mobius Transformations	69
7.12	Elementary Transformations	78
7.13	Bilinear Transformation	84
7.14	Cross Ratio	88
7.15	Fixed Points Of Bilinear Transformation	92

7.16	Special Bilinear Transformation	95
7.17	Contour And Contour Integrals	100
7.18	Contour And Contour Integrals	100
7.19	Anti Derivatives	122
7.20	Cauchy Goursat Theorem:	126
7.21	Power Series	128
7.22	Complex Integration	130
	7.22.1 Cauchy's Theorem	
	7.22.2 Morera's Theorem	
	7.22.3. Cauchy's Integral Formula	
	7.22.4. Liouville's Theorem	
7.23	Maximum Modulus Principle	140
7.24	Schwartz's Lemma	143
7.25	Taylor's Series	145
7.26	Laurent's Series	149
7.27	Calculus Of Residues	157
7.28	Residues Theorem	160
7.29	Evaluation Of Integrals And Definite Integral Of Function	163
7.30	Evaluation Of Integrals And Definite Integral Of Function	163
7.31	Argument Principal	172
7.32	Rouche's Theorem	174
7.33	Multiple Choice Questions	177

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UNIT - VII

COMPLEX ANALYSIS

ALGEBRA OF COMPLEX NUMBERS

7.1. FUNCTION OF A COMPLEX VARIABLE:



- We use the letters z and w to denote complex variables. Thus, to denote a complex valued function of a complex variable we use the notation $w = f(z)$. Throughout this chapter we shall consider functions whose domain of definition is a region of the complex plane.
- The function $w = iz + 3$ is defined in the entire complex plane.
- The function $w = \frac{1}{z^2 + 1}$ is defined at all points of complex plane except at $z = \pm i$
- The function $w = |z|$ is defined in the entire complex plane and this is a real values function of the complex variable z .
- If a_0, a_1, \dots, a_n are complex constants the function $p(z) = a_0 + a_1z + \dots + a_nz^n$ is defined in the entire complex plane and is called a polynomial in z .
- If $P(Z)$ and $Q(Z)$ are polynomials the quotient $\frac{P(Z)}{Q(Z)}$ is called a rational function and it is defined for all z with $Q(Z) \neq 0$
- The function $f(z) = x^4 + y^4 + i(x^2 + y^2)$ is defined over the entire complex plane.

- In general if $u(x, y)$ and $v(x, y)$ are real valued functions of two variables both defined on region S of the complex plane then $f(z) = u(x, y) + iv(x, y)$ is a complex valued function defined on S .
- Conversely each complex function $w = f(z)$ can be put in the form

$$w = f(z) = u(x, y) + iv(x, y)$$

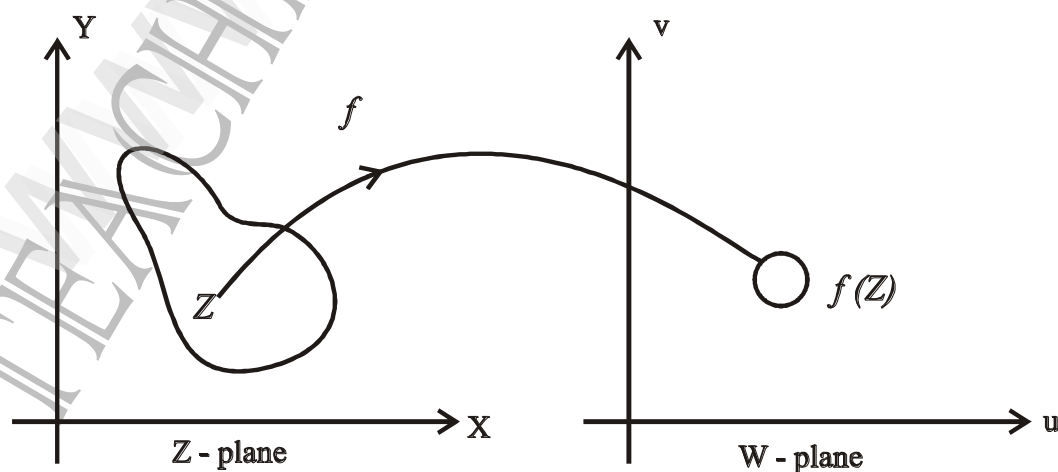
- When u and v are real valued functions of the real variables x and y $u(x, y)$ is called the real part and $v(x, y)$ is called the imaginary part of the function $f(z)$

For Example:

$$\begin{aligned} f(z) &= z^2 = (x + iy)^2 \\ &= x^2 + 2ixy + y^2(i^2) \\ &= (x^2 - y^2) + i(2xy) \end{aligned}$$


So that $u(x, y) = x^2 - y^2$ and $v(x, y) = 2xy$

- Thus, a complex function $w = f(z)$ can be viewed as a function of the complex variable z or as a function of two real variables x and y .
- To have a geometric representation of the function $w = f(z)$ it is convenient to draw separate complex planes for the variables z and w so that corresponding to each point $z = x + iy$ of the z -plane there is a point $w = u + iv$ in the w -plane.



Exercise Questions:

- The value of (iota) is _____.
 A) -1 B) 1 C) $(-1)^{\frac{1}{2}}$ D) $(-1)^{\frac{1}{4}}$
- Is i (iota) a root of $1+x^2=0$?
 A) True B) False


- In $z=4+i$, what is the real part?
 A) 4 B) i C) 1 D) $4+i$
- In $z=4+i$, what is the imaginary part?
 A) 4 B) i C) 1 D) $4+i$
- $(x+3)+i(y-2)=5+i2$, find the values of x and y .
 A) $x=8$ and $y=4$ B) $x=2$ and $y=4$
 C) $x=2$ and $y=0$ D) $x=8$ and $y=0$
- Find the domain of the function defined by $f(z)=\frac{z}{(z+\bar{z})}$
 A) $\text{Im}(z) \neq 0$ B) $\text{Re}(z) \neq 0$ C) $\text{Im}(z) = 0$ D) $\text{Re}(z) = 0$
- Let $f(z)=z+\frac{1}{z}$ what will be the definition of this function in polar form.
 A) $\left(r+\frac{1}{r}\right)\cos\theta+i\left(r-\frac{1}{r}\right)\sin\theta$ B) $\left(r-\frac{1}{r}\right)\cos\theta+i\left(r+\frac{1}{r}\right)\sin\theta$
 C) $\left(r+\frac{1}{r}\right)\sin\theta+i\left(r-\frac{1}{r}\right)\cos\theta$ D) $\left(r+\frac{1}{r}\right)\sin\theta-i\left(r-\frac{1}{r}\right)\cos\theta$
- For the function $f(z)=z^i$, what is the value of $|f(\omega)|+Arg f(\omega)$, ω being the cube root of unity with $\text{Im}(\omega) > 0$?
 A) $e^{-2\pi/3}$ B) $e^{2\pi/3}$ C) $e^{-2\pi/3}+2\pi/3$ D) $e^{-2\pi/3}-2\pi/3$

9. Let $f(z) = (z^2 - z - 1)^7$. If $a^2 + a + 1 = 0$ and $\text{Im}(\alpha) > 0$, then find $f(\alpha)$

- A) 128α B) -128α C) $128\alpha^2$ D) $-128\alpha^2$

10. For all complex numbers z satisfying $\text{Im}(z) \neq 0$, if $f(z) = z^2 + z + 1$ is a real value function the find its range

- A) $(-\infty, -1]$ B) $(-\infty, \frac{1}{3})$ C) $(-\infty, \frac{1}{2})$ D) $(-\infty, \frac{3}{4})$

7.2. LIMITS

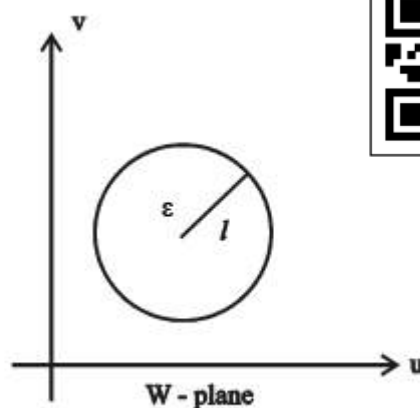
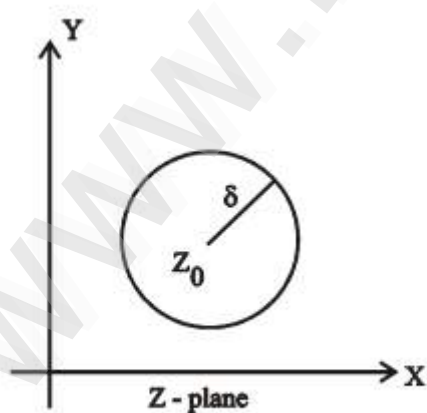
Definition:

- A function $w = f(z)$ is said to have the limit l as z tends to z_0 if given $\varepsilon > 0$ there exists $\delta > 0$ such that $0 < |z - z_0| < \delta$

$$\Rightarrow |f(z) - l| < \varepsilon$$

In this case we write $\lim_{z \rightarrow z_0} f(z) = l$

- Geometrically the definition states that given any open disc with centre l and radius ε , there exists an open disc with centre z_0 and radius δ such that for every point $z (\neq z_0)$ in the disc $|z - z_0| < \delta$ the image $w = f(z)$ lies in the disc $|w - l| < \varepsilon$



Lemma:

- When the limit of a function $f(z)$ exists as z tends to z_0 then the limit has a unique value.

Proof:

Suppose that $\lim_{z \rightarrow z_0} f(z)$ has two values l_1 and l_2

Then given $\varepsilon > 0$ there exists δ_1 and $\delta_2 > 0$ such that

$$0 < |z - z_0| < \delta_1 \Rightarrow |f(z) - l_1| < \frac{\varepsilon}{2} \text{ and}$$

$$0 < |z - z_0| < \delta_2 \Rightarrow |f(z) - l_2| < \frac{\varepsilon}{2}$$

Now let $\delta = \min\{\delta_1, \delta_2\}$

Then if $0 < |z - z_0| < \delta$ we have

$$|l_1 - l_2| = |l_1 - f(z) + f(z) - l_2|$$

$$\leq |f(z) - l_1| + |f(z) - l_2|$$

$$< \frac{\varepsilon}{2} + \frac{\varepsilon}{2}$$

$$= \varepsilon \quad (\text{Using triangle inequalities})$$

Since $\varepsilon < 0$ is arbitrary $|l_1 - l_2| = 0$

So that $l_1 = l_2$

Example – 1:

$$\text{Let } f(z) = \begin{cases} z^2 & \text{if } z \neq i \\ 0 & \text{if } z = i \end{cases}$$

As z approaches i , $f(z)$ approaches $i^2 = -1$

Hence, we expect that $\lim_{z \rightarrow i} f(z) = -1$

To prove that the given $\varepsilon > 0$ there exists $\delta > 0$ such that $0 < |z - i| < \delta$

$$\Rightarrow |z^2 + 1| < \varepsilon$$

$$\text{Now, } |z^2 + 1| = |(z+i)(z-i)| \Rightarrow |z+i||z-i| \quad \text{_____ (1)}$$

Note that if we can find a $\delta > 0$ satisfying the requirements of the definition then we can choose another $\delta \leq 1$ satisfying the requirements of the definition.

$$\text{Now } 0 < |z - i| < 1 \Rightarrow |z + i| = |z - i + 2i|$$

$$\leq |z - i| + |2i|$$

$$< 1 + 2 = 3$$

$$\therefore |z + i| < 3$$

Using this in (1) we obtain $0 < |z - i| < 1$

$$\Rightarrow |z^2 + 1| < 3|z - i|$$

Hence if we choose $\delta = \min\left\{1, \frac{\varepsilon}{3}\right\}$ we get

$$0 < |z - i| < \delta$$

$$\Rightarrow |z^2 + 1| < \varepsilon$$

$$\therefore \lim_{z \rightarrow i} f(z) = -1$$

Example – 2:

$$\lim_{z \rightarrow 2} \frac{z^2 - 4}{z - 2} = 4$$

$$\text{Let } f(z) = \frac{z^2 - 4}{z - 2}$$

Hence $f(z)$ is not defined at $z = 2$ and when $z \neq 2$ we have

$$f(z) = \frac{(z+2)(z-2)}{z-2}$$

$$= z + 2$$

$$\therefore |f(z) - 4| = |z + 2 - 4|$$

$$= |z - 2| \text{ when } z \neq 2$$

Now given $\varepsilon > 0$, we choose $\delta = \varepsilon$

Then $0 < |z - 2| < \delta \Rightarrow |f(z) - 4| < \varepsilon$

$$\therefore \lim_{z \rightarrow 2} f(z) = 4$$

Example – 3:

The function $f(z) = \frac{\bar{z}}{z}$ does not have a limit as $z \rightarrow 0$.

$$f(z) = \frac{\bar{z}}{z} = \frac{x - iy}{x + iy}$$

Suppose $z \rightarrow 0$ along the path $y = mx$

$$\begin{aligned} \text{Along this path } f(z) &= \frac{x - imx}{y + imx} \\ &= \frac{1 - im}{1 + im} \text{ as } x \neq 0 \end{aligned}$$

Hence if $z \rightarrow 0$ along the path $y = mx$, $f(z)$ tends to $\frac{1 - im}{1 + im}$ which is different for values of m .

Hence $f(z)$ does not have a limit as $z \rightarrow 0$

7.3. MAPPINGS

The mapping $w = z^2$

The transformation $w = z^2$ is conformal at all points except $z = 0$

Put $w = u + iv$ and $z = x + iy$

$$u + iv = (x + iy)^2$$

$$u + iv = x^2 - y^2 + i2xy$$

Equating real and imaginary parts, we get

$$u = x^2 - y^2 \qquad v = 2xy$$

Now we discuss the following cases,

Case (i):

The equation of real axis $y = 0$ in the z -plane

When $y = 0$, we have $u = x^2$ $v = 0$

The real axis $y = 0$ in the z -plane is mapped to positive u -axis in the w -plane

Case (ii):

The equation of imaginary axis $x = 0$ in the z -plane

When $x = 0$, we have $u = -y^2$ $v = 0$

∴ The imaginary axis $x = 0$ in the z -plane is mapped to negative u -axis in the w -plane

Case (iii):

The equation of the line parallel to x -axis in the z -plane is $y = 0$

Then, we have $u = x^2 - c^2$; $v = 2xc$

$$\Rightarrow x = \frac{v}{2c}$$

$$\therefore u = \frac{v^2}{4c^2} - c^2$$

$$u = \frac{v^2 - 4c^4}{4c^2}$$

$$4uc^2 - 4c^4 = v^2$$

$$4c^2(u + c^2) = v^2$$

This is a parabola with focus at the origin in the w -plane and u -axis as its axis.

For different values of c , we obtain a family of confocal parabola with u -axis as the axes.

Case (iv):

The equation of the line parallel to y -axis (i.e.,) $x = d$ we have

$$u = d^2 - y^2$$

$$v = 2dy$$

$$\Rightarrow y = \frac{v}{2d}$$

$$u = d^2 - \frac{v^2}{4d^2}$$

$$4d^2u = 4d^4 - v^2$$

$$v^2 = -4d^2u + 4d^4$$

$$v^2 = -4d^2[u - d^2]$$

- This is also a parabola with focus at the origin and u-axis as its axes in the w-plane.
- For different values of d, we get a family of focal parabola with u-axis as the axes and the common focus at the origin.

The mapping $w = \sin z$

Put $w = u + iv$ and $z = x + iy$

$$u + iv = \sin(x + iy)$$

$$= \sin x \cos iy + \cos x \sin iy$$

$$= \sin x \cosh y + \cos x (i \sinh y)$$

$$u + iv = \sin x \cosh y + i \cos x \sinh y$$

Equating real and imaginary parts, we get

$$u = \sin x \cosh y \quad v = \cos x \sinh y$$

Case (i):

The equation of real axis $y = 0$ in the z – plane

When $y = 0$, we have $u = \sin x$, $v = 0$

Since, $\sin x$ takes values between -1 and 1 , the image of the real axis $y = 0$ is the line segment $-1 \leq u \leq 1$ of the u – axis.

Case (ii):

The equation of imaginary axis $x = 0$ in the z -plane

When $x = 0$, we have $u = 0$, $v = \sin hy$

If $y = 0$, $\sin hy$ is positive and if $y < 0$, $\sin hy$ is negative

- Hence the upper – half of the imaginary axis in the z-plane maps into the upper half of the imaginary axis of the w-plane, while the lower halves of both corresponds with one another.

Case (iii):

The equation of any line parallel to x-axis in the z-plane is $y = c$

$$\text{From } u = \sin x \cosh y \qquad v = \cos x \sinh y$$

$$\Rightarrow \sin x = \frac{u}{\cosh y}, \cos x = \frac{v}{\sinh y}$$

$$\text{W.K.T } \sin^2 x + \cos^2 x = 1$$

$$\frac{u^2}{\cosh^2 y} + \frac{v^2}{\sinh^2 y} = 1$$

Put $y = c$ in above equation

$$\frac{u^2}{\cosh^2 c} + \frac{v^2}{\sinh^2 c} = 1$$

When $c \neq 0$ the above equation represent ellipse with semi-axes $\cosh c$ and $\sinh c$

Case (iv):

The equation of any line parallel to y-axis in the z-plane is $x = d$

$$\text{From } u = \sin x \cosh y, v = \cos x \sinh y$$

$$\cosh y = \frac{u}{\sin x}, \sinh y = \frac{v}{\cos x}$$

$$\text{W.K.T } \cosh^2 y - \sinh^2 y = 1$$

$$\frac{u^2}{\sin^2 x} - \frac{v^2}{\cos^2 x} = 1$$

Put $x = d$ in above equation

$$\frac{u^2}{\sin^2 d} - \frac{v^2}{\cos^2 d} = 1$$

- The above equation represents a system of hyperbola. Hence, the lines parallel to the imaginary axis of the z-plane map into confocal hyperbola.

The mapping $w = e^z$

The given transformation, $w = e^z$

$$\text{Since } \frac{dw}{dz} = e^z \neq 0$$

For any values of z , the mapping $w = e^z$ is conformal at all the points in z -plane.

Replace $z = x + iy$ and $w = u + iv$ in the mapping, we get

$$u + iv = e^{x+iy}$$

$$= e^x \cdot e^{iy}$$

$$u + iv = e^x (\cos y + i \sin y)$$

$$u + iv = e^x \cos y + ie^x \sin y$$

Equating real and imaginary parts we have

$$u = e^x \cos y \quad v = e^x \sin y$$

Eliminating y from the above equation, we get

$$\begin{aligned} u^2 + v^2 &= e^{2x} \cos^2 y + e^{2x} \sin^2 y \\ &= e^{2x} (\cos^2 y + \sin^2 y) \end{aligned}$$

$$u^2 + v^2 = e^{2x} \quad \text{--- (1)}$$

Eliminating x from the above equation, we have

$$\frac{v}{u} = \frac{e^x \sin y}{e^x \cos y}$$

$$\frac{v}{u} = \tan y$$

$$u \tan y = v \quad \text{--- (2)}$$

- Which represent a system of concentric circles with the origin.
- In particular, $x = 0$ transforms into a circle of unit radius with centre at the origin in the w -plane.

- Hence the lines parallel to y-axis transform into concentric circles with the centre and $w = 0$

When $y = \text{constant}$

- The equation (2) represent a line through the origin in the w-plane
- Hence the line parallel to x-axis Transforms into radial line

1. When $y = 0$ from the equation $u = e^x \cos y$ and $v = e^x \sin y$, we have $u = e^x, v = 0$

Since e^x is always positive for $u > 0, v = 0$. Hence x-axis transforms into positive u-axis in the w plane.

2. When $y = \frac{\pi}{2}$, we have $u = 0$ and $v = e^x$ Hence the line $y = \frac{\pi}{2}$, transforms into the v-axis in the w-plane.

3. When $y = \pi, v = 0$ and $u = -e^x < 0$

Hence the lines $y = \pi$ transforms into negative u-axis.

4. When $y = \frac{3\pi}{2}, u = 0$ and $v = -e^x < 0$

Hence the lines $y = \frac{3\pi}{2}$ transforms into the negative v-axis, in the w-plane.

5. When $y = 2\pi, v = 0$ and $u = e^x > 0$

Hence the lines $y = 2\pi$ transforms into the positive side of the u-axis in the w-plane.

Hence a ny horizontal strip of the z-plane of height 2π will cover the entire w-plane.

The mapping $w = z + d$

The transformation $w = z + d$, where d is complex constant, represent a translation,

Let $z = x + iy$ and $u + iv = w, d = a + ib$, then transformation becomes,

$$u + iv = x + iy + a + ib$$

$$u + iv = (x + a) + i(y + b)$$

Equating real and imaginary part

We get

$$u = x + a$$

$$v = y + b$$

- The point (x, y) in the z -plane is mapped onto the point $(x + a, y + b)$ in the w -plane.
- If we impose the w -plane on the z -plane, the figure of the w -plane is shifted to constant vector.
- Also, the region in the z and w planes will have the same shape, size and orientation.
- In particular, this transformations maps circles into circles.

Exercise Questions:

1. The function $f : N^+ \rightarrow N^+$, define on the set of (+ve) integers N^+ , satisfies the following properties

$$f(n) = f(n/2), \text{ if } n \text{ is even}$$

$$f(n) = f(n/5) \text{ if } n \text{ is odd}$$



Let $R = \{i/\exists j; f(j) = i\}$ be the set of distinct values that f takes. The maximum possible size of R is

- A) 5 B) 2 C) 0 D) - 1
2. The value of the limit $\lim_{x \rightarrow 0} (\cos x)^{\cot 2x}$ is
- A) 1 B) e C) $e^{\frac{1}{2}}$ D) $e^{-\frac{1}{2}}$
3. The value of the limit $\lim_{x \rightarrow 0} \{\sin(a+x) - \sin(a-x)\}/x$ is
- A) 0 B) 1 C) $2 \cos a$ D) $2 \sin a$
4. $\lim_{x \rightarrow -1} [1 + x + x^2 + \dots + x^{10}]$ is
- A) 0 B) 1 C) - 1 D) 2
5. The principal argument of $\frac{1}{2+3i}$ is _____.
- A) $\tan^{-1}(1.5)$ B) $\tan^{-1}(0.5)$ C) $\tan^{-1}(2.5)$ D) $\tan^{-1}(3.5)$

7.32. MULTIPLE CHOICE QUESTIONS

1. If $Z_1 = x_1 + iy_1$ and $Z_2 = x_2 + iy_2 \neq 0$ then $\frac{Z_1}{Z_2} = ?$
- A) $\frac{x_1x_2 - y_1y_2}{x_2^2 - y_2^2} + i \frac{y_1x_2 + x_1y_2}{x_2^2 + y_2^2}$ B) $\frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2} + i \frac{y_1x_2 - x_1y_2}{x_2^2 + y_2^2}$
- C) $\frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2} - i \frac{y_1x_2 - x_1y_2}{x_2^2 + y_2^2}$ D) $\frac{x_1x_2 + y_1y_2}{x_2^2 - y_2^2} + i \frac{y_1x_2 - x_1y_2}{x_2^2 - y_2^2}$
2. $\left[\frac{1+i}{1-i} \right]^5 - \left[\frac{1-i}{1+i} \right] = ?$
- A) i B) -i C) 2i D) -2i
3. The absolute value of $\frac{2+i}{4i(1+i)^2}$
- A) $\sqrt{2}$ B) $\sqrt{5}$ C) $\frac{\sqrt{5}}{b}$ D) $\frac{b}{\sqrt{5}}$
4. One value of $\arg Z$ when $Z = \frac{-2}{1+i\sqrt{3}}$
- A) $\frac{2\pi}{3}$ B) $\frac{\pi}{2}$ C) $-\frac{\pi}{2}$ D) $-\frac{2\pi}{3}$
5. The values of $(-i)^{\frac{1}{3}}$
- A) $\pm(1+i)$ B) $i, \frac{\sqrt{3}-i}{2}$ C) $i, \pm \frac{\sqrt{3}-i}{2}$ D) $i, \pm \frac{\sqrt{3}+i}{2}$
6. Find the complex numbers represented by the points $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right)$
- A) i B) -i C) 1 D) $\frac{1+i}{\sqrt{2}}$
7. Find the value of $\lim_{z \rightarrow i} \frac{\bar{z} + z^2}{1 - \bar{z}}$
- A) 1 B) i C) -1 D) -i



14. Which one is wrong?

A) $e^{iz} = 1 + \frac{iz}{1!} - \frac{z^2}{2!} - \frac{iz^3}{3!} + \dots$

B) $e^{iz} = 1 - \frac{iz}{1!} + \frac{z^2}{2!} - \frac{iz^3}{3!} + \dots$

C) $\cos z = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} \dots$

D) $\sin z = z - \frac{z^3}{3!} + \frac{z^5}{5!} \dots$

15. Which one is wrong?

A) $\cos z = \frac{e^{iz} + e^{-iz}}{2}$

B) $\sin z = \frac{e^{iz} - e^{-iz}}{2i}$

C) $\cosh z = \frac{e^z + e^{-z}}{2}$

D) $\sinh z = \frac{e^z - e^{-z}}{2}$

16. The function $f(x)$ is said to be continuous at a iff

A) $\lim_{x \rightarrow a} f(x) = f(a)$

B) $\lim_{x \rightarrow a^+} f(x) = f(a)$

C) $\lim_{x \rightarrow a} f(x)^{-1} = f(a)$

D) $\lim_{x \rightarrow a} f(x) = 0$



17. The function u which satisfies Laplace equation $\Delta u = 0$ is said to be

A) Homomorphic

B) Analytic

C) Harmonic

D) Conjugate

18. If $u = x^2 - y^2$ then the analytic function $f(z) =$

A) $2xy + c$

B) $z^3 + ic$

C) $z^2 + ic$

D) $z^3 - ic$

19. If $g(w)$ and $f(z)$ are analytic function then

A) $g(z)$ is analytic

B) $g(f(z))$ is analytic

C) $f(g(z))$ is analytic

D) $g(f(w))$ is analytic

20. The function $f(z)$ and $\overline{f(z)}$ are

A) harmonic

B) conjugate

C) analytic

D) constant

21. The Bilinear Transformation which map $\text{Im}Z \geq 0$ onto $|w| \leq 1$ are of the form

A) $w = e^{i\lambda} \frac{z - z_1}{z - \bar{z}_1}$

B) $w = \frac{z - \bar{z}_1}{z - z_1}$

C) $w = e^{-i\lambda} \frac{z - z_1}{z - z_1}$

D) $w = \frac{z + z_1}{z + \bar{z}_1}$



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UNIT VIII Mechanics

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INDEX

UNIT -VIII - MECHANICS: STATICS & DYNAMICS

S.N.	TOPIC	P. N.
STATICS		
8.1.	Moment of a force	1
8.2.	General motion of a rigid body	4
8.3.	Equivalent (or equi polent)	5
8.4.	Parallel forces	6
	8.4.1. Forces along the sides of a triangle	
8.5.	Resultant of several coplanar	11
8.6.	Equation of the line of action of the resultant	13
8.7.	Equilibrium of a rigid body under three coplanar forces	16
8.8.	Reduction of coplanar forces into a force and a couple	18
8.9.	Equilibrium of a uniform homogeneous string:	21
8.10.	Centenary	23
8.11.	Suspension bridge:	25
DYNAMICS		
8.12.	Velocity and Acceleration	28
8.13.	Coplanar motion	31
8.14.	Rectilinear motion under constant force	32
8.15	Acceleration and Retardation Thrust on plane	34
8.16.	Motion along a vertical line under gravity	36
8.17.	Motion along an inclined plane	38
8.18.	Motion of connected particles	40

8.19.	Newton's laws of motion	42
8.20.	Work power energy	43
8.21.	Conservative field of force	47
8.22.	Power	49
8.23.	Rectilinear Motion under varying force simple Harmonic motion.	50
8.24.	S.H.M along a Horizontal line	54
8.25.	S.H.M along vertical line	56
8.26.	Motion under gravity in a resisting medium	59
8.27.	Resistance proportional to velocity	63
8.28.	Path of a projects	67
8.29.	Particle projected on an inclined plane	71
8.30.	Analysis of force acting on particles and rigid to bodies on static equilibrium , equivalent	73
8.31.	Rigid bodies – equivalent system of force	74
8.32.	Friction	74
8.33.	Centroid of a system of particles	75
8.34.	Moment of Inertia	76
8.35.	Elastic medium , Impact	77
8.36.	Impulsive force	77
8.37.	Impact of spheres	79
8.38.	Impact two smooth spheres	80
8.39.	Impact of two smooth spheres of two smooth sphere on a plane	82
8.40.	Oblique impact of two smooth spheres	84

8.41.	Circular motion	89
8.42.	Conical pendulum, motion of a cyclist on a circular path	89
8.43.	Circular motion in a vertical plane	93
8.44.	Relative rest in a revolving cone	96
8.45.	Simple pendulum	98
8.46.	Central orbit	100
8.47.	Conic as a Central orbit	103
8.48.	Moment of inertia	107
8.49.	Multiple Choice Questions	115

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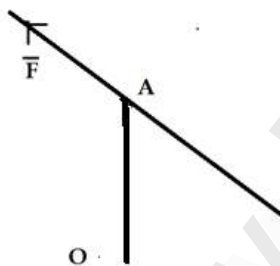
UNIT VIII : MECHANICS – STATICS & DYNAMICS

STATICS- PART - I

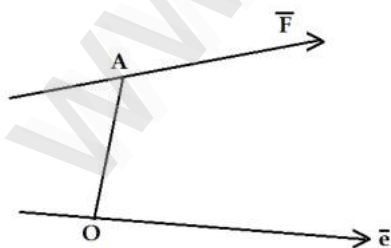
UNIT I - FORCES ON A RIGID BODY

8.1. MOMENT OF A FORCE

- Let \vec{F} be a force and A, a point on its on line of action. Let O be a point in space, then the vector
- $\vec{OA} \times \vec{F}$ is called the moment of \vec{F} about O.



Moment of a Force About a Line:

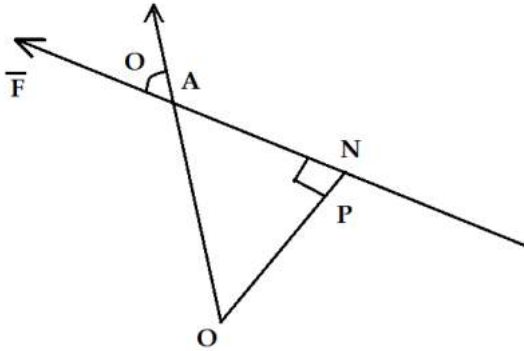


- Let \vec{F} be a force and A, a point on its line of a action. Let \vec{r} be a directed line through a point O, the direction of the line being specified by \vec{e} , then the scalar triple product

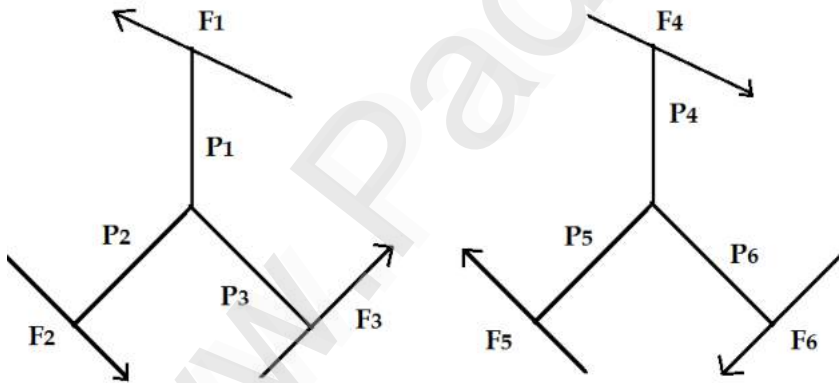
- $(\overline{OA} \times \overline{F}) \cdot \hat{e}$ is called the moment of the force \overline{F} about \hat{F} .

Scalar Moment:

- Let \overline{F} be a force in a plane. Let A be a point on its line of action and O, any point in the plane. Let ON be the perpendicular from O to the line and $ON = P$ then the moment \overline{F} about O is
- $\overline{OA} \times \overline{F} = OA \cdot F \sin \theta \hat{n} = PF \hat{n}$



- Where θ is the angle between \overline{OA} and \overline{F} , and \hat{n} is the unit vector perpendicular to \overline{OA} , \overline{F} such that $\overline{OA}, \overline{F}, \hat{n}$ form a right handed triad. Now we call p^F of the scalar moment of \overline{F} about O.



- the scalar moments of F_1, F_2, F_3 in the first figure are

$$P_1 F_1, P_2 F_2, P_3 F_3$$

- which are positive and the moments of F_4, F_5, F_6 in the second figure are

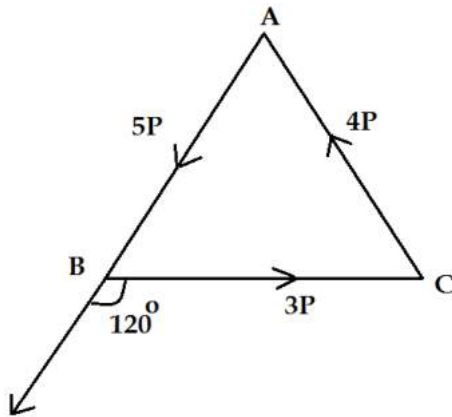
$$-P_4 F_4, -P_5 F_5, -P_6 F_6$$

- Which are negative, the first three forces are such as to cause on a rigid body a rotational motion in the anticlockwise sense and the other three to cause a rotational motion in the clockwise sense.



Example

- Forces of magnitudes $3P$, $4P$, $5P$, act along the sides BC , CA , AB of an equilateral triangle of side a . Find the moment of the resultant about A ,



- the moment of the resultant about A equals the sum of the moments of the individual forces about A . But the forces $4P$, $5P$ pass through A . So their moments about A are zero, the moment of $3P$ which passes through B is

$$\begin{aligned}\overline{AB} \times (3P\hat{BC}) &= AB \cdot 3P \sin 120^\circ \hat{n} \\ &= a \cdot 3P \cdot \frac{\sqrt{3}}{2} \hat{n}\end{aligned}$$

- So, this is the moment of the resultant about A .

Exercise - 1

- If three parallel forces are in equilibrium then each is proportional to the
 - Angle between the other two
 - n Distance between the other two
 - Cosine of the angle between the other two
 - None of these
- S is the circumcentre of a triangle ABC . Forces of magnitudes P , Q , R acting along SA , SB , SC respectively are in equilibrium. Then P , Q , R are in the ratio

(A) $\cos \frac{A}{2} : \cos \frac{B}{2} : \cos \frac{C}{2}$

(B) $a : b : c$

(C) $\sin 2A : \sin 2B : \sin 2C$

(D) $SA : SB : SC$



3. Maximum range on an inclined plane of inclination β is

- (A) $\frac{u^2}{g(1+\cos\alpha)}$ (B) $\frac{u^2}{g(1+\sin\beta)}$
 (C) $\frac{u^2}{g(1-\cos\alpha)}$ (D) $\frac{u^2}{g(1-\sin\beta)}$

8.2. GENERAL MOTION OF A RIGID BODY

- In this section we extend the Newton's laws of motion, N.1, N.2, N.3 to the motion of a rigid body.

Rigid Body:

- A system of particles such that the distance between any two of them is always constant, is called a rigid body.

Applied Forces:

- Forces applied on a body by external agencies are called applied forces on the body

Effective Forces:

- If a particle of mass m has an acceleration $\ddot{\mathbf{r}}$, then the quantity $m\ddot{\mathbf{r}}$ is called the effective force of the particle. With the nomenclature we have that the equation of motion of the particle, $m\ddot{\mathbf{r}} = \mathbf{F}$, is that the effective force on a particle = the applied force on a particle

Exercise - 2

- If a particle is projected with a velocity of 490 meters/sec at an elevation of 30° then the time of flight.
 (A) 5 seconds (B) 25 seconds (C) 50 seconds (D) 100 seconds
- A particle is thrown vertically upwards with a velocity u . The time taken by it to reach the maximum height is ____
 (A) $\frac{u^2}{g}$ (B) $\frac{2u}{g}$ (C) $\frac{u^2}{2g}$ (D) $\frac{u}{g}$
- Two forces of magnitude 7 and 8 act a point. If the magnitude of the resultant force is 13. Then angle between the two forces is ____
 (A) 30° (B) 45° (C) 60° (D) 90°

8.3. EQUIVALENT (OR EQUI POLENT)

Systems of Forces

- Two systems of forces, which produce the same motion on a given rigid body are equivalent or equipotent so, from the equations of the motion of the mass centre and motion of the body about the mass centre, we get that two systems of forces are equivalent or equipotent.
 - (i) If the vector sum of the forces of one system equals the Vector sum of forces of the other system and
 - (ii) If the Vector sum of the moments of the forces of one system, about any fixed point or other mass centre, equals the Vector sum of the moments of the forces of the other system about the same point on the mass centre
- In symbols, the system of forces \vec{F}_i acting at \vec{r}_i on a rigid body is equivalent to the system of forces \vec{F}_j acting at \vec{r}'_j on the rigid body if

$$\sum_i \vec{F}_i = \sum_j \vec{F}'_j,$$

$$\sum_i \vec{r}_i \times \vec{F}_i = \sum_j \vec{r}'_j \times \vec{F}'_j$$

Exercise - 3

1. The centre of parallel forces is _____

(A) Not a unique point	(B) Not a multi point
(C) a multi point	(D) a unique point
2. The ratio of the limiting friction to the normal reaction is called the ____

(A) coefficient of friction	(B) angle of friction
(C) cone of friction	(D) None of these
3. Two couples in the same plane whose moments are equal and of the same sign are _____

(A) not equivalent to one another	
(B) equivalent to one another	
(C) equivalent to a force	
(D) None of these	



8.4. PARALLEL FORCES

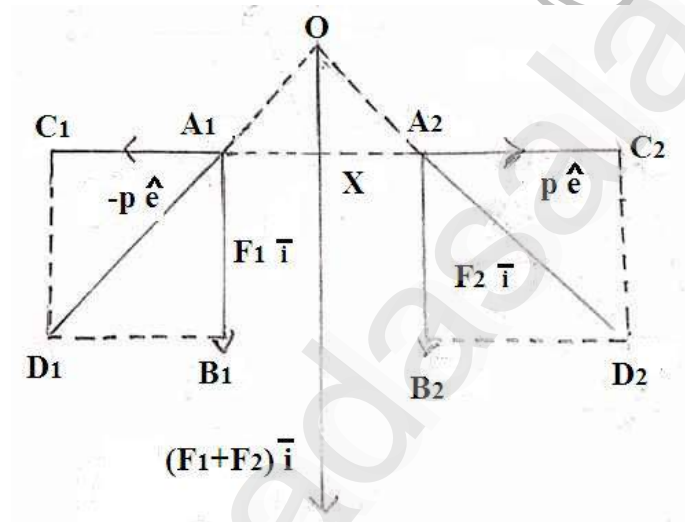
- Forces whose lines of action are parallel are called parallel forces. If their directions are in the same sense, then they are called like parallel forces otherwise they are called unlike parallel forces.

Book Work

- To find the resultant of two parallel forces acting on a rigid body

Case (i)

- Let the forces be like parallel forces, namely $F_1 \bar{i}$ and $F_2 \bar{i}$ acting at A_1 and A_2 respectively, where \bar{i} is the unit vector in the direction of the forces,



- Let \hat{e} be the unit Vector in the direction of $\overline{A_1A_2}$. Introduce a force $-p\hat{e}$ at A_1 and a force $p\hat{e}$ at A_2 . Since these two forces are equal in magnitude and opposite in direction and act along the same line, their introduction will not affect the effects of the given two forces,

$$\text{Let } \overline{A_1B_1} = F_1 \bar{i}, \overline{A_2B_2} = F_2 \bar{i}, \overline{A_1C_1}$$

$$= -p\hat{e}, \overline{A_2C_2}$$

$$= p\hat{e}$$

- Complete the parallelogram $A_1B_1C_1D_1$ and $A_2B_2D_2C_2$ then the resultant of two forces $F_1 \bar{i}$ and $-p\hat{e}$ acting at A_1 is

$$\overline{A_1D_1} = F_1 \hat{i} - p\hat{e}$$

and the resultant of the forces $F_2 \hat{i}$ and $p \hat{e}$ acting at A_2 is

$$\overline{A_2 D_2} = F_2 \hat{i} + p \hat{e}$$

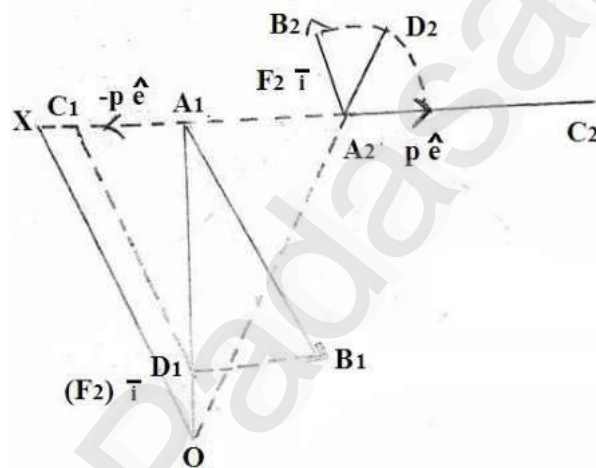
If the lines $A_1 D_1$ and $A_2 D_2$ intersect at O , then the resultant of these two resultants is

$$\begin{aligned} \overline{A_1 D_1} + \overline{A_2 D_2} &= (F_1 \hat{i} - p \hat{e}) + (F_2 \hat{i} + p \hat{e}) \\ &= (F_1 + F_2) \hat{i} \end{aligned}$$

acting at C . Note that their resultant is parallel to the original forces.

Cases (ii)

- Let the given forces be unlike parallel forces $F_1 \hat{i}$ and $F_2 (-\hat{i})$, ($F_1 > F_2$), acting at A_1 and A_2 respectively.



- If we adopt the procedure followed in case (i), we see that the steps of case (i) repeat with the only difference that instead of F_2 they have $-F_2$ their we get that the resultant of the forces $F_1 \hat{i}$ and $-F_2 \hat{i}$ acting at A_1 and A_2 is $\{F_1 + (-F_2)\} \hat{i}$ acting at the point which divides $A_1 A_2$ in the ratio $(-F_2):F_1$, that is, at the point which divides $A_1 A_2$ externally in the ratio $F_2:F_1$.

Example

- Two like parallel forces of magnitudes P, Q act on a rigid body. If Q is changed to $\frac{P^2}{Q}$, with the line of action being the same, show that line of the action of the resultant will be the same as it would be, if the forces were simply interchanged.

Solution

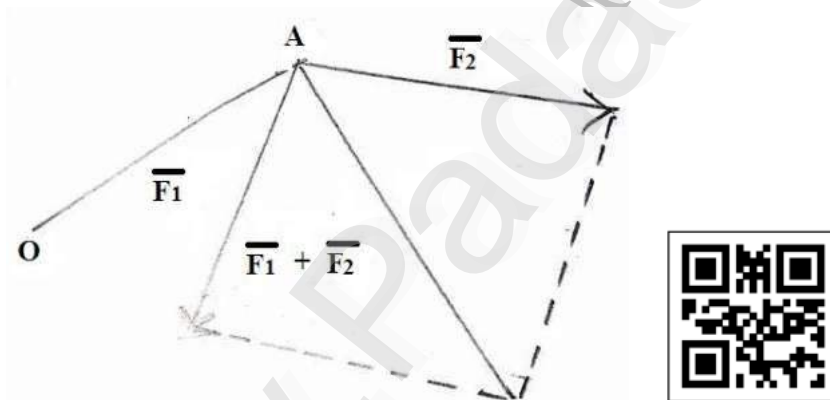
- If the forces, P and $\frac{P^2}{Q}$, act at A, B, then their resultant divides AB.
- Internally in the ratio

$$\frac{P^2}{Q} : P \text{ (or) } \frac{P}{Q} = 1 \text{ (or) } P : Q$$

- For the second case also, the ratio is the same P : Q. Further all the involved forces and the resultants are parallel to one another.

Varignon's Theorem

- The sum of the moments of two intersecting or parallel force about any point is equal to the moment of the resultant of the forces about the same point

Intersecting Forces**Case (i)**

- Let the lines of action of the forces \vec{F}_1 and \vec{F}_2 intersect at A, then the moment of \vec{F}_1 and \vec{F}_2 about any point O are

$$\vec{OA} \times \vec{F}_1, \vec{OA} \times \vec{F}_2$$

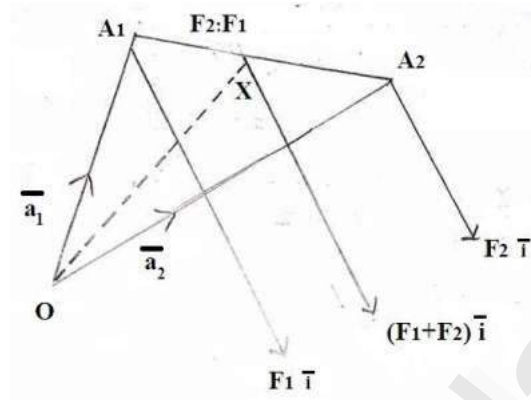
- and their sum is

$$\vec{OA} \times \vec{F}_1 + \vec{OA} \times \vec{F}_2$$

- But the resultant of \vec{F}_1 and \vec{F}_2 acting at A, so its moment about O is $\vec{OA} \times (\vec{F}_1 + \vec{F}_2)$
- Since $\vec{OA} \times \vec{F}_1 + \vec{OA} \times \vec{F}_2 = \vec{CA} \times (\vec{F}_1 + \vec{F}_2)$ the theorem follows for the intersecting forces

Case (ii)**Parallel Forces**

- Let the parallel forces be $\vec{F}_1 = F_1 \vec{i}$ and $\vec{F}_2 = F_2 \vec{i}$ acting at A_1 and A_2 . Let \vec{a}_1, \vec{a}_2 be the P.V's of A_1, A_2 with respect to O , then the moment of \vec{F}_1, \vec{F}_2 about O are



$$\vec{a}_1 \times F_1 \vec{i} + \vec{a}_2 \times F_2 \vec{i}$$

their sum

$$\vec{a}_1 \times F_1 \vec{i} + \vec{a}_2 \times F_2 \vec{i} = (F_1 \vec{a}_1 + F_2 \vec{a}_2) \times \vec{i}$$

- But the resultant of $F_1 \vec{i}$ and $F_2 \vec{i}$ is $(F_1 + F_2) \vec{i}$ acting at x , where x divides $A_1 A_2$ internally in the ratio $F_2 : F_1$ to the P.V of x is

$$\frac{F_1 \vec{a}_1 + F_2 \vec{a}_2}{F_1 + F_2} \quad \text{----- (1)}$$

- So, the moment of the resultant about O is

$$\vec{OX} \times (F_1 + F_2) \vec{i} = \frac{F_1 \vec{a}_1 + F_2 \vec{a}_2}{F_1 + F_2} \times (F_1 + F_2) \vec{i}$$

$$(F_1 \vec{a}_1 + F_2 \vec{a}_2) \times \vec{i} \quad \text{----- (2)}$$

- From (1) and (2) we get the theorem for parallel forces.

Example

- Three like parallel forces P, Q, R act at the vertices of a triangle ABC , show that their resultant passes through

(i) The centroid if $P = Q = R$,

(ii) the in centre if $\frac{P}{a} = \frac{Q}{b} = \frac{R}{c}$

Let $\bar{a}, \bar{b}, \bar{c}$ be the P. V's of A, B, C, then the resultant passes through the point whose P.V is

$$\frac{P\bar{a} + Q\bar{b} + R\bar{c}}{P + Q + R}$$

(i) If $P=Q=R$, then

$$\frac{P\bar{a} + Q\bar{b} + R\bar{c}}{P + Q + R} = \frac{\bar{a} + \bar{b} + \bar{c}}{3}$$

Which is the P.V of the centroid

(ii) If $\frac{P}{a} = \frac{Q}{b} = \frac{R}{c} = k$, then

$$\frac{P\bar{a} + Q\bar{b} + R\bar{c}}{P + Q + R} = \frac{R(\bar{a}a + \bar{b}b + \bar{c}c)}{k(a + b + c)}$$

$$\frac{\bar{a}a + \bar{b}b + \bar{c}c}{a + b + c}$$

Which is the P.V of the incentre

Exercise - 4

- The centre of gravity of a triangle is ____
 (A) orthocentre (B) incentre
 (C) centroid (D) circumcentre
- On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4m. When its is 6m away from the point of projection. Finally it reaches the ground 12m away from the starting point. The angle of projection at the origin is ____
 (A) $\tan^{-1}\left(\frac{3}{4}\right)$ (B) $\tan^{-1}\left(\frac{1}{3}\right)$
 (C) $\tan^{-1}\left(\frac{1}{4}\right)$ (D) $\tan^{-1}\left(\frac{4}{3}\right)$
- A particle is tossed up vertically with velocity of 19.6 m/sec. The time taken to reach the maximum height is ____
 (A) 4 secs (B) 1 sec
 (C) 2 secs (D) 2/3 sec



8.4.1. FORCES ALONG THE SIDES OF A TRIANGLE

Example

- Three forces P, Q, R act along the sides BC, CA, AB of a triangle ABC. If their resultant passes through the incentre and centroid, show that

$$\frac{P}{a(b-c)} = \frac{Q}{b(c-a)} = \frac{R}{c(a-b)}$$

Since the resultant passes through the incentre and centroid. We have respectively

$$P+Q+R=0 \text{-----(1)}$$

$$\frac{P}{a} + \frac{Q}{b} + \frac{R}{c} = 0 \text{-----(2)}$$

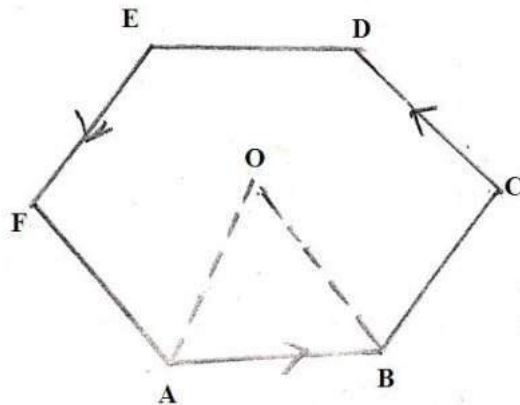
Solving (1) and (2)

$$\begin{vmatrix} P & Q & R \\ 1 & 1 & 1 \\ \frac{1}{b} & \frac{1}{c} & \frac{1}{a} \end{vmatrix} = \begin{vmatrix} Q & R \\ 1 & 1 \\ \frac{1}{c} & \frac{1}{a} \end{vmatrix} = \begin{vmatrix} R \\ 1 & 1 \\ \frac{1}{a} & \frac{1}{b} \end{vmatrix}$$

$$\frac{P}{c} - \frac{1}{b} = \frac{Q}{a} - \frac{1}{c} = \frac{R}{b} - \frac{1}{a}$$

8.5. RESULTANT OF SEVERAL COPLANAR FORCES

- Show that the forces $\overline{AB}, \overline{CD}, \overline{EF}$ acting respectively at A, C, E of a regular hexagon ABCDEF, are equivalent to a couple of moment equal to the area of the hexagon.



- Let O be the centre of the hexagon
- Now the sum of the forces is

$$\vec{AB} + \vec{CD} + \vec{EF}$$

$$\vec{AB} + \vec{BD} + \vec{OA}$$

- It is evident that is zero, so either the system is in equilibrium or it reduces to a couple.
- Multiplying the denominators by abc, we get the result.
- But the moment of \vec{AB} about O is

$$\vec{OA} \times \vec{AB} = OA \cdot AB \sin \angle OAB \vec{k}$$

$$= 2\Delta \vec{k}$$

- Where Δ is the area of $\triangle AOB$. By symmetry the sum of the moments of all the forces is $3(2\Delta)\vec{k}$ (or) $6O\Delta\vec{k}$, so the system reduces to a couple of moment 6Δ . But the area of the hexagon also is 6Δ .

Exercise - 5

- The resolved part of a force in its own direction is the force itself ____
 - when $\theta = \pi$
 - when $\theta = 0$
 - when $\theta = \frac{\pi}{2}$
 - when $\theta = \frac{3\pi}{2}$
- O is the orthocentre and S is the circumcentre of a triangle AB. The resultant of forces OA, OB, OC is
 - AB
 - BC
 - OS
 - 2OS
- Three like parallel forces P, Q, R act at the corners of a triangle ABC. Then their centre is the orthocentre of the triangle if

$$(A) \frac{P}{OA} = \frac{Q}{OB} = \frac{R}{OC}$$

$$(B) \frac{P}{\tan A} = \frac{Q}{\tan B} = \frac{R}{\tan C}$$

$$(C) P \tan A = Q \tan B = R \tan C$$

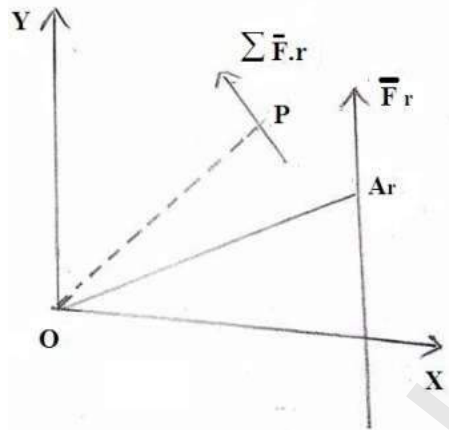
$$(D) \frac{P}{\sin(Q,R)} = \frac{Q}{\sin(P,R)} = \frac{R}{\sin(P,Q)}$$



8.6. EQUATION OF THE LINE OF ACTION OF THE RESULTANT

Book Work

- When a system of Coplanar forces $\vec{F}_1, \vec{F}_2, \dots, \vec{F}_n$, acting at A_1, A_2, \dots, A_n , reduce to a single force, to find the equation of line of action



- Choose any two perpendicular lines Ox, Oy in the plane of the forces as the x, y axes and let \vec{i}, \vec{j}
 - Let the unit Vectors in their direction. Let $p(x, y)$ be any point on the line of action of the resultant force $\sum \vec{F}_r$ of the system. Then any relation is x, y is the equation of the line .
- Now

$$\vec{OP} = x\vec{i} + y\vec{j}$$

- Let P_r, Q_r be the components of \vec{F}_r in the \vec{i}, \vec{j} directions, then

$$\vec{F}_r = P_r\vec{i} + Q_r\vec{j}$$

- since the sum of the moments of the forces about any point, say O , equals the moment of their resultant about O ,

$$\sum (\vec{OA}_r \times \vec{F}_r) = \vec{OP} \times (\sum \vec{F}_r)$$

(or)

$$\vec{OP} \times (\sum \vec{F}_r) - \sum (\vec{OA}_r \times \vec{F}_r) = 0$$

i.e.,

$$(x\vec{i} + y\vec{j}) \times \sum (P_r\vec{i} + Q_r\vec{j}) - \sum (\vec{OA}_r \times \vec{F}_r) = \vec{0}$$

i.e.,

$$(\bar{x}\bar{i} + \bar{y}\bar{j}) \times \{(\sum P_r)\bar{i} + (\sum Q_r)\bar{j}\} - \sum(\bar{O}A_r \times \bar{F}_r) = \bar{O}$$

i.e.,

$$x(\sum Q_r)\bar{k} - y(\sum P_r)\bar{k} - (\sum P_r F_r)\bar{k} = \bar{O}$$

- The sum of the moment about with the usual meaning for \bar{k} and P_r being the perpendicular distance of O from E_r such that its value is positive or negative according as the sense of rotation of \bar{F}_r about O is anticlockwise or not thus the equation of the line of action of the resultant is

$$(\sum Q_r)x - (\sum P_r)y - \sum P_r F_r = 0 \text{-----(1)}$$

(or)

$$(\sum Q_r)x - (\sum P_r)y - \sum G_r = 0$$

Where $G_r = P_r F_r$

this equation can be put in the elegant form

$$Y_x - X_y - G = 0 \text{-----(2)}$$

Where

$X = \sum P_r$ = sum of the components of the forces in the x direction

$Y = \sum Q_r$ = sum of the component of the forces in the y direction

$G = \sum G_r = \sum P_r F_r$ = sum of the scalar moments of the forces about the origin.

Now we have that the resultant force is

$$\bar{F}_1 + \bar{F}_2 + \dots + \bar{F}_n$$

(or)

$$X\bar{i} + Y\bar{j}$$

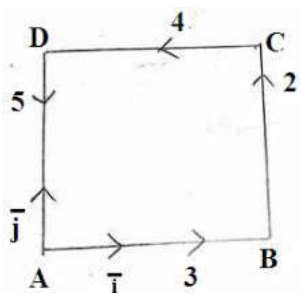
Whose magnitude is $\sqrt{X^2 + Y^2}$ and the line of action is

$$y_x - X_y = G$$

the slope of the line is $\frac{Y}{X}$.

Examples

- Forces 3,2,4,5 Kg. wt. act along the sides AB, BC, CD, CA of a square. Find their resultant and its line of action.



Let \bar{i}, \bar{j} be the unit Vectors parallel to $\overline{AB}, \overline{AD}$ and $AB = a\bar{j}$.

Let AB, AD be the x, y axes, the vector sum of the forces is

$$(3\bar{i}) + (2\bar{j}) + (-4\bar{i}) + (-5\bar{j}) = -\bar{i} - 3\bar{j}$$

Let X, Y, be the sum of \bar{i}, \bar{j}

Components of the forces and G, the sum of the moments about the origin A, then

$$X = -1, Y = -3$$

the magnitude of the resultant force is

$$\sqrt{X^2 + Y^2} = \sqrt{(-1)^2 + (-3)^2} = \sqrt{10}$$

$$G = 0 \times 3 + a(2) + a(4) + 0 \times 5 = 6a$$

the equation of line of action of the resultant forces is

$$\begin{vmatrix} X & Y \\ x & y \end{vmatrix} + G = 0$$

(or)

$$\begin{vmatrix} -1 & -3 \\ x & y \end{vmatrix} + 6a = 0$$

i.e.,

$$-y + 3x + 6a = 0$$

Exercise – 6

- A solid sphere of mass m rolls down a plane inclined to the horizon at an angle α . The acceleration is

(A) $\frac{g \sin \alpha}{7}$	(B) $\frac{3g \sin \alpha}{7}$
(C) $\frac{4g \sin \alpha}{7}$	(D) $\frac{5g \sin \alpha}{7}$
- A 100 gm cricket ball moving horizontally at 24 m/s was hit straight back with a speed of 15 m/s. If the contact lasted $\frac{1}{20}$ second. The average force exerted by the bat is ____

(A) 78000 Dynes	(B) 8000 Dynes
(c) 90000 Dynes	(D) 1500 Dynes
- Let u and v be two velocities at the point A then their resultant direction is ____

(A) $\tan \theta = \frac{v \cos \alpha}{u+v \sin \alpha}$	(B) $\tan \theta = \frac{u \cos \alpha}{v+u \sin \alpha}$
(C) $\tan \theta = \frac{v \sin \alpha}{u+v \cos \alpha}$	(D) $\tan \theta = \frac{v \sin \alpha}{v+u \sin \alpha}$

8.7. EQUILIBRIUM OF A RIGID BODY UNDER THREE COPLANAR FORCES**Book Work**

- If three coplanar forces keep a rigid body in equilibrium, then either they all are parallel to one another or they are concurrent.
- Let the forces be $\vec{F}_1, \vec{F}_2, \vec{F}_3$ considering only \vec{F}_1 and \vec{F}_2 , we get the following two cases
 - \vec{F}_1 and \vec{F}_2 are parallel
 - \vec{F}_1 and \vec{F}_2 are not parallel

Case (i)

- Suppose $\vec{F}_1 = F_1 \vec{i}$ and $\vec{F}_2 = F_2 \vec{i}$ act at A_1 and A_2, \dots then their resultant is $(F_1 + F_2) \vec{i}$. Consequently their resultant $(F_1 + F_2) \vec{i}$ and \vec{F}_3 keep the body in equilibrium, this implies not only that these two forces act along the same line but also that $\vec{F}_3 = -(F_1 + F_2) \vec{i}$ so \vec{F}_3 is parallel to \vec{F}_1 and \vec{F}_2 that is the given three forces are parallel to one another.



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UG TRB MATHEMATICS 2023-2024

UNIT IX

Operations Research

Your Success is Our Goal....

UNIT – IX - OPERATIONS RESEARCH		
INDEX		
S.NO	CHAPTER NAME	P.NO
1.1	INTRODUCTION	1
1.2	SCOPE OR USES OR APPLICATIONS OF O.R.	1
1.3	ROLE OF OPERATINS RESEARCH IN BUSINESS AND MANAGEMENT	2
1.4	CLASSIFICATION OF MODELS	3
1.5	SOME CHARACTERISTICS OF A GOOD MODEL	5
1.6	GENERAL METHODS FOR SOLVING O.R. MODELS	5
1.7	MAIN PHASES OF O.R.	5
1.8	LIMITATION	6
2.	LINEAR PROGRAMMING FORMULATION	8
2.1	INTRODUCTION	8
2.2	MATHEMATICAL FORMULATION OF L.P.P	8
2.3	PROCEDURE FOR FORMING A LPP MODEL	8
2.4	BASIC ASSUMPTIONS	12
2.5	GRAPHICAL METHOD OF THE SOLUTION OF A L.P.P	13
2.6	SOME MORE CASES	16
2.7	ADVANTAGE OF LINEAR PROGRAMMING	20
2.8	LIMITATIONS OF LINEAR PROGRAMMING	21
2.9	GENERAL LINEAR PROGRAMMING PROBLEMS – SIMPLEX METHOD	21
	2.9.1 CANONICAL FORM OF LPP	23

	2.9.2	CHARACTERISTICS OF THE CANONICAL FORM	23
	2.9.3	THE STANDARD FORM	24
	2.9.4	CHARACTERISTICS OF THE STANDARD FORM	24
2.10	THE SIMPLEX METHOD		24
	2.10.1	THE SIMPLEX ALGORITHM	25
2.11	ARTIFICIAL VARIABLES TECHNIQUES		35
	2.11.1	THE BIG M – METHOD	35
	2.11.2	THE TWO-PHASE METHOD:	42
2.12	DISADVANTAGE OF BIG-M METHOD OVER TWO-PHASE METHOD:		50
2.13	REVISED SIMPLEX METHOD		50
	2.13.1	REVISED SIMPLEX ALGORITHM	51
2.14	DUALITY IN LPP		59
	2.14.1	FORMULATION OF DUAL PROBLEMS	59
	2.14.2	TO CONSTRUCT THE DUAL PROBLEM, WE ADOPT THE FOLLOWING GUIDELIN	60
	2.14.3	UNSYMMETRIC FORM	62
2.15	DUAL SIMPLEX METHOD		62
	2.15.1	WORKING PROCEDURE FOR DUAL SIMPLEX METHOD	62
3.	SENSITIVITY ANALYSIS:		72
3.1	I. VARIATIONS AFFECTING FEASIBILITY		72
	3.1.1	(1) VARIATIONS IN THE RIGHT SIDE OF CONSTRAINTS	72
	3.1.2	(2) ADDITION OF NEW CONSTRAINT	73
3.2	II. CHANGES AFFECTING OPTIMALITY		73

3.3	III. VARIATION IN THE CO-EFFICIENTS a_{ij} OF THE CONSTRAINTS (OR) VARIATION IN THE COMPONENTS a_{ij} OF THE CO-EFFICIENT MATRIX A:	74
3.4	IV. ADDITION OF A NEW ACTIVITY (OR VARIABLE)	74
3.5	(V) DELETION OF A VARIABLE	77
3.6	(VI) DELETION OF CONSTRAINT	78
4.	TRANSPORTATION PROBLEM	78
4.1	INTRODUCTION	78
4.2	MATHEMATICAL FORMULATION OF A TRANSPORTATION PROBLEM:	79
4.3	STANDARD TRANSPORTATION TABLE	80
4.4	BALANCED AND UNBALANCED TRANSPORTATION PROBLE	81
4.5	METHODS FOR FINDING INITIAL BASIC FEASIBLE SOLUTION	81
4.5.1	NORTH WEST CORNER RULE:	81
4.5.2	LEAST COST METHOD (OR) MATRIX MINIMA METHOD (OR) LOWEST COST ENTRY METHOD:	82
4.5.3	VOGEL'S APPROXIMATION METHOD (VAM) OR UNIT COST PENALTY METHOD:	83
4.6	TRANSPORTATION ALGORITHM (OR) MODI METHOD (MODIFIED DISTRIBUTION METHOD) TEST FOR OPTIMAL SOLUTION)	85
4.6.1	TO FIND THE OPTIMAL SOLUTION	86
4.7	DEGENERACY IN TRANSPORTATION PROBLEMS	89
5.	ASSIGNMENT PROBLEM	91
5.1	MATHEMATICAL FORMULATION OF AN ASSIGNMENT PROBLEM	92

5.2	DIFFERENCE BETWEEN THE TRANSPORTATION PROBLEM AND THE ASSIGNMENT PROBLEM		93
5.3	ASSIGNMENT ALGORITHM (OR) HUNGARIAN METHOD		93
5.4	MAXIMIZATION CASE IN ASSIGNMENT PROBLEMS		96
6.	QUEUING MODEL		98
6.1	QUEUING SYSTEM		98
6.2	TRANSIENT AND STEADY STATES:		100
6.3	KENDAL'S NOTATION FOR REPRESENTING QUEUING MODELS:		100
6.4	DISTRIBUTION OF ARRIVALS "THE PASSION PROCESS" DISTRIBUTION THEOREM (PURE BIRTH PROCESS)		100
6.4.1	MODEL I: (M/M/I) (∞ /FCFS) –BIRTH AND DEATH MODEL		102
6.4.2	MEASURE OF MODEL I		104
6.4.3	MODEL II {MULTI – SERVICE MODEL} (M/M/S): (∞ /FCFS)		107
6.4.4	MEASURES OF MODEL II:		110
6.4.5	MODEL III: (M/M/I); (N/FCFS)		112
6.4.6	MODEL IV: (M/M/S): FCFS/ N)		113
7.	SCHEDULING BY PERT AND CPM		115
7.1	BASIC TERMINOLOGIES:		116
7.2	RULES FOR CONSTRUCTING A PROJECT NETWORK		117
7.3	NODES MAY BE NUMBERED USING THE RULE GIVEN BELOW		117
7.4	NETWORK COMPUTATIONS		118
7.4.1	TO COMPUTE THE LATEST FINISH AND LATEST START OF EACH		119

7.5	FLOATS	119
7.6	USES OF FLOATS	122
7.7	PROGRAMME EVALUATION REVIEW TECHNIQUE: (PERT)	122
7.7.1	TWO MAIN ASSUMPTIONS MADE IN PERT IN CALCULATIONS ARE:	122
7.7.2	PERT PROCEDURE	122
7.8	BASIC DIFFERENCES BETWEEN PERT AND CPM	123
7.9	CPM	123
8.	INVENTORY MODEL:	126
8.1	TYPES OF INVENTORY	127
8.2	REASONS FOR MAINTAINING INVENTORY:	127
8.3	COSTS INVOLVED IN INVENTORY PROBLEMS:	127
8.4	VARIABLES IN INVENTORY PROBLEM:	128
8.5	ECONOMIC ORDER QUANTITY (E.O.Q) OR ECONOMIC LOT SIZE FORMULA:	129
8.6	DETERMINISTIC INVENTORY MODELS:	129
8.6.1	MODEL I: PURCHASING MODEL WITH NO SHORTAGES.	129
8.6.2	MODEL II: MANUFACTURING MODEL WITH NO SHORTAGES.	131
8.6.3	MODEL IV:	135
8.7	INVENTORY MODELS WITH PRICE BREAKS	136
8.7.1.	MODEL VII	136
8.7.2	PURCHASE INVENTORY MODEL WITH SINGLE PRICE – BREAK	137
9.	GAME THEORY	139

9.1	A COMPETITIVE SITUATION IS CALLED A GAME IF IT HAS THE FOLLOWING PROPERTIES.	139
9.2	TWO PERSON ZERO – SUM GAMES	139
9.3	PURE STRATEGIES:	140
9.4	MAIN CHARACTERISTICS OF GAME THEORY:	140
9.5	SADDLE POINT AND VALUE OF THE GAME:	141
9.6	MIXED STRATEGY:	141
9.7	GAMES WITHOUT SADDLE POINTS, MIXED STRATEGIES SOLUTION OF 2×2 GAMES WITHOUT SADDLE POINT	142
9.7.1	LET THE PAY OFF MATRIX BE AS FOLLOWS	142
9.7.2	MODEL 2	144
9.8	DOMINANCE PROPERTY	146
9.8.1.	GENERAL RULE:	146
9.9.	GRAPHICAL METHOD FOR $2 \times N$ OR $M \times 2$ GAMES	148
10.	INTEGER PROGRAMMING	151
10.1	CUTTING METHODS	152
10.1.1	GOMARY'S FRACTIONAL CUT ALGORITHM (OR) CUTTING PLANE METHOD FOR PURE (ALL) I.P.P.	152
10.2	IMPORTANCE OF INTEGER PROGRAMMING	153
10.3	APPLICATIONS OF INTEGER PROGRAMMING	154
10.4	PITFALLS IN ROUNDING THE OPTIMUM SOLUTION OF AN I.P.P.	154
10.5	METHODS OF INTEGER PROGRAMMING	154
10.6	GOMARY'S MIXED INTEGER METHOD	162
11	BRANCH AND BOUND METHOD	166
12	ONE MARK SET - I	177

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UG TRB – MATHS – 2022-23

UNIT - IX

OPERATIONS RESEARCH

1.1. Introduction:

- ❖ Operations Research is the study of optimisation techniques. It is applied decision theory. The existence of optimisation techniques can be traced at least to the days of Newton and Lagrange. Rapid development and invention of new techniques occurred since the World War II essentially, because of the necessary to win the war with the limited resources available.
- ❖ Different teams had to do research on military operations in order to invent techniques to manage with available resources so as to obtain the desired objective. Hence the nomenclature Operations Research or Resource Management Techniques.

1.2. Scope or Uses or Applications of O.R.:

O.R. is useful for solving.

- Resource allocation problems.
- Inventory control problems.
- Maintenance and Replacement problems.
- Sequencing and scheduling problems.
- Assignment of jobs to applicants to maximise total profit or minimize total cost.
- Transportation problems.
- Shortest route problems like travelling sales person problems.
- Marketing Management problems.



- Finance Management problems.
- Production, planning and control problems.
- Design problems
- Queuing problems, etc. to mention a few.

1.3. Role of Operations Research In Business And Management:

1. Marketing management Operations research techniques have definitely a role to play in

- (a) Product selection
- (b) Competitive strategies
- (c) Advertising strategy etc

2. Production Management:

- (a) Production scheduling
- (b) Project scheduling
- (c) Allocation of resources
- (d) Location of factories and their sizes
- (e) Equipment replacement and maintenance
- (f) Inventory policy etc.

3. Finance Management

- (a) Cash flow analysis
- (b) Capital requirement
- (c) Credit policies
- (d) Credit risks etc.

4. Personal Management

- (a) Recruitment policies and
- (b) Assignment of jobs are some of the areas of personnel management where O.R. techniques are useful.



5. Purchasing and procurement:

- (a) Rules for purchasing
- (b) Determining the quality
- (c) Determining the time of purchaser are some of the areas where O.R. techniques can be applied.

6. Distribution

- (a) Location of warehouses
- (b) Size of the ware houses
- (c) Rental outlets
- (d) Transportation strategies

1.4. Classification of Models:

- ❖ The first thing one has to do to use O.R. techniques after formulating a practical problem is to construct a suitable model to represent practical problem. A model is a reasonably simplified representation of a real-world situation. It is an abstraction of reality. The models can broadly be classified as.

Iconic Model

- ❖ This is physical, or pictorial representation of various aspects of a system.

Example:

- ❖ Toy, Miniature model of a building, scaled up model of a cell in biology etc.

Analogue or schematic model:

- ❖ This uses one set of properties to represent another set of properties which a system under study has

Example:

- ❖ A network of water pipes to represent the flow of current in an electrical network or graphs organisational charts etc.

Mathematical model symbolic Model:

- ❖ This uses a set of mathematical symbols (letters, numbers, etc) to represent the decision variables of a system under consideration. These variables related by mathematical equations or inequalities which describes the properties of the system.

Example:

- ❖ A linear programming model, A system of equations representing an electrical network or differential equations representing dynamic systems etc.

Static model:

- ❖ This is a model which does not take time into account. It assumes that the values of the variables do not change with time during a certain period of time horizon.

Example:

- ❖ A linear programming problem, an assignment problem, transportation problem etc

Dynamic Model:

- ❖ This is a model which considers time as one of the important variables.

Example:

- ❖ A dynamic programming problem, A replacement problem.

Deterministic Model:

- ❖ This is a model which does not take uncertainty into account.

Example:

- ❖ A linear programming problem, an assignment problem etc.

Stochastic Model:

- ❖ This is a model which considers uncertainty as an important aspect of the problem.

Example:

- ❖ Any stochastic programming problem, stochastic inventory models etc.

Descriptive model:

- ❖ This is one which just describes a situation or system.

Example

- ❖ An opinion poll, any survey

Predictive Model:

- ❖ This is one which predicts something based on some data. Predicting election results before actually the counting is completed.

Prescriptive model:

- ❖ This is one which prescribes or suggests a course of action for a problem.

Example:

- ❖ Any programming (linear, nonlinear, dynamic, geometric etc.) problem.

Analytic model:

- ❖ This is a model in which exact solution is obtained by mathematical methods in closed form.

Simulation model:

- ❖ This is a representation of reality through the use of a model or device which will react in the same manner as reality under a given set of conditions.
- ❖ Once a simulation model is designed, it takes only a little time, in general, to run a simulation on a computer.
- ❖ It is usually less mathematical and less time consuming and generally least expensive as well, in many situations.

Example:

- ❖ Queuing problems, Inventory problems

1.5. Some Characteristics of A Good Model:

- ❖ It should be simple
- ❖ Assumptions should be as small as possible
- ❖ Number of variables should be minimum
- ❖ The models should be open to parametric treatment
- ❖ It is easy and economical to construct.

**1.6. General methods for Solving O.R. Models:****(1) Analytic Procedure:**

Solving models by classical mathematical techniques like differential calculus, finite differences etc. to obtain analytic solutions.

(2) Iterative Procedure:

Starts with a trial solution and a set of rules for improving it by repeating the procedure until further improvement is not possible.

(3) Monte-Carlo Technique:

Taking sample observations, computing probability distributions for the variable using random numbers and constructing some functions to determine values of the decision variables.

1.7. Main Phases of O.R.:**(i) Formulation of the Problems:**

- ❖ Identifying the objective, the decision variables involved and the constraints that arise involving the decision variables.

(ii) Construction of a Mathematical Model:

- ❖ Expressing the measure of effectiveness which may be total profit, total cost, utility etc. to be optimised by a mathematical function called objective function
- ❖ Representing the constraints like budget constraints, raw materials, constraints, resource constraints, quality constraints etc, by means of mathematical equations or inequalities.

(iii) Solving the Model Constructed:

- ❖ Determining the solution by analytic or iterative or Monte-Carlo method depending upon the structure of the mathematical model.

(iv) Controlling and Updating:

- ❖ A solution which is optimum today may not be so tomorrow. The values of the variables may change, new variables may emerge. The structural relationship between the variables may also undergo a change. All these are determined in updating.
- ❖ Controls must be established to indicate the limits within which the model and its solution can be considered as reliable. This is called controlling.

(v) Testing the Model and its Solution (i.e.,) Validating the Model

- ❖ Checking as far as possible either from the past available data or by expertise and experience whether the model gives a solution which can be used in practice.

(vi) Implementation

- ❖ Implement using the solution to achieve the desired goal.

1.8. Limitation:

- ❖ Mathematical models which are the essence of OR do not take into account qualitative or emotional or some human factors which are quite real and influence the decision making.
- ❖ All such influencing factors find no place in O.R. This is the main limitation of O.R.
- ❖ Hence O.R is only an aid in decision making.

EXERCISES:

1. Operation research is the _____ of providing executive with analytical and objective basic for decision

(A) scientific method	(B) economic method
(C) both a and b	(D) none of these



2. The objective of _____ is to identifies the significant factors and interrelationships.
(A) OR (B) models (C) both a and b (D) none of these
3. _____ model is to describe and predict the facts and relationships among the various activities of the problem.
(A) descriptive (B) predictive (C) optimization (D) Iconic
4. _____ models are used in predictive analysis is involving a variety of statistical techniques used to analyze the current and historical facts to make predictions about future events.
(A) optimization (B) descriptive (C) Analogue (D) predictive
5. _____ are prescriptive in nature and develop objective decisions rules for optimum solution.
(A) descriptive (B) predictive (C) optimization (D) Analogue
6. One set of properties to represent another set of properties which a system under study, then the model is _____
(A) Iconic model (B) Analogue model (C) static model (D) dynamic model
7. _____ is a model which does not take time into account.
(A) Iconic model (B) symbolic model (C) dynamic model (D) static model
8. _____ is a model which considers time as one of the important variables.
(A) Iconic model (B) mathematical model
(C) dynamic model (D) static model
9. _____ technique is to taking samples observations, computing probability distributions for the variable using random numbers and constructing some functions to determine values of the variables.
(A) Monte- carlo (B) analytic (C) Iterative (D) none of these
10. If solving models by classical mathematical techniques like differential calculus, finite difference etc., to obtain analytic solution is known as _____.
(A) Monte- carlo technique (B) analytic procedure
(C) Iterative procedure (D) none of these
11. If starts with a trial solution and a set of rules for improving it by repeating the procedure until further improvement is not possible is _____.
(A) Monte- carlo technique (B) analytic procedure
(C) Iterative procedure (D) none of these



2. LINEAR PROGRAMMING FORMULATION

2.1. Introduction:

- ❖ Linear Programming problems deal with determining optimal allocations of limited resources to meet given objectives.
- ❖ The objective is usually maximizing profit. Minimizing total cost, maximizing utility etc.
- ❖ Linear programming problem deals with the optimization (maximization or minimization) of a function of decision variables known as objective function.
- ❖ Subject to a set of simultaneous linear equations (or inequalities) known as constraints.
- ❖ The term linear means that all the variables occurring in the objective function and the constraints are of the first degree in the problems under consideration and the term programming means the process of determining a particular course of action.
- ❖ Linear programming techniques are used in many industrial and economic problems.

2.2. Mathematical Formulation of L.P.P:

If x_j ($j=1,2,\dots,n$) are the n decision variables of the problem and if the system is subject to m constraints, the general mathematical model can be written in the form:

$$\text{Optimize } Z = f(x_1, x_2, \dots, x_n)$$

$$\text{Subject to } g_i(x_1, x_2, \dots, x_n) \leq, =, \geq b_i \quad (i=1,2,\dots,m) \text{ and } x_1, x_2, \dots, x_n \geq 0$$

2.3. Procedure for Forming a LPP Model:

Step 1: Identify the unknown decision variables to be determined and assign symbols to them.

Step 2: Identify all the restrictions or constraints in the problem and express them as linear or inequalities of decision variables.

Step 3: Identify the objective or aim and represent it also as a linear function of decision variables.

Step 4: Express the complete formulation of LPP as a general mathematical model.

Problem 1:

- ❖ A firm manufactures two types of products A and B and sells them at a profit of Rs. 2 on type A and Rs. 3 on type B. Each product is processed on two machines M_1 and M_2 . Type A requires 1 minute to processing time on M_1 and two minutes on M_2 . Type B requires 1 minute on M_1 and 1 minute on M_2 . Machine M_1 is available for not more than 6 hours 40 minutes while machine M_2 is available for 10 hours during any working day. Formulate the problem as a LPP so as to maximize the profit.

Solution:

Formulation of LPP is

$$\text{Maximize } Z = 2x_1 + 3x_2$$

Subject to the constraints

$$x_1 + x_2 \leq 400$$

$$2x_1 + x_2 \leq 600$$

$$\text{and } x_1, x_2 \geq 0$$

**Problem 2:**

- ❖ A company makes two types of leather products A and B. Product A is of high quality and product B is of lower quality. The respective profits are Rs. 4 and Rs. 3 per product. Each product A requires twice as much time as product B and if all products were of type B, the company could make 1000 per day. The supply of leather is sufficient for only 800 products per day (Both A and B combined), Product A requires a special spare part and only 400 per day are available. There are only 700 special spare parts a day available for product B. Formulate this as a LPP.

Solution:

$$\text{Maximize } Z = 4x_1 + 3x_2$$

Subject to,

$$2x_1 + x_2 \leq 1000$$

$$x_1 + x_2 \leq 800$$

$$x_1 \leq 400$$

$$x_2 \leq 700$$

$$\text{and } x_1, x_2 \geq 0$$

Problem 3:

- ❖ A firm engaged in producing two models A and B performs three operations – painting, Assembly and testing. The relevant data are as follows:

Model	Units Sale Price	Hours required for each unit		
		Assembly	Painting	Testing
A	Rs. 50	1.0	0.2	0.0
B	Rs. 80	1.5	0.2	0.1

- ❖ Total number of hours available are: Assembly 600, painting 100, testing 30. Determine weekly production schedule to maximize the profit.

Solution:

$$\text{Maximize } Z = 50x_1 + 80x_2$$

Subject to,

$$x_1 + 1.5x_2 \leq 600$$

$$0.2x_1 + 0.2x_2 \leq 100$$

$$0.1x_2 \leq 30$$

$$\text{and } x_1, x_2 \geq 0$$

**Problem 4:**

- ❖ A person wants to decide the constituents of a diet which will fulfil his daily requirements of proteins, fats and carbohydrates at the minimum cost. The choice is to be made from four different types of foods. The yields per unit of these foods are given in the following table.

Food type	Yield/unit			Cost/unit (Rs.)
	Proteins	Fats	Carbohydrates	
1	3	2	6	45
2	4	2	4	40
3	8	7	7	85

4	6	5	4	65
Maximum Requirement	800	200	700	

- ❖ Formulate the L.P model for the problem

Solution:

$$\text{Minimize } Z = 45x_1 + 40x_2 + 85x_3 + 65x_4$$

Subject to,

$$3x_1 + 4x_2 + 8x_3 + 6x_4 \geq 800$$

$$2x_1 + 2x_2 + 7x_3 + 5x_4 \geq 200$$

$$6x_1 + 4x_2 + 7x_3 + 4x_4 \geq 700$$

$$\text{and } x_1, x_2, x_3, x_4 \geq 0$$

Problem 5:

- ❖ A television company operates two assembly sections, section A and section B. Each section is used to assemble the components of three types of televisions: colour, standard and Economy. The expected daily production on each section is as follows:

T.V. Model	Section A	Section B
Colour	3	1
Standard	1	1
Economy	2	6

- ❖ The daily running costs for two sections average Rs. 6000 for section A and Rs. 4000 for section B. It is given that the company must produce atleast 24 colours, 16 standard and 40 Economy TV sets for which an order is pending. Formulate this as a L.P.P so as to minimize the total cost.

Solution:

$$\text{Maximize } Z = 6000x_1 + 4000x_2$$

Subject to

$$3x_1 + x_2 \geq 24$$

$$x_1 + x_2 \geq 16$$

$$2x_1 + 6x_2 \geq 40$$

$$\text{and } x_1, x_2 \geq 0$$

Problem 6:

- ❖ A company produces refrigerators in Unit I and heaters in Unit II. The two products are produced and sold on a weekly basis. The weekly production cannot exceed 25 in Unit I and 36 in Unit II, due to constraints 60 workers are employed. A refrigerator requires 2 man-week of labour, while a heater requires 1 man-week of labour. The profit available is Rs. 600 per refrigerator and Rs. 400 per heater. Formulate the LPP problem.

Solution:

$$\text{Maximize } Z = 600x_1 + 400x_2$$

Subject to,

$$2x_1 + x_2 \leq 60$$

$$x_1 \leq 25$$

$$x_2 \leq 36$$

$$\text{and } x_1, x_2 \geq 0$$

2.4. Basic Assumptions:

The linear programming problems are formulated on the basis on the following assumptions:

1. **Proportionality:** The contribution of each variable in the objective function or its usage of the resources is directly proportional to the value of the variable.
2. **Additivity:** Sum of the resources used by different activities must be equal to the total quantity of resources used by each activity for all the resources individually or collectively.
3. **Divisibility:** The variables are not restricted to integer values.
4. **Certainty or Deterministic:** Co-efficients in the objective function and constraints are completely known and do not change during the period under study in all the problems considered.
5. **Finiteness:** Variables and constraints are finite in number.
6. **Optimality:** In a linear programming problem we determine the decision variables so as to extremise (optimize) the objective function of the LPP.
7. The problem involves only one objective namely profit maximization or cost minimization.

2.5. Graphical Method of the Solution of a L.P.P:

- ❖ Linear programming problems involving only two variables can be effectively solved by a graphical method which provides a pictorial representation of the problems and its solutions and which gives the basic concepts used in solving general L.P.P. which may involve any finite number of variables. This method is simple to understand and easy to use.
- ❖ Graphical method is not a powerful tool of linear programming as most of the practical situations do involve more than two variables. But the method is really useful to explain the basic concepts of L.P.P to the persons who are not familiar with this. Though graphical method can deal with any number of constraints but since each constraint is shown as a line on a graph a large constraint is shown as a line on a graph, a large number of lines makes the graph difficult to read.

Problem 1:

- ❖ Solve the following L.P.P by the graphical method.

$$\text{Maximize } Z = 3x_1 + 2x_2$$

Subject to,

$$-2x_1 + x_2 \leq 1$$

$$x_1 \leq 2$$

$$x_1 + x_2 \leq 3$$

$$\text{and } x_1, x_2 \geq 0$$

Solution:

- ❖ First consider the inequality constraints as equalities.

$$-2x_1 + x_2 = 1 \quad \text{_____ (1)}$$

$$x_1 = 2 \quad \text{_____ (2)}$$

$$x_1 + x_2 = 3 \quad \text{_____ (3)}$$

$$\text{and } x_1 = 0 \quad \text{_____ (4)}$$

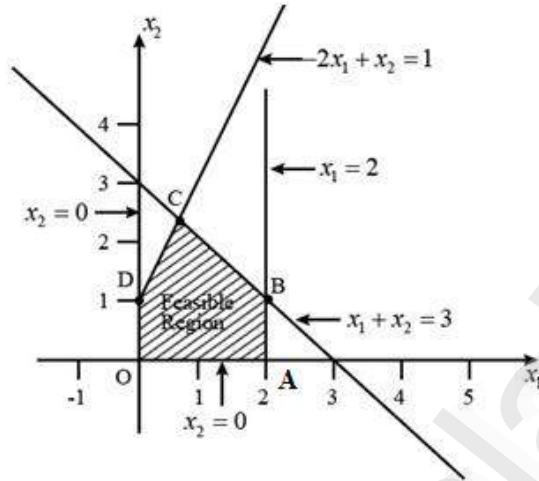
$$x_2 = 0 \quad \text{_____ (5)}$$

For the line $-2x_1 + x_2 = 1$

Put $x_2 = 0 \Rightarrow x_1 = 1 \Rightarrow (0,1)$

Put $x_2 = 0 \Rightarrow -2x_1 = 1 \Rightarrow x_1 = -0.5 \Rightarrow (-0.5,0)$

- ❖ The vertices of the solution space are O (0, 0), A (2, 0), B (2, 1), C $\left(\frac{2}{3}, \frac{7}{3}\right)$ and D (0,1)



- ❖ The value of Z at these vertices are given by $\therefore (z = 3x_1 + 2x_2)$



Vertex	Value of Z
O(0, 0)	0
A (2, 0)	6
B (2, 1)	8
C $\left(\frac{2}{3}, \frac{7}{3}\right)$	$\frac{20}{3}$
D(0,1)	2

- ❖ Since the problem is of maximization type, the optimum solution to the L.P.P is
maximum $Z = 8$, $x_1 = 2$, $x_2 = 1$

Problem 2:

Solve the following L.P.P by the graphical method.

Maximize $Z = 3x_1 + 5x_2$

Subject to,

$$-3x_1 + 4x_2 \leq 12$$

$$x_1 \leq 4$$

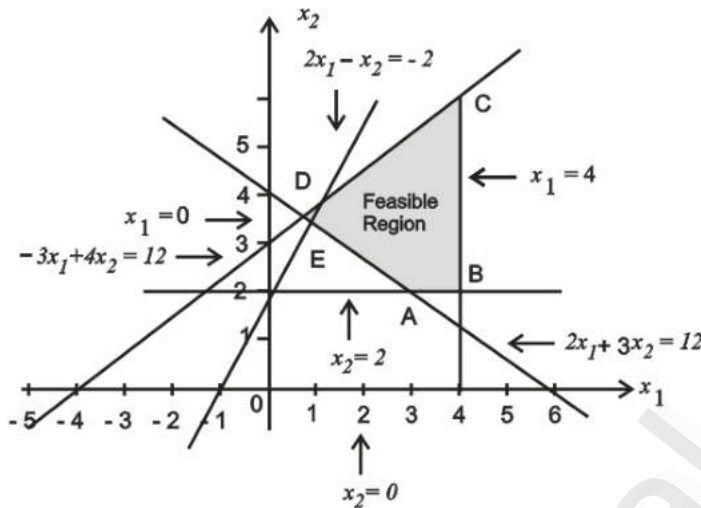
$$2x_1 - x_2 \geq -2$$

$$x_2 \geq 2$$

$$2x_1 + 3x_2 \geq 12 \quad \text{and} \quad x_1, x_2 \geq 0$$

Solution:

The vertices of the solution space are A (3, 2), B (4, 2), C (4, 6), D $\left(\frac{4}{5}, \frac{18}{5}\right)$ and E $\left(\frac{3}{4}, \frac{7}{2}\right)$



The value of Z at these vertices are given by $\therefore (z = 3x_1 + 5x_2)$

Vertex	Value of Z
A (3, 2)	19
C (4, 2)	22
C (4, 6)	42
D $\left(\frac{4}{5}, \frac{18}{5}\right)$	$\frac{102}{5}$
E $\left(\frac{3}{4}, \frac{7}{2}\right)$	$\frac{79}{4}$

Since the problem is of minimization type, the optimum solution is,

$$\text{Minimum } Z = 19, \quad x_1 = 3, \quad x_2 = 2$$

Problem 3:

Apply graphical method to solve the L.P.P

$$\text{Maximize } Z = x_1 - 2x_2$$

Subject to,

$$-x_1 + x_2 \leq 1$$

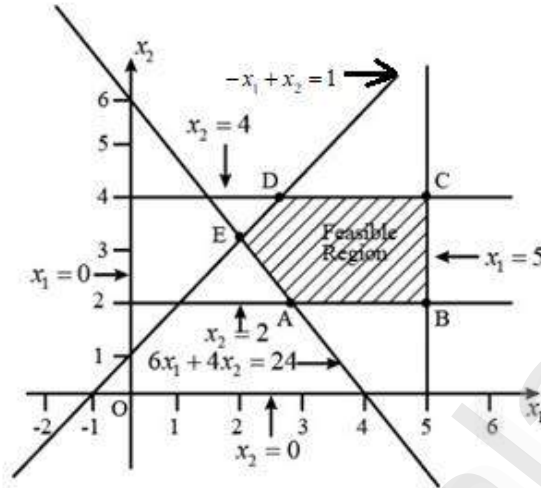
$$6x_1 + 4x_2 \geq 24$$

$$0 \leq x_1 \leq 5$$

$$2 \leq x_2 \leq 4$$

Solution:

- By using graphical method, the solution space is given below with shaded area ABCDE with vertices $A\left(\frac{8}{3}, 2\right)$, $B(5, 2)$, $C(5, 4)$, $D(3, 4)$ and $E(2, 3)$



- The value of Z at these vertices are given by $\because (z = x_1 - 2x_2)$

Vertex	Value of Z
$A\left(\frac{8}{3}, 2\right)$	$\frac{4}{3}$
$B(5, 2)$	1
$C(5, 4)$	-3
$D(3, 4)$	-5
$E(2, 3)$	-4

Since the problem is of maximization type, the optimum solution is,

$$\text{Maximum } Z = 1, x_1 = 5, x_2 = 2$$

2.6. Some More Cases:

The constraints generally, give region of feasible solution which may be bounded or unbounded. However, it may not be true for every problem. In general, a linear programming problem may have;

- A unique optimal solution
- an infinite number of optimal solutions
- an unbounded solution
- no solution.

EXERCISES

- Branch and Bound method is applicable to _____ IPP.
 A) pure B) mixed C) both a& b D) None of these
- If sometimes a few or all the variables of an IPP are constrained by their upper or lower bounds, then the most general method for the solution of optimization problem is called _____
 A) Branch and Bound method B) Gomary's cutting plane –method
 C) simplex method D) Big – M method

12. SET – I - ONE MARKS

- Operations research is the application of _____ methods to arrive at the optimal solutions to the problems.
 A) economical B) scientific
 C) both (a) and (b) D) none of the above
- In operations research the _____ are prepared for situations.
 A) mathematical models B) iconic model
 C) static model D) dynamic model
- _____ is a physical or pictorial representation of various aspects of a system.
 A) mathematical models B) iconic model
 C) static model D) dynamic model
- Analytic model is a model in which exact solution is obtained by _____ in closed form.
 A) static B) iconic
 C) simulation D) mathematical
- Operations research started just before World War II in Britain with the establishment of teams of scientists to study the strategic and tactical problems involved in military operations.
 A) True B) False
- OR can be applied only to those aspects of libraries where mathematical models can be prepared.
 A) True B) False



7. OR has a characteristic that it is done by a team of
- A) Scientists
B) mathematicians
C) Academics
D) All the above
8. OR uses models to help the management to determine is _____.
- A) Policies
B) Actions
C) Both (A) and (B)
D) None of the above
9. Linear programming problem deals with the _____ of a function of decision variables.
- A) maximization
B) minimization
C) optimization
D) None of the above
10. The variables whose values determine the solution of a problem are called _____ of the problem.
- A) decision variables
B) objective function
C) constraints
D) non-negativity restrictions
11. In LPP optimization of a function of decision variables is known as
- A) decision variables
B) objective function
C) constraints
D) non-negativity restrictions
12. Linear programming techniques are used in many problems.
- A) industrial
B) economic
C) both (A) and (b)
D) none of the above
13. LPP Technique requires
- A) objective function
B) constraints
C) non-negativity restrictions
D) all the above
14. LPP involving only two variables can be effectively solved by a _____ which provides a pictorial representation of the problems.
- A) formulation method
B) graphical method
C) simplex method
D) Big – M – method
15. In graphical method, if there exists an optimal solution of an L.P.P, it will be at one of the vertices of the _____.
- A) feasible region
B) unique optimal solution
C) an unbounded solution
D) no solution



16. In graphical method, the problem is of maximization type and the maximum value of Z is attained at a single vertex, then the solution is _____.
- A) unique optimal solution B) an unbounded solution
C) infinite number of optimal solution D) no solution
17. An LPP having more than one optimal solution is said to have _____ solution.
- A) feasible B) unique
C) multiple optimal D) no solution
18. An L.P.P, the maximum value of Z occurs at infinity, then the solution is _____ solution.
- A) feasible B) unique
C) multiple optimal D) unbounded
19. In graphical method, the given LPP cannot be solved, then the solution is _____ solution.
- A) unique B) unbounded
C) infinite D) no feasible
20. A set of values x_1, x_2, \dots, x_n which satisfies the constraints of the LPP is called its
- A) feasible solution B) solution
C) optimal solution D) no solution
21. Any solution to a LPP which satisfies the non-negativity restrictions of the LPP is called its
- A) feasible solution B) solution
C) optimal solution D) unique solution
22. Any feasible solution which optimizes the objective function of the LPP is called its
- A) feasible solution B) solution
C) optimal solution D) unbounded solution
23. In simplex method, to convert the inequalities into equalities for \leq type constraints to introduce _____ variables.
- A) optimum B) slack
C) surplus D) none of the above
24. In simplex method, to convert the inequalities into equalities for \geq type constraints to introduce variables.
- A) optimum B) slack
C) surplus D) none of the above



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UG TRB MATHEMATICS 2023-2024

UNIT X

Statistics / Probability

Your Success is Our Goal....

UG TRB 2022-23 MATHEMATICS		
UNIT - X - STATISTICS PROBABILITY		
INDEX		
S.NO	CHAPTER NAME	P.NO
1.	MEASURES OF CENTRAL TENDENCY	1
1.1	Characteristics of An Average	1
1.2	Arithmetic Mean	1
1.3	Mathematical Characteristics	8
1.4	Merits of Arithmetic Mean	13
1.5	Demerits (Limitations)	13
1.6	Uses of Arithmetic Mean	13
1.7	Median	13
1.8	Graphic Location of Median	18
1.9	Merits of Median	18
1.10	Demerits of Median	18
1.11	Quartiles	18
1.11.1	Deciles	
1.11.2	Percentile	
1.12	Mode	19
1.13	Graphic Location of Mode	22
1.14	Relationship Between Different Averages	23
1.15	Merits of Mode	23
1.16	Demerits of Mode	24
1.17	Uses of Mode	24
1.18	Geometric Mean:	24
1.19	Merits of G.M	27

1.20	Demerits of G.M	27
1.21	Uses Of G.M	28
1.22	Harmonic Mean	28
1.23	Merits Of H.M	31
1.24	Demerits Of H.M	31
1.25	Relationship Between Mean, Geometric Mean and Harmonic Mean	31
2.	MEASURES OF DISPERSION	33
2.1	Definition	33
2.1.2	Properties of A Good Measure Of Variation	
2.1.3	Methods of Measuring Dispersion	
2.2.1	(I) Range	
2.2.1.1	Merits of Range	
2.2.1.2	Demerits of Range	
2.2.2	(ii) Inter Quartile Range and Quartile Deviation	35
2.2.2.1	Merits of Quartile Deviation	
2.2.2.2	Demerits of Quartile Deviation	
2.2.3	Mean Deviation or Average Deviation	
2.2.3	Mean Deviation or Average Deviation	39
2.2.3.1	Merits of Mean Deviation	
2.2.3.2	Demerits of Mean Deviation	
2.2.3.3	Uses of Mean Deviation	
2.2.4	Standard Deviation:	44
2.2.4.1	Mathematical Properties of Standard Deviation	
2.2.4.2	Merits of Standard Deviation	
2.2.4.3	Demerits of Standard Deviation	
2.2.4.4.	Coefficient of Variation	

2.3	Lorenz Curve	50
2.4	Moments	51
	2.4.1	Raw Moments.
	2.4.2	Central Moments
2.5	Skewness	53
	2.5.1	Characteristics
	2.5.2	Absolute Measure:
	2.5.3	Relative Measures
2.6	Kurtosis	56
3.	CORRELATION	60
3.1	Types of Correlation	60
	3.1.1	Positive and Negative Correlation
	3.1.2	Simple and Multiple
	3.1.3	Partial and Total:
	3.1.4	Linear and Non –Linear
3.2	Methods of Studying Correlation.	30
	3.2.1.1	Scatter Diagram Method
	3.2.2	Karl Pearson's Co Efficient Of Correlation (R)
3.3	Properties	65
3.4	Standard Error and Probable Error	65
3.5	Merits	66
3.6	Demerits	66
3.7	Rank Correlation Coefficient	66
	3.7.1	Types of Rank Correlation: Coefficient
	3.7.1.2	Merits of Rank Correlation Coefficient:
	3.7.1.3	Demerits of Rank Correlation Coefficient

3.8	Concurrent Deviation Method		70
	3.8.1	Merits	
	3.8.2	Demerits	
4.	REGRESSION		74
4.2	Differences Between Correlation and Regression Are		
4.3	Methods of Forming The Regression Equations		
	4.3.1. Methods - Regression Equations on the Basis of Normal Equations		
5.	INDEX NUMBERS		84
5.1	Definition		84
5.2	Characteristics of Index Numbers		84
5.3	Uses		84
5.4	Types of Index Numbers:		85
	5.4.1	A) Price Index	
	5.4.2	B) Quantity Index	
	5.4.3	C) Value Index	
5.5	Problems in The Construction of Index Numbers		86
5.6	Choice of Formulae		87
	5.6.1	Merits of Simple Average of Price Relative Method	
	5.6.2	Demerits	
5.7	Weighted Index Numbers:		91
	5.7.1	Weighted Aggregate Index Numbers	
	5.7.2	Weight Average of Price Relative	96
5.8	Value Index Number		96
5.9	Test of Consistency of Index Numbers		
	5.9.1	Time Reversal Test	

	5.9.2	Factor Reversal Test	
5.10	Unit Test		99
5.11	Circular Test		99
5.12	Chain Base Method		99
5.13	Construction of Chain Indices		100
	5.13.1	Merits of Chain Base Method	
5.14	Difference Between Chain Base Method and Fixed Base Method		101
5.15	Conversion of Chain Index into Fixed Index		102
5.16	Consumer Price Index		103
5.17	Uses of Consumer Price Index		103
5.18	Construction of A Consumer Price Index		103
5.19	Method of Constructing Consumer Price Index		104
5.20	Precautions in The Use of Cost of Living Index Numbers		104
5.21	Limitations of Index Numbers		106
6.	CURVE FITTING		109
6.1	Principle of Least Square		109
6.2	Fitting A Straight Line		109
6.3	Fitting A Second-Degree Parabola		112
6.4	Fitting of a Power Curve		114
7.	THEORY OF ATTRIBUTES		117
7.1	Introduction		117
7.2	Classification		117
7.3	Correlation and Association		117
7.4	Uses of Terms and Notation		117
7.5	Positive and Negative Classes		117
7.6	Relationship		118

7.7	Determination of Frequencies	118
7.8	Consistency of Data	120
7.9	Types of Association:	121
	7.9.1 Positive Association	
	7.9.2 Negative Association (Disassociation)	
	7.9.3 Independent Association	
	7.9.4 Limitations	
7.10	Method of Determining Association	124
	7.10.1 Comparison of Observed and Expected Frequencies	
	7.10.2 Comparison of Proportions	
	7.10.3 Yule's Coefficient of Association	
	7.10.4 Yule's Coefficient of Colligation	
	7.10.5. Pearson's Coefficient of Contingency	
8.	THEORY OF PROBABILITY	131
8.1	Basic Terminology	132
	8.1.1 Random Experiment	
	8.1.2 Outcome	
	8.1.3 Trial and Event	
	8.1.4 Exhaustive Events of Causes	
	8.1.5 Favourable Events or Cases	
	8.1.6 Mutually Exclusive Events	
	8.1.7 Equally Likely Events	
	8.1.8 Independent Events	
8.2	Mathematical (Or) Classical (Or) 'A Priori' Probability	134
	8.2.1 Definition	
	8.2.2 Statistical (Or) Empirical Probability	

	8.2.3	Subjective Probability	
	8.2.4	Axiomatic Probability	
8.3	Theorem		138
8.4	Theorem		138
8.5	Theorem:		139
8.6	Addition Theorem of Probability		139
8.7	Boole' S in Equality:		140
8.8	Conditional Probability:		141
	8.8.1	Multiplication Theorem of Probability	
8.9	Independent Events:		142
	8.9.1	Multiplication Theorem of Probability for Independent Events	
8.10	Baye's Theorem		144
9.	RANDOM VARIABLES		146
9.1	Definition		146
9.2	Distribution Function		146
9.3	Properties of Distribution Function		146
9.4	Discrete Random Variable		147
	9.4.1	Probability Mass Function	
9.5	Continuous Random Variable		150
	9.5.1	Probability Density Function	
9.6	Various Measures of Central Tendency, Dispersion, Skewness and Kurotsis For Continuous Probability Distribution		150
9.7	Continuous Distribution Function		154
	9.7.1	Properties of Distribution Function	
9.8	Two-Dimensional Random Variables		155
	9.8 .1	Joint Probability Mass Function	

	9.8 .2	Marginal Probability Function	
	9.8 .3	Conditional Probability Function	
9.9	Two-Dimensional Distribution Function		157
	9.9.1	Marginal Distribution Function	
9.10	Independent Random Variables		158
10.	MATHEMATICAL EXPECTATION		161
10.1	Properties of Expectation:		162
	10.1.1	Property 1	
	10.1.2	Property 2: Multiplication Theorem of Expectation	
	10.1.3	Property 3	
	10.1.4	Property 4	
	10.1.5	Property 5	
	10.1.6	Property 6	
	10.1.7	Property 7	
10.2	Properties of Variance		164
10.3	Covariance		164
10.4	Variance of A Linear Combination of Random Variables		165
10.5	Cauchy' Schwartz Inequality		165
10.6	Jenson's Inequality		165
10.7	Moments of Bivariate Probability Distributions		165
	10.7.1.	Moment Generating Function	
10.8	Properties of Moment Generating function		166
	10.8.1.	Property 1	
	10.8.2	Property 2	
	10.8.3	Property 3	
10.9	Uniqueness Theorem of Moment Generating Function:		168

10.10	Cumulantes	168
10.11	Properties of Cumulants	169
	10.11.1	Property 1: Additive Property of Cumulants
	10.11.2	Property 2: Effect of Change of Origin and Scale on Cumulants
10.12	Characteristic Function	169
	10.12.1	Property 1
	10.12.2	Property 2
	10.12.3	Property 3
	10.12.4	Property 4
	10.12.5	Property 5
	10.12.6	Property 6
	10.12.7	Property 7
	10.12.8	Property 8
10.13	Uniqueness Theorem of Characteristic Functions	171
11.	THEORETICAL DISTRIBUTION	171
11.1	Binomial Distribution	172
	11.1.2	Properties of Binomial Distribution
	11.1.3	Fitting of Binomial Distribution
11.2	Poisson Distribution	176
	11.2.1	Fitting A Poisson Distribution
11.3	Normal Distribution	180
	11.3.1	Characteristic of The Normal Curve
12.	TESTS OF SIGNIFICANCE	186
12.1	Standard Error	187
12.2	Statistical Hypothesis	187
	12.2.1	One Tailed Test

	12.2.2	Two – Tailed Test	
12.3	Large Sample (Test of Significance)		188
	12.3.1	Test of Significance for Single Mean	
	12.3.2	Test of Significance for Difference of Mean	
	12.3.3	Test of Significance for Single Proportion	
	12.3.4	Test of Significance for Difference of Proportion	
	12.3.5	(Difference of Proportion)	
12.4	Small Samples (Test of Significance)		194
	12.4.1	T – Test for Testing the Significance of The Difference Between Population Mean and Sample Mean	
	12.4.2	T – Test for Difference of Means	
	12.4.3	Paired Test	
	12.4.4	T – Test for Correlation Co-Efficient	
12.5	F – Test for Equality of Population Variance		202
12.6	x^2 Test		204
	12.6.1. Test for Single Variance		
	12.6.2. Goodness of Fit		
	12.6.3. x^2 Test for Independent of Attributes		
12.7	Theorem		209
13	IMPORTANT QUESTIONS (MCQ)		212

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UG TRB – MATHEMATICS – 2022-23

UNIT - X

STATISTICS / PROBABILITY

1. MEASURES OF CENTRAL TENDENCY

- ❖ An average is a value which is typical or representative of a set of data. The measures of central tendency are also known as “measures of location”.
- ❖ Various measures of central tendency are the following
 1. Arithmetic mean, 2. Median, 3. mode, 4. Geometric mean and, 5. Harmonic mean

1.1 Characteristics of An Average:

1. It should be rigidly defined
2. It should be based on all the items
3. It should not be unduly affected by extreme items.
4. It should lend itself for algebraic manipulation.
5. It should be simple to understand and easy to calculate.
6. It should have sampling stability.



1.2 Arithmetic Mean:

- Arithmetic mean is the total of the value of the items divided by their number.
- It is denoted by \bar{x}

Type - I: Individual observations or Raw data)

Formula: $A.M = \frac{\text{Total of the observations}}{\text{No. of the observations}}$

$$(i.e) A.M = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{\sum X}{n}$$

Problem: the expenditure of 10 families in rupees are given below:

Family	A	B	C	D	E	F	G	H	I	J
Expenditure	30	70	10	75	500	8	42	250	40	36

Calculate the arithmetic mean:

Solution: x- Expenditure: N=10

Family	Expenditure (Rs)
	X
A	30
B	70
C	10
D	75
E	500
F	8
G	42
H	250
I	40
J	36
TOTAL	$\sum x = 1061$



$$\bar{X} = \frac{\sum X}{n}$$

$$= \frac{1061}{10}$$

$$\bar{X} = 106.1$$

Type - II: (Discrete series)

$$\bar{X} = \frac{\sum fX}{\sum f}$$

Problem:

Calculate the mean number of persons per house

Given

No. of persons per house	2	3	4	5	6	Total
No. of houses	10	25	30	25	10	100

Solution:

x- No. of persons per house

f - No. of houses

No. of persons per house X	No. of houses f	fx
2	10	20
3	25	75
4	30	120
5	25	125
6	10	60
	$\sum f = 100$	$\sum fx = 400$

$$\bar{X} = \frac{\sum fX}{\sum f}$$

$$= \frac{400}{100}$$

$$\bar{X} = 4$$

Type - III: (Continuous Series): Exclusive class Intervals

$$\bar{X} = \frac{\sum fm}{\sum f}; m = \text{mid point of the class interval}$$

Problem: calculate A.M for the following

Marks	20-30	30-40	40-50	50-60	60-70	70-80
No. of students	5	8	12	15	6	4

Marks	No. of students	m	fm
20-30	5	25	125
30-40	8	35	280
40-50	12	45	540
50-60	15	55	825
60-70	6	65	390
70-80	4	75	300
	$\sum f = 50$		$\sum fm = 2460$

$$\bar{X} = \frac{\sum fm}{\sum f}$$

$$= \frac{2460}{50}$$

$$\bar{X} = 49.20$$



Continuous series: Inclusive class Intervals

Problem: The annual profits of 90 companies are given below. Find the arithmetic mean.

Annual profit (Rs. lakhs)	0-19	20-39	40-59	60-79	80-99
No. of companies	5	17	32	24	12

Solution:

Annual profit (Rs. lakhs)	No. of companies f	Mid value m	fm
0-19	5	19.5	47.5
20-39	17	29.5	501.5
40-59	32	49.5	1584.0
60-79	24	69.5	1668.0
80-99	12	89.5	1074.0
	$\Sigma f = 90$		$\Sigma fm = 4875.0$

$$\bar{X} = \frac{\Sigma fm}{\Sigma f}$$

$$= \frac{4875.0}{90}$$

$$\bar{X} = \text{Rs. } 54.17 \text{ lakhs}$$

Problem:

- Average rainfall of a city from Monday to Saturday was 1.2 cms. Due to heavy rainfall on Sunday, the average rainfall on Sunday, the average rainfall increased to 2cms. What was the rain fall on Sunday?

Solution:

Total rain fall on 6 days = Number \times Average

$$= 6 \times 1.2$$

$$= 7.2 \text{ cms}$$

Total rain fall on 7 days = $7 \times 2 = 14 \text{ cms}$

Total rain fall on 7th days, Sunday = $14 - 7.2 = 6.8 \text{ cms}$

Formula for combined means:

If two means are given,

$$\bar{X}_{12} = \frac{N_1 \bar{X}_1 + N_2 \bar{X}_2}{N_1 + N_2}$$

If three means are given, $\bar{X}_{123} = \frac{N_1\bar{X}_1 + N_2\bar{X}_2 + N_3\bar{X}_3}{N_1 + N_2 + N_3}$

Problem:

- There are two branches of an establishment employing 100 and 80 persons respectively. If the arithmetic means of the monthly salaries paid by the two branches are Rs.275 and Rs.225 respectively. Find the arithmetic mean of the salaries of the employees of the establishment as a whole.

Solution:

Given $N_1 = 100, N_2 = 80, \bar{X}_1 = 275, \bar{X}_2 = 225$

$$\begin{aligned}\bar{X}_{12} &= \frac{N_1\bar{X}_1 + N_2\bar{X}_2}{N_1 + N_2} \\ &= \frac{(100 \times 275) + (80 \times 225)}{100 + 80}\end{aligned}$$

$$\bar{X}_{12} = \text{Rs.}252.78$$

Problem:

- The average mark in mathematics of foundation course students of three centers, Kolkata, Mumbai and Delhi is 50. The number candidates in Kolkata, Mumbai and Delhi are respectively 100, 120 and 150. The average marks of Kolkata and Mumbai are 70 and 40 respectively. Find the average mark of Delhi.

Solution:

Given $\bar{X}_{123} = 50, N_1 = 100, N_2 = 120, N_3 = 150; \bar{X}_1 = 70, \bar{X}_2 = 40$

$$\begin{aligned}\bar{X}_{123} &= \frac{N_1\bar{X}_1 + N_2\bar{X}_2 + N_3\bar{X}_3}{N_1 + N_2 + N_3} \\ 50 &= \frac{(100 \times 70) + (120 \times 40) + (150 \times \bar{X}_3)}{100 + 120 + 150}\end{aligned}$$

$$\bar{X}_3 = \frac{6700}{150} = 44.67$$

Corrected Arithmetic Mean:**Problem:**

- The mean of 20 marks is found to be 40. Later on it was discovered that a mark 53 was misread as 83, Find the correct mean.

Solution:

Given $N = 20, \bar{X}_w = 40, X_c = 53, X_w = 83$

$$\bar{X}_w = \frac{(\sum X)_w}{N}$$

$$\therefore \text{Wrong total } (\sum X)_w = N\bar{X}_w$$

$$= 20 \times 40 = 800$$

$$\therefore \text{Correct total } (\sum X)_c = (\sum X)_w - X_w + X_c$$

$$= 800 - 83 + 53$$

$$= 770$$

$$\therefore \text{Correct mean } \bar{X}_c = \frac{(\sum X)_c}{N}$$

$$= \frac{770}{20} = 38.5$$

Problem:

- A student found the mean of 50 items as 38.6. when checking the work he found that he had taken one item as 50 while it should correctly read as 40. Also the number of items turned out to be only 49. In the circumstances, what should be the correct mean?

Solution:

Given $N_w = 50; \bar{X}_w = 38.6, X_w = 50, X_c = 40; N_c = 49$

$$\therefore \text{Wrong total } (\sum X)_w = N_w \bar{X}_w$$

$$= 50 \times 38.6 = 1930$$

$$\therefore \text{Correct total } (\sum X)_c = (\sum X)_w - X_w + X_c$$

$$= 1930 - 50 + 40 = 1920$$

$$\begin{aligned} \therefore \text{Correct mean } \bar{X}_c &= \frac{(\sum X)_c}{N} \\ &= \frac{1920}{49} = 39.18 \end{aligned}$$

Missing frequencies:

Problem:

Find the missing frequency from the following frequency distribution if mean is 38.

Marks	10	20	30	40	50	60	70
No. of students	8	11	20	25	-	10	3

Solution: Let the missing frequency be f

$$\sum f = 8 + 11 + 20 + 25 + f + 10 + 3 = 77 + f$$

$$\sum f_x = 2710 + 50f$$

$$\text{Consider, } \bar{X} = \frac{\sum fx}{\sum f}$$

$$38 = \frac{2710 + 50f}{77 + f} \Rightarrow 38f + 2926 = 2710 + 50f$$

$$50f - 38f = 2926 - 2710$$

$$f = \frac{216}{12} = 18$$



1.3 Mathematical Characteristics:

1. The algebraic sum of the deviations, of all the items from their arithmetic mean is zero.

$$\text{(ie) } \sum (X - \bar{X}) = 0$$

2. The sum of the standard deviations of the items from mean is a minimum.

3. If all the items of a series are increased (or) decreased by any constant number, the arithmetic mean will also increase (or) decrease by the same constant.

Discrete series: (Direct method)

$$\bar{X} = \frac{\sum fX}{N}$$

\bar{X} = Arithmetic mean; $\sum fX$ = the sum of product;

N= total number of items

Problem: Calculate mean from the following data

Value	1	2	3	4	5	6	7	8	9	10
Frequency	21	30	28	40	26	34	40	9	15	57

Solution:

X	f	f_x
1	21	21
2	30	60
3	28	84
4	40	160
5	26	130
6	34	204
7	40	280
8	9	72
9	15	135
10	57	570
	N=300	$\sum fX = 1716$

$$\bar{X} = \frac{\sum fX}{N}$$

$$= \frac{1716}{300}$$

$$\bar{X} = 5.72$$

Short Cut Method:

$$\bar{X} = A \pm \frac{\sum fd}{N}$$

\bar{X} = Mean, A = Assumed mean, $\sum fd$ = sum of total deviations, N = total frequency

Problem: (solving the previous problem)**Solution:**

X	f	$d = (X - A)$	fd
1	21	-4	-84
2	30	-3	-90
3	28	-2	-56
4	40	-1	-40
5	26	0	0
6	34	1	34
7	40	2	80
8	9	3	27
9	15	4	60
10	57	5	285
	$\sum f = 300$		$\sum fd = +216$

Continuous Series:**1. Direct method**

$$\bar{X} = \frac{\sum fm}{\sum f};$$

\bar{X} = mean, m - mid value,

Problem: From the following find out the mean profits:

Profits per shop Rs	100-200	200-300	300-400	400-500	500-600	600-700	700-800
Number of shops	10	18	20	26	30	28	18

Solution:

X	f	M	fm
100-200	10	150	1500
200-300	18	250	4500
300-400	20	350	7000
400-500	26	450	11700
500-600	30	550	16500
600-700	28	650	18200
700-800	18	750	13500
	$\Sigma f = 150$		$\Sigma fm = 72900$

$$\bar{X} = \frac{\Sigma fm}{\Sigma f}$$

$$= \frac{72900}{150}$$

$$\bar{X} = 486$$

2. Short cut method

$$\bar{X} = A \pm \frac{\Sigma fd}{N}$$

A= Assumed mean, Σfd = sum of total deviations, N =Number of items

3) step deviation method

$$\bar{X} = A \pm \frac{\sum fd'}{N}$$

\bar{X} = Mean, A = Assumed mean, $\sum fd'$ = sum of total deviations, N = Number of items, C = common factor.

Note:

- If we use any method to find the arithmetic mean for continues series, we can get the same answer for same problem.

Problem:

Find mean of the following data:

Class - Interval	0-9	10-19	20-29	30-39	40-49	50-59
Frequency	2	15	10	8	3	1

Solution:

- The given problem is to be convert into exclusive class interval series (ie. Left side C.I subtract 0.5 and right side C.I add 0.5 to given data)

C.I	True C.I	f	m	$d' = \frac{m-34.5}{10}$	fd'
0-9	0.5-9.5	1	4.5	2	2
10-19	9.5-19.5	3	14.5	1	3
20-29	19.5-29.5	8	24.5	0	0
30-39	29.5-39.5	10	34.5	-1	-10
40-49	39.5-49.5	15	44.5	-2	-30
50-59	49.5-59.5	2	54.5	-3	-6
	$\sum f = 40$				$\sum fd' = -41$

$$\bar{X} = A \pm \frac{\sum fd'}{N} \times c$$

$$A=34.5, \sum fd' = -41, N=40, C=10$$

$$\bar{X} = 34.5 - \frac{(-41)}{40} \times 10$$

$$\bar{X} = 24.25$$

1.4. Merits of Arithmetic Mean:

1. It is easy to understand
2. It is easy to calculate
3. It is rigidly defined
4. It is based on the value of every item in the series
5. It provides a good basis for comparison.
6. It can be used for further analysis and algebraic treatment.
7. The mean is a more stable measure of central tendency.

1.5. Demerits (Limitations)

1. The mean is unduly affected by the extreme items.
2. It is unrealistic.
3. It may lead to a false conclusion.
4. It cannot be accurately determined even if one of the values is not known.
5. It cannot be located by observations or the graphic method.
6. It gives greater importance to bigger items of a series and lesser importance to smaller items.

1.6. Uses of Arithmetic Mean:

It is used in social economic and business problem.

1.7. Median:

- Median is the value of item that goes to divided the series into equal parts. Median may be defined as the value of that item which divides the series into two equal parts, one half containing values greater than it and the other half containing values less than it. Therefore, the series has to be arranged in ascending or descending order, before finding the median. It is also called positional average.

Individual Series:**Problem (odd number problem)**

Find the median of the following series.

X: 10 15 9 25 19

Solution:

Size of the item ascending order(x)	Size of the item descending order(x)
9	25
10	19
15	15
19	10
25	9

$$\text{Median} = \text{size of } \left(\frac{N+1}{2}\right)^{\text{th}} \text{ item}$$

$$= \text{size of } \left(\frac{5+1}{2}\right)^{\text{th}} \text{ item}$$

$$= \text{Size of } 3^{\text{rd}} \text{ item}$$

$$\text{median} = 15$$

Problem (even number problem)

Find the value of median from the following series.

X: 8 10 5 9 12 11

Solution:

X
5
8
9
10
11
12

$$\text{Median} = \text{size of } \left(\frac{N+1}{2} \right)^{\text{th}} \text{ items}$$

$$= \text{size of } \left(\frac{6+1}{2} \right)^{\text{th}} \text{ items}$$

$$= \text{Size of } 3.5^{\text{th}} \text{ item}$$

$$= \text{Size of } \left(\frac{3^{\text{rd}} \text{ item} + 4^{\text{th}} \text{ item}}{2} \right)$$

$$= \frac{9+10}{2}$$

$$\text{median} = 9.5$$

Discrete Series:

Problem: Find out the median from the following:

Size of shoes	5	5.5	6	6.5	7	7.5	8
Frequency	10	16	28	15	30	40	34

Solution:

Size of shoes	f	Cf
5	10	10
5.5	16	26
6	28	54
6.5	15	69
7	30	99
7.5	40	139
8	34	173

$$\text{Median} = \text{size of } \left(\frac{N+1}{2} \right)^{\text{th}} \text{ item}$$

13. MULTIPLE CHOICE QUESTIONS

1. _____ is a typical value of the entire group or data.
- A) Mean
B) Median
C) Mode
D) Measure of central tendency
2. Arithmetic average is also called as _____.
- A) Mean
B) Median
C) Mode
D) G.M.
3. In continuous series, the formula for A.M. is
- A) $\bar{X} = \frac{\sum fx}{N}$
B) $\bar{X} = \frac{\sum fm}{N}$
C) $\bar{X} = \frac{\sum X}{N}$
D) None of these
4. The sum of the deviations taken from A.M is
- A) Minimum
B) Maximum
C) zero
D) None of these
5. The sum of squares of deviations from A.M is
- A) zero
B) Maximum
C) Minimum
D) one
6. The best measure of central tendency is
- A) A.M
B) Median
C) G.M
D) H.M
7. For dealing with qualitative data the best average is
- A) Mean
B) Median
C) Mode
D) H.M
8. Median is a _____ average.
- A) Positional
B) Locational
C) both (a) and (b)
D) None of these
9. _____ is the most unstable average.
- A) Mean
B) Median
C) Mode
D) G.M.
10. _____ average is affected by extreme observations.
- A) H.M
B) A.M
C) G.M.
D) Median
11. Harmonic mean is the _____ of the arithmetic average of the reciprocal of values.
- A) reciprocal
B) non-reciprocal
C) neither a nor b
D) equal



12. If the items in a distribution have the same value then,

A) $\bar{X} \neq G.M \neq H.M$

B) $\bar{X} > G.M > H.M$

C) $\bar{X} < G.M < H.M$

D) $\bar{X} = G.M = H.M$

13. ____ is the measure of the variation of the items.

A) dispersion

B) range

C) Q.D

D) S.D

14. Range is the best measure of dispersion.

A) True

B) False

15. Quartile deviation is more suitable in case of open – end distribution.

A) True

B) False

16. Mean deviation can never be negative

A) True

B) False

17. Formula for standard deviation in discrete series is,

A) $\sigma = \sqrt{\frac{\sum X^2}{N} - \left(\frac{\sum X}{N}\right)^2}$

B) $\sigma = \sqrt{\frac{\sum fX^2}{N} - \left(\frac{\sum fX}{N}\right)^2}$

C) $\sigma = \sqrt{\frac{\sum fm^2}{N} - \left(\frac{\sum fm}{N}\right)^2}$

D) None of these

18. Standard deviation is always _____ than range.

A) Maximum

B) Minimum

C) less

D) more

19. Variance is _____ of S.D.

A) equal

B) square

C) both a and b

D) None of these

20. Formula for combined mean is,

A) $\bar{X}_{12} = \frac{N_1\bar{X}_1 + N_2\bar{X}_2}{N_1 + N_2}$

B) $\bar{X}_{12} = \frac{N_2\bar{X}_1 + N_1\bar{X}_2}{N_1 + N_2}$

C) $\bar{X}_{12} = \frac{\bar{X}_1 + \bar{X}_2}{N_1 + N_2}$

D) $\bar{X}_{12} = \frac{N_1 + N_2}{\bar{X}_1 + \bar{X}_2}$



21. The coefficient of skewness is zero, then distribution is,
 A) J-shaped B) U-shaped C) Z-shaped D) symmetrical
22. A negative coefficient of skewness implies that
 A) Mean > Mode B) Mean < Mode
 C) Mean = Mode D) Mean \neq Mode
23. For a symmetrical distribution the coefficient of skewness is
 A) + 1 B) - 1 C) + 3 D) - 3
24. The first central moment is always zero
 A) True B) False
25. The second central moment does not indicate the variance.
 A) True B) False
26. β_2 must always be positive
 A) True B) False
27. If β_2 is greater than 3, then curve is called,
 A) mesokurtic B) Leptokurtic C) Platykurtic D) None of these
28. If β_2 is less than 3, the curve is called
 A) mesokurtic B) Leptokurtic C) Platykurtic D) None of these
29. The coefficient of correlation.
 A) cannot be positive B) cannot be negative
 C) can be either positive or negative D) none of these
30. The coefficient of correlation is independent of
 A) change of scale only B) change of origin only
 C) both change of scale and origin D) none of these
31. The study of two variables excluding some other variables is called ____ correlation.
 A) positive B) negative C) multiple D) partial



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