* Result / Algebra / Test / UNIT-5 (Mathematics)

① Every composit number can be resolved into prime factors uniquely. ஒவ்வொரு பகு எண்ணையும் பகா எண்களின் பெருக்கலாக ஒரே ஒரு முறையாக மட்டுமே எழுத இயலும்.

Eg: 1) $180 = 2^2 \cdot 3^2 \cdot 5^1$ 2) $236 = 2^2 \cdot 59$

② Divisors of a given number N : N-ன் வகுப்பான்கள்

Let $N = p^a q^b r^c$ எனில். p, q, r are all primes. [include 1 & N]

a) Number of divisors of N $= (a+1)(b+1)(c+1)$
N-ன் வகுப்பான்களின் எண்ணிக்கை

b) Sum of the divisors of N $= \dfrac{p^{a+1}-1}{p-1} \times \dfrac{q^{b+1}-1}{q-1} \times \dfrac{r^{c+1}-1}{r-1}$
N-ன் வகுப்பான்களின் கூட்டல்

c) Product of the divisors of N $= N^{\frac{1}{2}(a+1)(b+1)(c+1)}$
N-ன் வகுப்பான்களின் பெருக்கல்

Eg:- $N = 24 = 2^3 \times 3^1$, Divisors : 1, 2, 3, 4, 6, 8, 12, 24
a) 8 b) 60 c) 331,776

For, a) Number of divisors of N $= (3+1)(1+1) = 4 \times 2 = 8$

b) Sum of the divisors of N $= \left(\dfrac{2^{3+1}-1}{2-1}\right) \times \left(\dfrac{3^{1+1}-1}{3-1}\right) = \dfrac{15}{1} \times \dfrac{8}{2} = 60$

c) Product of the divisors of N $= 24^{\frac{1}{2}(3+1)(1+1)} = 24^4 = 331,776$.

③ Euler's function :- $\phi(N)$ ஆய்லரின் சார்பு

$\phi(N)$ = Number of +ve integers $< N$ and relative prime to it. N-ஐ விட சிறியதாகவும் N-உடன் பகா எண்ணாகவும் உள்ள மிகை எண்களின் எண்ணிக்கை $\phi(N)$ ஆகும்.

ie $\phi(N) = p^a q^b r^c \Rightarrow \phi(N) = N\left(1-\frac{1}{p}\right)\left(1-\frac{1}{q}\right)\left(1-\frac{1}{r}\right)$

Eg:- 1) $N = 16 = 2^4 \Rightarrow \phi(N) = 16\left(1-\frac{1}{2}\right) = 16 \times \frac{1}{2} = 8$.

2) $N = 24 = 2^3 \times 3^1 \Rightarrow \phi(N) = 24\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right) = 24 \times \frac{1}{2} \times \frac{2}{3} = 8$

3) $N = 720 = 2^4 \cdot 3^2 \cdot 5^1 \Rightarrow \phi(N) = 720 \times \left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right)$
$= 720 \times \frac{1}{2} \times \frac{2}{3} \times \frac{4}{5}$
$= 192$

$\therefore \phi(16) = 8$
$\phi(24) = 8$
$\phi(720) = 192$.

④ If $N = ab$, where a, b are relatively prime, then
$\phi(N) = \phi(a)\phi(b)$ (e.g) $\phi(60) = \phi(4)\phi(15)$
$16 = 2 \times 8$

⑤ Sum of the integers less than N and prime to $N = \frac{N}{2}\phi(N)$

⑥ Product of r consecutive integer is divisible by $r!$

⑦ nC_r is divisible by n.

⑧ Highest power of a prime p contained in $n!$ is
$\left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \cdots$

(e.g) ① Highest power of 2 in 10! is 8.

$\left[\frac{10}{2}\right] + \left[\frac{10}{4}\right] + \left[\frac{10}{8}\right] = 5 + 2 + 1 = 8.$

For, $10! = 1 \times 2 \times \cdots \times 10$

2 4 6 8 10

$\Rightarrow 2^1 \; 2^2 \; 2^1 \; 2^3 \; 2^1$

$\Rightarrow 2^8 \Rightarrow$ ⑧

2 | 10, 2 | ⑤, 2 | ②, ①

$5 + 2 + 1 = 8$

② Highest power of 3 in 120!

$= \left[\frac{120}{3}\right] + \left[\frac{120}{9}\right] + \left[\frac{120}{27}\right] + \left[\frac{120}{81}\right]$

$= 40 + 13 + 4 + 1 = 58$

3 | 120, 3 | ㊵, 3 | ⑬, 3 | ④, ①

$40 + 13 + 4 + 1 = 58.$

③ Highest power of 6 in 150!

$= 72$

$\because 6 = 2 \times 3$

3 | 150, 3 | (50), 3 | (16), 3 | ⑤, ①

$50 + 16 + 5 + 1 = 72.$

Questions:-

① The number of binary operations that can be defined on a set of n elements is
(A) n^n (B) $n^{\frac{n(n+1)}{2}}$ (C) n^{n^2} (D) n^2

② The number of commutative binary operations that can be defined on a set of n elements is
(A) n^n (B) $n^{\frac{n(n+1)}{2}}$ (C) n^{n^2} (D) $n^{\frac{n+1}{2}}$

③ Which of the following is a semigroup but not a monoid.
(A) $(N, +)$ (B) $(N, \cdot)$ (C) $(Z, \cdot)$ (D) $(Z, +)$

(4) If $a * b = a^2 + b^2$, then $(4 * 3) * 5$ is (A) 650 (B) 120 (C) 620 (D) 50

(5) Which of the following is an infinite non-abelian group?
(A) Set of all integers under addition.
(B) Set of all real numbers under addition.
(C) Set of all non-zero complex numbers under addition.
(D) Set of all (2×2) non-singular real matrices under multiplication.

(6) Which of the following group is cyclic?
(A) $(Z, +)$ (B) $(Q, +)$ (C) $(R, +)$ (D) $(C, +)$

(7) Which of the following is not true?
(A) Every cyclic group is abelian
(B) Every abelian group is cyclic
(C) Subgroup of an cyclic group is cyclic.
(D) Subgroup of an abelian group is abelian.

(8) Which of the following binary operations is not associative and not commutative?
(A) $*$ on Z^+ by $a * b = a + b \ \forall a, b \in Z^+$
(B) $a * b = a^b \ \forall a, b \in N$
(C) $a * b = \frac{ab}{2} \ \forall a, b \in Q$
(D) $A * B = AB \ \forall A, B \in G$ where G is the set of all 2×2 non-singular real matrices.

(9) Which is not a group under multiplication modulo?
(A) $(Z_3^*, \cdot)$ (B) $(Z_5^*, \cdot)$ (C) $(Z_4^*, \cdot)$ (D) $(Z_7^*, \cdot)$

(10) A non-abelian group of least order contains
(A) 2 (B) 3 (C) 4 (D) 6

(11) A non-cyclic group must have at least
(A) 2 elements (B) 3 elements (C) 4 elements (D) 6 elements.

(12) In the Klein-4 group $\{e, a, b, c\}$ the number of elements satisfying the equation $x^2 = e$ is
(A) 1 (B) 2 (C) 3 (D) 4

(13) Let $G = \{\bar{1}, \bar{3}, \bar{5}, \bar{7}\}$. The inverse of $\bar{7}$ under multiplication modulo 8 is
(A) $\bar{1}$ (B) $\bar{3}$ (C) $\bar{5}$ (D) $\bar{7}$

(14) Let $G = \{f_1, f_2, f_3, f_4\}$. Where $f_1(x) = x$, $f_2(x) = -x$, $f_3(x) = \frac{1}{x}$, $f_4(x) = -\frac{1}{x}$ which of the following is/are true?

1) G is a group under composition of functions.
2) G is abelian
3) The inverse of f_1, f_2, f_3, f_4 are respectively f_1, f_2, f_3, f_4.
4) G is cyclic.

(A) 1, 2, 3, 4 (B) 1, 2, 3 only (C) 1, 2, 4 only (D) 1, 3, 4 only.

(15) Which of the following is/are true?

1) $(Z, +)$ is a cyclic group with generators ± 1.
2) $G = \{\pm 1, \pm i\}$ is a cyclic group with generators $\pm i$
3) $(R, +)$ is a cyclic group with generators ± 1.
4) A cyclic group of order 'n' has $\phi(n)$ number of generators.

(A) 1, 2, 3 only (B) 1, 2 only (C) 1, 2, 4 only (D) All are true

(16) Let Z_n denote the set of all congruence classes of modulo n and Z_n^* the set of all non-zero congruence classes modulo n where n is any integer.

1) Z_n is a group under addition modulo n
2) Z_n is a group under addition modulo n only when n is a prime.
3) Z_n^* is a group under multiplication modulo n only when n is a prime.
4) (3) is valid for all integers 'n'.

(A) 1, 2 only (B) 2, 3 only (C) 1, 4 only (D) 1, 3 only.

(17) $o(a) = n$ only when $a^n = e$ for

(A) any integer n (B) n is a prime (C) n is the least +ve integer.
(D) n is a composite number.

(18) In the Group $(Z_5^*, \cdot_5)$ where $Z_5^* = \{\bar{1}, \bar{2}, \bar{3}, \bar{4}\}$ then the order of $\bar{4}$ is

(A) 1 (B) 2 (C) 4 (D) 3

(19) If for $a \in G$ $a^m = e$, which of the following is/are true?

(A) $o(a) = m$ (B) m is a multiple of $o(a)$
(C) $o(G) = m$ (D) $o(a) > m$

(20) If $a \in G$ and $o(a) = n$ and $a^m = e$ which of the following is/are true?

1) $n = m$ 2) $m \leq n$ 3) $o(a) = o(a^{-1}) = n$ 4) $n | m$

A) 1, 3 only (B) 2, 3 only (C) 1, 3, 4 only (D) 4 only.

(21) Which of the following is/are true?

1) $aH = H \Leftrightarrow a \in H$ 2) $aH \cap bH \neq \phi \Rightarrow aH = bH$

3) $G = \bigcup_{a \in G} aH$ 4) $o(aH) = o(H)$

(A) 1, 2 only (B) 2, 4 only (C) 1, 2, 4 only (D) 1, 2, 3, 4.

(22) A subset H of a group G is a subgroup if!
which of the following is/are satisfied?

1) H itself is a group under the same operation of G.

2) (i) $a, b \in H \Rightarrow ab \in H$ (ii) $a \in H \Rightarrow a^{-1} \in H$

3) $a, b \in H \Rightarrow ab^{-1} \in H$

4) H is closed under the same operation of G.

(A) 2, 3 only (B) 4 only (C) 1 only (D) 1, 2, 3 only.

(23) Which of the following is not true?

A) Union of two subgroups is also a subgroup.

B) Intersection of two subgroups is also a subgroup.

C) If H and K are subgroups, then HK is a subgroup iff $HK = KH$

D) $o(HK) = \dfrac{o(H)\, o(K)}{o(H \cap K)}$

(24) If H is a subgroup of a finite group G, then by Lagrange's theorem.

(A) $o(G) \mid o(H)$ (B) $o(H) \mid o(G)$ (C) $o(H) = o(G)$ (D) None of these.

(25) G is a group and H is a subgroup. Then $\dfrac{o(G)}{o(H)}$ is

(A) number of subgroups of G (B) number of left cosets of H in G

(C) number of elements in H (D) None of these.

(26) Which of the following is/are true?

1) $o(a) \mid o(G)$ (2) $o(a) = o(a^{-1})$ (3) $a^{o(G)} = e$ (4) $o(a) = o(G) \Leftrightarrow G = \langle a \rangle$

(A) 1, 2 only (B) 3, 4 only (C) All are true (D) 1, 2, 3 only.

(27) If H and K are two finite subgroups of a group G and if $o(H)$ and $o(K)$ are relatively prime. Then

(A) $H \cap K = \phi$ (B) $H \cap K = G$ (C) $H \cap K = \{e\}$ (D) $H \cup K = G$.

28) If a group G has no proper subgroups then which of the following is/are true?

1) $o(G)$ is prime 2) G is Cyclic 3) $\{e\}$ and G are the only subgroups of G.
4) G is Abelian.

(A) 1, 2, 3, 4 (B) 1, 2, 3 only (C) 2, 3, 4 only (D) 1, 2, 4 only.

29) Let G be a group and N be a subgroup. Which of the following is/are satisfied for N to be a normal subgroup in G.

1) $gng^{-1} \in N \ \forall g \in G, \forall n \in N$ 2) $gNg^{-1} \subseteq N \ \forall g \in G$
3) $gN = Ng \ \forall g \in G$ (Every left coset of N is a right cosets of N)
4) product of two left cosets of N is again a left coset of N.

(A) 1, 2 only (B) 1, 2, 3 only (C) All (D) 1 only.

30) Let G be a group and N be normal in G. Then (G/N) is also a group under

(A) the same operation of G (B) any operation
(C) product of two left cosets of N. (D) None of these.

31) Which of the following is/are true?

1) Every subgroup of an abelian group is Normal.
2) If H is of index 2 in G, then H is normal.
3) Intersection of two normal subgroups is also a normal subgroup.
4) A group of prime order is a simple group.

(A) 1, 2, 3, 4 (B) 1, 2, 3 only (C) 4 only (D) 1, 2 only.

32) Which of the following is not true?

(A) Centre of a group $[Z(G) = \{x \in G \mid gx = xg \ \forall g \in G\}]$ is a normal subgroup.
(B) If N and M are normal subgroup of G, then NM is also a normal subgroup.
(C) N is a normal subgroup and H is a subgroup of G, then $H \cap N$ is normal in G
(D) If N is a normal subgroup of G. Then (G/N) is abelian iff G is abelian.

(33) If G is a finite group and N is a normal subgroup of G then

(a) (G/N) is an abelian group (b) $o(G/N) = o(G)\cdot o(N)$

(c) $o(G/N) = \frac{o(G)}{o(N)}$ (d) (G/N) is a cyclic group.

(34) Let G be a group. Then which of the following is/are true?

1) $a, x \in G \Rightarrow o(a) = o(xax^{-1})$

2) If $G = (a)$, then $G = (a^{-1})$ [ie if a is a generator then a^{-1} is also a generator]

3) If G is cyclic and $o(G) = n$ then G has $\phi(n)$ number of generators.

4) $o(a) = o(a^{-1})$ if $a \in G$

(A) 1, 2, 4 only (B) 1, 2, 3, 4 (C) 1, 2, 3 only (D) 2, 3 only.

(35) If G is a group of even order.

(A) G is abelian (B) $\exists\, a \in G$ such that $a^2 = e$.

(C) $\exists\, a \in G$ such that $a^3 = e$ (D) there is atleast one $a \in G$ which do not possess inverse element.

(36) If G is abelian group, then which of the following is/are satisfied?

1) $(ab)^2 = a^2b^2 \ \forall a, b \in G$ 2) $a^{-1} = a \ \forall a \in G$ (Every element is its own inverse).

3) $a^2 = e \ \forall a \in G$ (or) $o(a) = 2 \ \forall a \in G$ 4) $o(G)$ is even

(A) 1, 2 only (B) 1, 2, 3 only (C) All are true (D) 1 only.

(37) Which of the following is a cyclic group?

(A) additive group of integers. (C) Multiplicative group of non-zero reals.

(B) additive group of reals. (D) Multiplicative group of non-zero complex.

(38) If $a^2 = e$, $b^6 = e$, $ab = b^4a$ is a group of G for $a, b \in G$, then $o(ab)$ is

(A) 2 (B) 4 (C) 6 (D) 8

(39) Which of the following is a non-abelian group.

(A) $(Z_5^*, \cdot_5)$ (B) Additive group of complex numbers

(C) $G = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle/ \begin{matrix} ad-bc \neq 0 \\ a,b,c,d \text{ reals} \end{matrix} \right\}$ under matric multiplication.

(D) Additive group of all 2x2 real matrices.

(40) Which of the following is not Correct ?

(A) Every group is a monoid (B) Every monoid is a semigroup.

(C) Every semigroup is a group (D) Every group is a semigroup.

(41) Which of the following axioms are satisfied to Constitute a Ring ?

(1.) $(R, +)$ is an abelian group. (2) $a \cdot b \in R \ \forall a, b \in R$

(3) $a \cdot (b \cdot c) = (a \cdot b) \cdot c$ (4) $a \cdot (b+c) = a \cdot b + a \cdot c$ & $(a+b) \cdot c = a \cdot c + b \cdot c$

(5) $a \cdot a^{-1} = a^{-1} \cdot a = 1$.

(A) 1, 2, 3, 5 only (B) 1, 2, 3, 4 only (C) 2, 3, 4, 5 only (D) All are satisfied.

(42) Which of the following is not true? for $(R, +, \cdot)$.

A) $1 \cdot a = a \cdot 1 = a \Rightarrow R$ is a ring with unity.

B) $a \cdot b = 0 \Rightarrow a = 0$ (or) $b = 0 \Rightarrow R$ is a ring without zero divisors.

c) For every $a \in R$, $\exists\, a^{-1} \in R$ such that $a \circ a^{-1} = a^{-1} \cdot a = 1 \Rightarrow R$ is division ring (or) Skew field.

D) A commutative ring, with zero divisors is an Integral domain.

(43) A field is a

(A) Commutative division ring (B) Commutative integral domain

(C) A ring with unity (D) $(R, +)$ is abelian & $(R, \cdot)$ is abelian.

(44) Which of the following is a field ?

(A) $(Z_5, +_5, \cdot_5)$ (B) $(Z_6, +_6, \cdot_6)$ (C) $(Z, +, \cdot)$

(D) Set of all 2x2 real matrices under addition & Multiplication of matrices.

(45) Which of the following is/are true ?

1) Any field is an integral domain 2) Any finite integral domain is a field.

3) $Z_p = \{\bar{0}, \bar{1}, \bar{2}, \ldots, \overline{p-1}\}$ is a field for any integer p.

4) Z_p is a field only when p is a prime.

(A) 1, 3 only (B) 1, 2, 4 only (C) 1, 4 only (D) 2, 3 only.

(46) M is the ring of all 2x2 real matrices with respect to matrix addition and matrix multiplication. which of the following is/are true?

1) M is a commutative ring 2) M is a ring with unity
3) M is a ring without zero divisors 4) M is not a division ring.

(A) 2, 4 only (B) 1, 3 only (C) 1, 4 only (D) 2, 3 only

(47) Let R be a ring with unity. An element $a \in R$ is called a unit in R if

(A) $a \cdot 1 = 1 \cdot a = a$ (B) $\exists\, a^{-1} \in R$ such that $a \cdot a^{-1} = a^{-1} \cdot a = 1$

(C) $ab = ba \ \forall b \in R$ (D) None of these.

(48) which of the following is/are true?

1) In $(Z, +, \cdot)$ 1, −1 are the only units.
2) In $(R, +, \cdot)$ 1, −1 are the only units
3) In $(C, +, \cdot)$ all non-zero elements are units.
4) In $(Z_6, +_6, \cdot_6)$, [1], [5] are the units.

(A) 1, 2, 3, 4 (B) 3, 4 only (C) 1, 2 only (D) 1, 3, 4 only.

(49) which of the following is/are true?

1) A finite commutative ring without zero divisors is a field.
2) Integral domain with 6 elements does not exist.
3) A field has no zero divisors.
4) A field need not be an integral domain

(A) 1, 2, 3, 4 (B) 1, 2, 3 only (C) 1, 2 only (D) 3, 4 only.

(50) An element x of a ring R is said to be idempotent.

(A) $x^2 = 0$ (B) $x^2 = 1$ (C) $x^2 = x$ (D) $x^2 \notin R$

(51) A boolean Ring R is a ring in which every $x \in R$ is such that

(A) $x^2 = 0$ (B) $x^2 = 1$ (C) Every element is idempotent (D) None of these.

(52) Let J be the set of all Gaussian Integers. Then which of the following is/are true for $(J, +, \cdot)$. $\left(J = \{a+ib \mid a, b \in Z \text{ \& } i = \sqrt{-1}\}\right)$

1) Commutative ring, with unity and without zero divisors.

2) Commutative ring, with zero divisors.

3) Division Ring 4) Integral domain.

(A) 1, 4 only (B) 2, 3 only (C) 1, 2, 4 only (D) 3, 4 only.

(53) Which of the following is not an integral domain?

(A) $(Z, +, \cdot)$ (B) $(Z_5, +_5, \cdot_5)$ (C) $Z(\sqrt{5}) = \{a + \sqrt{5}b \mid a, b \in Z\}$ under addition & multiplication.

(D) $R = \left\{ \begin{pmatrix} 0 & a \\ 0 & b \end{pmatrix} \middle| a, b \in \text{Reals} \right\}$ under addition & multiplication of matrices.

(54) Let R be the ring of all 2×2 real matrices w.r.to addition and multiplication of matrices. Then which of the following is not true?

(A) R is not a commutative ring (B) R is a ring with unity.

(C) R is without zero divisors. (D) R is not an integral domain.

(55) A positive integer 'n' is said to be the characteristic of a ring R if

(A) $na = 0 \; \forall a \in R$ (B) n is the least +ve integer such that $na = 0 \; \forall a \in R$.

(C) $na \neq 0 \; \forall a \in R$ (D) None of these.

(56) For the ring $(Z_5, +_5, \cdot_5)$ the characteristic is

(A) 0 (B) 1 (C) 4 (D) 5

(57) For an infinite ring $(R, +, \cdot)$ the characteristic is

(A) 0 (B) ∞ (C) 2 (D) 3

58) The characteristic of a finite integral domain is
(A) infinite (B) finite (C) 0 (D) does not exist.

59) The characteristic of an integral domain is
(A) zero only (B) infinite only (C) prime number only. (D) zero (or) prime number

60) A subset S of a ring R is said to be a subring of R if
(A) $a, b \in S \Rightarrow a-b \in S$ (B) $a, b \in S \Rightarrow ab \in S$
(C) $a, b \in S \Rightarrow a-b \in S$ and $a, b \in S \Rightarrow ab \in S$
(D) $a, b \in S \Rightarrow a-b \in S$ (or) $a, b \in S \Rightarrow ab \in S$

61) R is a ring. I is a subring of R. Then which one of the following is not true?
A) $a \in I$, $r \in R \Rightarrow ar \in I$ then I is a right ideal.
B) $a \in I$, $r \in R \Rightarrow ra \in I$ then I is a left ideal.
c) $a \in I$, $r \in R \Rightarrow ar, ra \in I$ then I is an ideal of R.
D) $a \in I$, $r \in R \Rightarrow ar = ra$ then I is an ideal of R.

62) Which of the following is not true? If I and J are two ideals of a ring R.
A) $I \cap J$ is also an ideal. (B) $I \cup J$ is also an ideal.
c) $I + J = \{x+y \mid x \in I, y \in J\}$ (D) $IJ = \{xy \mid x \in I, y \in J\}$

63) The units of the ring $(J, +, \cdot)$ of Gaussian integers are.
(A) ± 1 only (B) $\pm i$ only (C) $\pm 1, \pm i$ only. (D) $1, i$ only.

64) Let D be an integral Domain, $0 \neq a$, $0 \neq b \in D$ are called associates if
(A) $a|b$ (B) $b|a$ (C) $a|b$ & $b|a$ (D) None of these.

65) Let a be an arbitrary element of a ring R. Then $S = \{x \in R : ax = 0\}$ is
(A) an ideal of R (B) a left ideal of R (C) a right ideal of R
(D) None of these.

(66) Let a be an arbitrary element of a ring R. Then $S = \{x \in R : xa = 0\}$ is

(A) an ideal of R (B) a left ideal of R (C) a right ideal of R (D) None of these.

(67) If N is a normal subgroup and H is a subgroup of G, then NH is

(A) Normal subgroup of G (B) not a subgroup

(C) a subgroup (D) None of these.

(68) If N and M are normal subgroups of a group G then NM is

(A) a subgroup but not a normal subgroup.

(B) not a subgroup. (C) a normal subgroup (D) None of these.

(69) If N is a normal subgroup and M is a subgroup of G, then $N \cap M$ is

(A) normal in G (B) a normal subgroup in H

(C) not a subgroup of G (D) None of these.

(70) A Boolean ring is of characteristic

(A) 1 (B) 3 (C) 2 (D) 4

(71) Which of the following is/are to be satisfied for a non-empty set V to be a vector space over the field F.

1) $(V, +)$ is an abelian group 2) $\alpha \in F, v \in V \Rightarrow \alpha v \in V$

3) $(\alpha+\beta)v = \alpha v + \beta v$, $\alpha(u+v) = \alpha u + \alpha v$ $(\alpha, \beta \in F, u, v \in V)$

4) $\alpha(\beta v) = (\alpha\beta)v$, $1 \cdot v = v$ $(\alpha, \beta \in F, v \in V)$

(A) 2, 3, 4 only (B) 1, 2, 3, 4 (C) 1, 2, 3 only (D) 3, 4 only.

(72) Let F be a subfield of a field K.

(A) F is a vector space over K (B) K is a vector space over the field F

(C) F is not a vector space over the field K (D) None of these.

73) Let V be a vector space over F. Then $W \subset V$ is vector subspace of V if .

(A) W is an additive subgroup of V

(B) $\alpha \in F, w \in W \Rightarrow \alpha w \in W$

(C) $\alpha, \beta \in F, u, v \in W \Rightarrow \alpha u + \beta v \in W$

(D) $\alpha; \beta \in F, u, v \in W \Rightarrow \alpha - \beta \in F$ & $u - v \in W$.

74) If W_1 and W_2 are subspaces of V, then which of the following is/are true for subspace of V.

1) $W_1 + W_2$ 2) $W_1 \cap W_2$ 3) $W_1 \cup W_2$ 4) W_1 / W_2

(A) 1, 2 only (B) 1, 2, 3 only (C) 1, 2, 3, 4 only (D) 2 only.

75) Which of the following is not a subspace of $V = R^3(R)$?

(A) $W = \{(a, b, 0) \mid a, b \in V\}$ (B) $W = \{(a, b, a-b) \mid a, b \in V\}$

(C) $W = \{(a, b, ab) \mid a, b \in V\}$ (D) $W = \{(a, 0, 0) \mid a \in V\}$

76) The vector space V over F is said to be the direct sum of the two subspaces W_1 and W_2 if:

(A) $V = W_1 + W_2$ & $W_1 \cap W_2 = \{0\}$ (B) $V = W_1 + W_2$ & $W_1 \cap W_2 \neq \phi$

(C) $V = W_1 + W_2$ & $W_1 \cap W_2 = \phi$ (D) $V = W_1 \cup W_2$ & $W_1 \cap W_2 = \{0\}$.

77) Let S be a non-empty subset of the vector space V over F. Then L(S), the linear span of S which of the following is/are true?

(1) L(S) is a subspace of V (2) $S \subseteq T \Rightarrow L(S) \subseteq L(T)$

(3) $S \subseteq T \Rightarrow L(S) \supseteq L(T)$ (4) $L(L(S)) = L(S)$.

(A) 1, 3, 4 only (B) 1, 2, 4 only (C) 1, 4 only (D) 4 only.

78) Let V be a vector space over F and $v_1, v_2, \ldots, v_n$ be elements of V. Then $v_1, v_2, \ldots, v_n$ are said to be linearly independent if:

(A) $\alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_n v_n = 0$, not all α_i's are zero (B) $\alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_n v_n = 0$ all v_i's are zero

(C) $\alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_n v_n = 0 \Rightarrow \alpha_1 + \alpha_2 + \cdots + \alpha_n = 0$ (D) $\alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_n v_n = 0 \Rightarrow \alpha_i = 0 \ \forall i$

(79) In the above problem $v_1, v_2, \ldots, v_n$ are linearly dependent if:

(A) $\alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_n v_n = 0$, not all α_i are zero

(B) $\alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_n v_n = 0$, all v_i are zero

(C) $\alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_n v_n = 0$, $\alpha_1 + \alpha_2 + \alpha_3 + \cdots + \alpha_n = 0$

(D) $\alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_n v_n = 0$, $\alpha_i = 0 \ \forall i$.

(80) Which of the following vectors are linearly dependent.

A) $e_1 = (1,0,\ldots 0), e_2 = (0,1,0,\ldots 0) \ldots e_n = (0,0,\ldots 1)$ in $R^n(R)$

B) $v_1 = (1,1)$, $v_2 = (-3,2)$ in $R^2(R)$

C) $v_1 = (1,3,2)$, $v_2 = (1,-7,-8)$, $v_3 = (2,1,-1)$ in $R^3(R)$

D) $v_1 = (2,1,1,1)$, $v_2 = (1,2,-1,3)$, $v_3 = (1,3,1,-2)$ in $R^4(R)$

(81) A subset $S = \{v_1, v_2, \ldots, v_n\}$ is said to form a basis of a vector space V over F if:

(A) V is generated by S (B) S is linearly independent.

(C) $L(S) = V$ and S is Linearly independent (D) $L(S) = V$ & S is L.D.

(82) The dimension of a vector space V is n if:

(A) If there is a linearly independent subset of n vectors.

(B) If V has a basis of n vectors.

(C) If V is generated by a subset of n vectors.

(D) None of these.

(83) Which of the following is/are true for a vector space of dimension n.

(1) any (n+1) vectors in V are L.I

(2) any set of n L.I vectors is a basis of V.

(3) V cannot be generated by fewer than n vectors.

(4) If $\{v_1, v_2, \ldots, v_m\}$ is linearly independent then $m \leq n$.

(A) 1, 2, 3 only (B) 2, 3 only (C) 1, 2, 3, 4 (D) 2, 3, 4 only.

(84) If W is a subspace of a vectorspace V. Then

(A) $\dim W = \dim V$ (B) $\dim W < \dim V$ (C) $\dim W > \dim V$ (D) $\dim W \leq \dim V$

(85) If V is a vector space and W is a subspace of V. Then $\dim(V/W) =$ ___ .

(A) $\dim V - \dim W$ (B) $\frac{\dim V}{\dim W}$ (C) $\dim V + \dim W$ (D) $\dim V$ or $\dim W$

(86) If W_1 and W_2 are subspace of a finite dimensional vectorspace V. Then $\dim(W_1 + W_2)$ is

(A) $\dim W_1 + \dim W_2$ (B) $\dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$

(C) $\dim W_1 - \dim W_2 + \dim(W_1 \cap W_2)$. (D) $\dim W_1 - \dim W_2$

(87) If V is the direct sum of the subspaces W_1 and W_2, then

(A) $\dim(W_1 + W_2) = \dim W_1 + \dim W_2$

(B) $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$

(C) $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 + \dim(W_1 \cap W_2)$

(D) None of them.

(88) which of the following set of vectors do not form a basis for R^3?

(A) (2, 1, 4) (1, -1, 2) (3, 1, -2)

(B) (1, 1, 0) (2, -1, 3) (-1, 0, 1) (2, 1, -2)

(C) (1, 1, 2), (3, -1, 0), (2, 0, -1)

(D) (1, 0, 0), (1, 1, 0), (1, 1, 1).

(89) The set of vectors $S = \{(a_1, b_1, c_1), (a_2, b_2, c_2), (a_3, b_3, c_3)\}$ in $R^3(R)$. Then S is satisfied (is/are)?

(1) linearly independent if $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$

(2) linearly dependent if $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$

(3) linearly dependent if $\Delta \neq 0$

(4) linearly independent if $\Delta \neq 0$.

(A) 1, 3 only (B) 2, 4 only (C) 2, 3 only (D) 1, 2 only.

(90) In a vector space V of dimension 'n' which of the following is/are true?

1) Any set of m vectors ($m \leq n$) can be extended to form a basis.

2) Any two bases have the same number of vectors.

3) $\{v_1, v_2, \ldots, v_n, v_{n+1}\}$ is linearly dependent.

4) Any set of vectors $\{w_1, w_2, \ldots, w_n\}$ form a basis.

(A) 1, 2, 3 only (B) 1, 2 only (C) 4 only (D) 2, 3, 4 only.

(91) Which of the following is not a subspace of $V = R^3$

(A) $\{(a, 2b, 3c) \mid a, b, c \in R\}$ (B) $\{(x, x, x) \mid x \in R\}$

(C) $\{(x, y, z) \mid x, y, z$ are rationals (D) $\{(0, y, z) \mid y, z \in R\}$.

(92) which of the following is an integral domain?

(A) Ring of integers (B) $(Z_6, +_6, \cdot_6)$

(C) Ring of real quaternion (D) None of them.

(93) The characteristic of the ring of integers is

(A) ∞ (B) 0 (C) -1 (D) 2

(94) If M is the ring of 2×2 matrices over the integers then $K = \left\{\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} \mid a, b \in Z\right\}$ is

(A) a sub ring (B) an ideal (C) a right ideal (D) a left ideal.

(95) The order of element $a, a^{-1} \in G$ are n and k, then

(A) $n = k = 0$ (B) $n \neq k$ (C) $n = k$ (D) All of them.

(96) Any subgroup of a cyclic group is

(A) a normal group (B) a cyclic group.

(C) a finite group (D) an abelian group.

97) An integral domain R is a Euclidean ring then

(A) $d(a) \leq d(ab)$ $\forall a, b \in R$ (B) it possess a unit element.

(C) every element in R is either a unit in R or can be written as the product of a finite number of prime elements of R.

(D) All of the above.

98) zero divisors of a Ring means, for every $a, b \in R$.

(A) $a = 0$ such that $ab = 0$ (B) $a \neq 0, b \neq 0$ such that $ab \neq 0$

(C) $a \neq 0, b \neq 0$ such that $ab = 0$ (D) None of these.

99) How many vectors will make $x, 4x^2, 8x^3$ into a basis of $F_n[x]$.

(A) 1 (B) $n-1$ (C) $n-2$ (D) $n+1$

100) which one of the following statement is not true?

A) Number of maximal ideal in $\frac{Z[i]}{\langle 2+3i \rangle}$ is 1

B) Number of prime ideal in $\frac{Z[i]}{\langle 2+3i \rangle}$ is 1.

(C) Number of Maximal ideals in Z_{52} is 2

(D) Z_p is integral domain if $p = 2024$.

All the Best.

SAMPLE OMR SHEET

Class & Sec. ____________ Name of Exam TRB - MATHEMATICS Date ____________

NAME OF CANDIDATE (IN CAPITAL LETTERS)

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

CLASS: TRB

SECTION: A B C D E F G H I J

ROLL No.: 0 1 2 3 4 5 6 7 8 9

STREAM: PCM, PCB, COM, HUM

SET NUMBER: 1, 2, 3

SUBJECT :

SUBJECT CODE: 0 1 2 3 4 5 6 7 8 9

ADMISSION No.: 0 1 2 3 4 5 6 7 8 9

1 A B C D	26 A B C D	51 A B C D	76 A B C D
2 A B C D	27 A B C D	52 A B C D	77 A B C D
3 A B C D	28 A B C D	53 A B C D	78 A B C D
4 A B C D	29 A B C D	54 A B C D	79 A B C D
5 A B C D	30 A B C D	55 A B C D	80 A B C D
6 A B C D	31 A B C D	56 A B C D	81 A B C D
7 A B C D	32 A B C D	57 A B C D	82 A B C D
8 A B C D	33 A B C D	58 A B C D	83 A B C D
9 A B C D	34 A B C D	59 A B C D	84 A B C D
10 A B C D	35 A B C D	60 A B C D	85 A B C D
11 A B C D	36 A B C D	61 A B C D	86 A B C D
12 A B C D	37 A B C D	62 A B C D	87 A B C D
13 A B C D	38 A B C D	63 A B C D	88 A B C D
14 A B C D	39 A B C D	64 A B C D	89 A B C D
15 A B C D	40 A B C D	65 A B C D	90 A B C D
16 A B C D	41 A B C D	66 A B C D	91 A B C D
17 A B C D	42 A B C D	67 A B C D	92 A B C D
18 A B C D	43 A B C D	68 A B C D	93 A B C D
19 A B C D	44 A B C D	69 A B C D	94 A B C D
20 A B C D	45 A B C D	70 A B C D	95 A B C D
21 A B C D	46 A B C D	71 A B C D	96 A B C D
22 A B C D	47 A B C D	72 A B C D	97 A B C D
23 A B C D	48 A B C D	73 A B C D	98 A B C D
24 A B C D	49 A B C D	74 A B C D	99 A B C D
25 A B C D	50 A B C D	75 A B C D	100 A B C D

123456789

INSTRUCTIONS FOR MARKING OMR SHEET

1. Use Black/Blue ball point pen for writing
2. Darken only one circle for marking response against the question to answer
3. To record response.

CORRECT METHOD

WRONG METHOD

MARKS OBTAINED	EXAMINER'S SIGNATURE

Signature of Candidate

Signature of Invigilator