

# 10th Maths

## Centum Marks

### Task On 1

### Chapter 1

## Chapter 1

### Relations and Functions

#### EXERCISE 1.1

Ques 1 Find  $A \times B$ ,  $A \times A$  and  $B \times A$ .

- (i)  $A = \{2, -2, 3\}$  and  $B = \{1, -4\}$       (ii)  $A = B = \{p, q\}$       (iii)  $A = \{m, n\}; B = \emptyset$

**Solution :**

- (i)  $A = \{2, -2, 3\}$  and  $B = \{1, -4\}$

$$A \times B = \{2, -2, 3\} \times \{1, -4\}$$

$$= \{(2, 1), (2, -4), (-2, 1), (-2, -4), (3, 1), (3, -4)\}$$

$$A \times A = \{2, -2, 3\} \times \{2, -2, 3\}$$

$$= [(2, 2), (2, -2), (2, 3), (-2, 2), (-2, -2), (-2, 3), (3, 2), (3, -2), (3, 3)]$$

$$B \times A = \{1, -4\} \times \{2, -2, 3\}$$

$$= \{(1, 2), (1, -2), (1, 3), (-4, 2), (-4, -2), (-4, 3)\}$$

- (ii)  $A = B = \{p, q\}$

$$A \times B = \{p, q\} \times \{p, q\}$$

$$= \{(p, p), (p, q), (q, p), (q, q)\}$$

$$A \times A = \{p, q\} \times \{p, q\} = \{(p, p), (p, q), (q, p), (q, q)\}$$

$$B \times A = \{p, q\} \times \{p, q\}$$

$$= \{(p, p), (p, q), (q, p), (q, q)\}$$

- (iii)  $A = \{m, n\}; B = \emptyset$

$$A \times B = \{m, n\} \times \{\}$$

$$= \{\} \text{ or } \emptyset$$

$$A \times A = \{m, n\} \times \{m, n\}$$

$$= \{(m, m), (m, n), (n, m), (n, n)\}$$

$$B \times A = \{\} \times \{m, n\}$$

$$= \{\} \text{ or } \emptyset$$

Ques 2 Let  $A = \{1, 2, 3\}$  and  $B = \{x \mid x \text{ is a prime number less than } 10\}$ . Find  $A \times B$  and  $B \times A$ .

**Solution :**

$$A = \{1, 2, 3\}$$

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By definition of a prime number,

$$B = \{2, 3, 5, 7\}$$

$$A \times B = \{1, 2, 3\} \times \{2, 3, 5, 7\}$$

$$= \{(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7)\}$$

$$B \times A = \{2, 3, 5, 7\} \times \{1, 2, 3\}$$

$$= \{(2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (5, 1), (5, 2), (5, 3), (7, 1), (7, 2), (7, 3)\}$$

Q3) If  $B \times A = \{(-2,3), (-2,4), (0,3), (0,4), (3,3), (3,4)\}$  find  $A$  and  $B$ . X 2mark

Solution :

$$B \times A = \{(-2,3), (-2,4), (0,3), (0,4), (3,3), (3,4)\}$$

$B$  = Set of all first coordinates of elements of  $B \times A$   
 $= \{-2, 0, 3\}$

$A$  = Set of all second coordinates of elements of  $B \times A$   
 $= \{3, 4\}$

Thus,  $A = \{3, 4\}$ ,  $B = \{-2, 0, 3\}$

Q4) If  $A = \{5, 6\}$ ,  $B = \{4, 5, 6\}$ ,  $C = \{5, 6, 7\}$ , show that  $A \times A = (B \times B) \cap (C \times C)$ .

Solution :

$$A = \{5, 6\}, B = \{4, 5, 6\}, C = \{5, 6, 7\}$$

$$A \times A = \{5, 6\} \times \{5, 6\}$$

$$A \times A = \{(5,5), (5,6), (6,5), (6,6)\} \quad \dots (1)$$

$$B \times B = \{4, 5, 6\} \times \{4, 5, 6\}$$

$$B \times B = \{(4,4), (4,5), (4,6), (5,4), (5,5), (5,6), (6,4), (6,5), (6,6)\}$$

$$C \times C = \{5, 6, 7\} \times \{5, 6, 7\}$$

$$C \times C = \{(5,5), (5,6), (5,7), (6,5), (6,6), (6,7), (7,5), (7,6), (7,7)\}$$

$$(B \times B) \cap (C \times C) = \{(5,5), (5,6), (6,5), (6,6)\} \quad \dots (2)$$

From (1) and (2), we get

$$A \times A = (B \times B) \cap (C \times C)$$

5. Given  $A = \{1, 2, 3\}$ ,  $B = \{2, 3, 5\}$ ,  $C = \{3, 4\}$  and  $D = \{1, 3, 5\}$ , check whether

$$(A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$$
 is true?

Solution :

$$(A \cap C) = \{1, 2, 3\} \cap \{3, 4\}$$

$$(A \cap C) = \{3\}$$

$$B \cap D = \{2, 3, 5\} \cap \{1, 3, 5\}$$

$$B \cap D = \{3, 5\}$$

$$(A \cap C) \times (B \cap D) = \{3\} \times \{3, 5\}$$

$$(A \cap C) \times (B \cap D) = \{(3,3), (3,5)\} \quad \dots (1)$$

$$A \times B = \{1, 2, 3\} \times \{2, 3, 5\}$$

$$= \{(1,2), (1,3), (1,5), (2,2), (2,3), (2,5), (3,2), (3,3), (3,5)\}$$

$$C \times D = \{3, 4\} \times \{1, 3, 5\}$$

$$= \{(3,1), (3,3), (3,5), (4,1), (4,3), (4,5)\}$$

$$(A \times B) \cap (C \times D) = \{(3,3), (3,5)\} \quad \dots (2)$$

From (1) and (2) we get,

$$(A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D).$$

6. Let  $A = \{x \in \mathbb{W} | x < 2\}$ ,  $B = \{x \in \mathbb{N} | 1 < x \leq 4\}$  and  $C = \{3, 5\}$ . Verify that

- (i)  $A \times (B \cup C) = (A \times B) \cup (A \times C)$
- (ii)  $A \times (B \cap C) = (A \times B) \cap (A \times C)$
- (iii)  $(A \cup B) \times C = (A \times C) \cup (B \times C)$

**Solution :**

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$$(i) A \times (B \cup C) = (A \times B) \cup (A \times C)$$

$$A = \{0, 1\}, B = \{2, 3, 4\}, C = \{3, 5\}$$

$$B \cup C = \{2, 3, 4\} \cup \{3, 5\}$$

$$= \{2, 3, 4, 5\}$$

$$A \times (B \cup C) = \{0, 1\} \times \{2, 3, 4, 5\}$$

$$= \{(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)\} \dots (1)$$

$$A \times B = \{0, 1\} \times \{2, 3, 4\}$$

$$= \{(0, 2), (0, 3), (0, 4), (1, 2), (1, 3), (1, 4)\}$$

$$A \times C = \{0, 1\} \times \{3, 5\}$$

$$= \{(0, 3), (0, 5), (1, 3), (1, 5)\}$$

$$(A \times B) \cup (A \times C) = \{(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)\} \dots (2)$$

From (1) and (2) we get,

$$A \times (B \cup C) = (A \times B) \cup (A \times C)$$

$$(ii) A \times (B \cap C) = (A \times B) \cap (A \times C)$$

$$B \cap C = \{2, 3, 4\} \cap \{3, 5\}$$

$$= \{3\}$$

$$A \times (B \cap C) = \{0, 1\} \times \{3\}$$

$$= \{(0, 3), (1, 3)\} \dots (3)$$

$$A \times B = \{0, 1\} \times \{2, 3, 4\}$$

$$= \{(0, 2), (0, 3), (0, 4), (1, 2), (1, 3), (1, 4)\}$$

$$A \times C = \{0, 1\} \times \{3, 5\}$$

$$= \{(0, 3), (0, 5), (1, 3), (1, 5)\}$$

$$(A \times B) \cap (A \times C) = \{(0, 3), (1, 3)\} \dots (4)$$

From (3) and (4) we get,

$$A \times (B \cap C) = (A \times B) \cap (A \times C).$$

$$(iii) (A \cup B) \times C = (A \times C) \cup (B \times C)$$

$$A \cup B = \{0, 1\} \cup \{2, 3, 4\}$$

$$= \{0, 1, 2, 3, 4\}$$

$$(A \cup B) \times C = \{0, 1, 2, 3, 4\} \times \{3, 5\}$$

$$= \{(0, 3), (0, 5), (1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5), (4, 3), (4, 5)\} \dots (5)$$

$$A \times C = \{0, 1\} \times \{3, 5\}$$

$$= \{(0, 3), (0, 5), (1, 3), (1, 5)\}$$

$$B \times C = \{2, 3, 4\} \times \{3, 5\}$$

$$= \{(2, 3), (2, 5), (3, 3), (3, 5), (4, 3), (4, 5)\}$$

$$(A \times C) \cup (B \times C) = \{(0, 3), (0, 5), (1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5), (4, 3), (4, 5)\} \quad \dots(6)$$

From (5) and (6) we get,

$$(A \cup B) \times C = (A \times C) \cup (B \times C).$$

Q. Let  $A$  be the set of all natural numbers less than 8,  $B$  be the set of all prime numbers less than 8 and  $C$  be the set of all even prime numbers. Verify that

$$(i) (A \cap B) \times C = (A \times C) \cap (B \times C)$$

$$(ii) A \times (B - C) = (A \times B) - (A \times C)$$

Solution :

$$A = \{1, 2, 3, 4, 5, 6, 7\}, \quad B = \{2, 3, 5, 7\}, \quad C = \{2\}$$

$$(i) (A \cap B) \times C = (A \times C) \cap (B \times C)$$

$$\begin{aligned} A \cap B &= \{1, 2, 3, 4, 5, 6, 7\} \cap \{2, 3, 5, 7\} \\ &= \{2, 3, 5, 7\} \end{aligned}$$

$$(A \cap B) \times C = \{2, 3, 5, 7\} \times \{2\}$$

$$= \{(2, 2), (3, 2), (5, 2), (7, 2)\} \quad \dots(1)$$

$$A \times C = \{1, 2, 3, 4, 5, 6, 7\} \times \{2\}$$

$$= \{(1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2), (7, 2)\}$$

$$B \times C = \{2, 3, 5, 7\} \times \{2\}$$

$$= \{(2, 2), (3, 2), (5, 2), (7, 2)\}$$

$$(A \times C) \cap (B \times C) = \{(2, 2), (3, 2), (5, 2), (7, 2)\} \quad \dots(2)$$

From (1) and (2) we get,

$$(A \cap B) \times C = (A \times C) \cap (B \times C)$$

$$(ii) A \times (B - C) = (A \times B) - (A \times C)$$

$$B - C = \{2, 3, 5, 7\} - \{2\}$$

$$B - C = \{3, 5, 7\}$$

$$A \times (B - C) = \{1, 2, 3, 4, 5, 6, 7\} \times \{3, 5, 7\}$$

$$\{(1, 3), (1, 5), (1, 7), (2, 3), (2, 5), (2, 7), (3, 3), (3, 5),$$

$$= (3, 7), (4, 3), (4, 5), (4, 7), (5, 3), (5, 5), (5, 7), (6, 3), \dots(3)$$

$$(6, 5), (6, 7), (7, 3), (7, 5), (7, 7)\}$$

$$A \times B = \{1, 2, 3, 4, 5, 6, 7\} \times \{2, 3, 5, 7\}$$

$$= \{(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7),$$

$$(3, 2), (3, 3), (3, 5), (3, 7), (4, 2), (4, 3), (4, 5), (4, 7),$$

$$(5, 2), (5, 3), (5, 5), (5, 7), (6, 2), (6, 3), (6, 5), (6, 7),$$

$$(7, 2), (7, 3), (7, 5), (7, 7)\}$$

$$A \times C = \{1, 2, 3, 4, 5, 6, 7\} \times \{2\}$$

$$= \{(1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2), (7, 2)\}$$

$$(A \times B) - (A \times C) = \{(1,3), (1,5), (1,7), (2,3), (2,5), (2,7), (3,3), (3,5), (3,7), (4,3), (4,5), (4,7), (5,3), (5,5), (5,7), (6,3), (6,5), (6,7), (7,3), (7,5), (7,7)\} \quad \dots (4)$$

From (3) and (4) we get,

$$A \times (B - C) = (A \times B) - (A \times C).$$

### EXERCISE 1.2

1. Let  $A = \{1, 2, 3, 7\}$  and  $B = \{3, 0, -1, 7\}$ . Which of the following are relation from  $A$  to  $B$ ?

- (i)  $R_1 = \{(2,1), (7,1)\}$
- (ii)  $R_2 = \{(-1,1)\}$
- (iii)  $R_3 = \{(2,-1), (7,7), (1,3)\}$
- (iv)  $R_4 = \{(7,-1), (0,3), (3,3), (0,7)\}$

**Solution**

$$A \times B = \{1, 2, 3, 7\} \times \{3, 0, -1, 7\}$$

$$A \times B = \{(1,3), (1,0), (1,-1), (1,7), (2,3), (2,0), (2,-1), (2,7), (3,3), (3,0), (3,-1), (3,7), (7,3), (7,0), (7,-1), (7,7)\}$$

- (i)  $R_1 = \{(2,1), (7,1)\}$

Here  $(2,1) \in R_1$ . But  $(2,1) \notin A \times B$ . So  $R_1$  is not a relation from  $A$  to  $B$ .

- (ii)  $R_2 = \{(-1,1)\}$

Here  $(-1,1) \in R_2$ . But  $(-1,1) \notin A \times B$ . So  $R_2$  is not a relation from  $A$  to  $B$ .

- (iii)  $R_3 = \{(2,-1), (7,7), (1,3)\}$

All the elements of  $R_3$  belong to  $A \times B$ . i.e.,  $R_3 \subseteq A \times B$ . Thus  $R_3$  is a relation from  $A$  to  $B$ .

- (iv)  $R_4 = \{(7,-1), (0,3), (3,3), (0,7)\}$

Here  $(0,3) \in R_4$  but  $(0,3) \notin A \times B$ .

So  $R_4$  is not a relation from  $A$  to  $B$ .

Important Example Sums:

**Note :** Even if a single element in  $R$  is not in  $A \times B$  then  $R$  is not a relation.

2. Let  $A = \{1, 2, 3, 4, \dots, 45\}$  and  $R$  be the relation defined as "is a square of" on  $A$ . Write the maximum possible  $R$  as a subset of  $A \times A$ . Also, find the domain and range of  $R$ .

**Solution :**

$$A = \{1, 2, 3, 4, \dots, 45\}$$

$R$  is a relation defined as "is a square of" on  $A$ . i.e.,  $x^2 R x$

$$1^2 R 1 \Rightarrow 1R1 \qquad 4^2 R 4 \Rightarrow 16R4$$

$$2^2 R 2 \Rightarrow 4R2 \qquad 5^2 R 5 \Rightarrow 25R5$$

$$3^2 R 3 \Rightarrow 9R3 \qquad 6^2 R 6 \Rightarrow 36R6$$

But for 7 to 45 the squares do not belong to  $A$ .

$$\therefore R = \{(1,1), (4,2), (9,3), (16,4), (25,5), (36,6)\}$$

$$\text{Domain of } R = \{1, 4, 9, 16, 25, 36\}$$

$$\text{Range of } R = \{1, 2, 3, 4, 5, 6\}$$

3. A Relation  $R$  is given by the set  $\{(x, y) / y = x + 3, x \in \{0, 1, 2, 3, 4, 5\}\}$ . Determine its domain and range.

**Solution :**

$$R = \{(x, y) / y = x + 3, x \in \{0, 1, 2, 3, 4, 5\}\}$$

Given  $y = x + 3$

Put  $x = 0, y = 0 + 3 = 3$

$$x = 1, y = 1 + 3 = 4$$

$$x = 2, y = 2 + 3 = 5$$

$$x = 3, y = 3 + 3 = 6$$

$$x = 4, y = 4 + 3 = 7$$

$$x = 5, y = 5 + 3 = 8$$

$$R = \{(0, 3), (1, 4), (2, 5), (3, 6), (4, 7), (5, 8)\}$$

Domain of  $R = \{0, 1, 2, 3, 4, 5\}$

Range of  $R = \{3, 4, 5, 6, 7, 8\}$ .

4. Represent each of the relation by

(a) an arrow diagram, (b) a graph and (c) a set in roster form, wherever possible.

(i)  $\{(x, y) | x = 2y, x \in \{2, 3, 4, 5\}, y \in \{1, 2, 3, 4\}\}$

(ii)  $\{(x, y) | y = x + 3, x, y \text{ are natural numbers } < 10\}$

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**Solution :**

(i)  $\{(x, y) | x = 2y, x \in \{2, 3, 4, 5\}, y \in \{1, 2, 3, 4\}\}$

Given  $x = 2y$

$$\Rightarrow y = \frac{x}{2}$$

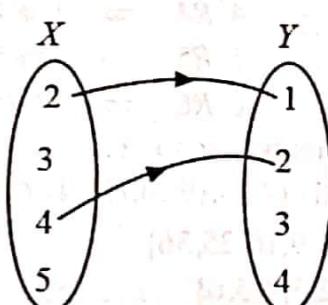
Put  $x = 2, y = \frac{2}{2} = 1$   $\times$   $A$  is  $\{2, 3, 4, 5\}$  and  $A$  is  $\{1, 2, 3, 4\}$  so  $i$

$x = 3, y = \frac{3}{2}$   $\times$   $A$  is  $\{2, 3, 4, 5\}$  and  $A$  is  $\{1, 2, 3, 4\}$  so  $i$

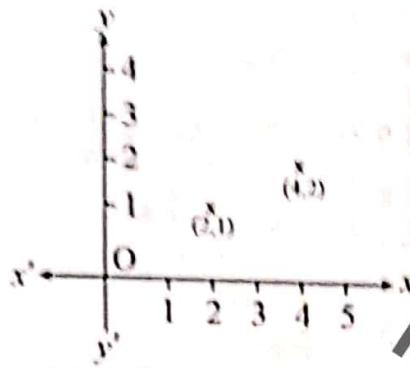
$x = 4, y = \frac{4}{2} = 2$   $\times$   $A$  is  $\{2, 3, 4, 5\}$  and  $A$  is  $\{1, 2, 3, 4\}$  so  $i$

$x = 5, y = \frac{5}{2}$   $\times$   $A$  is  $\{2, 3, 4, 5\}$  and  $A$  is  $\{1, 2, 3, 4\}$  so  $i$

(a) Arrow diagram :



(b) Graph : The graph consists of the points  $(2,1)$  and  $(4,2)$ .



(c) Roster form :

$$R = \{(2,1), (4,2)\}$$

(ii)  $\{(x,y) / y = x+3, x, y \text{ are natural numbers} < 10\}$

$$x \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$y \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

Given  $y = x+3$

Put  $x = 1, y = 1+3=4$

$$x = 2, y = 2+3=5$$

$$x = 3, y = 3+3=6$$

$$x = 4, y = 4+3=7$$

$$x = 5, y = 5+3=8$$

$$x = 6, y = 6+3=9$$

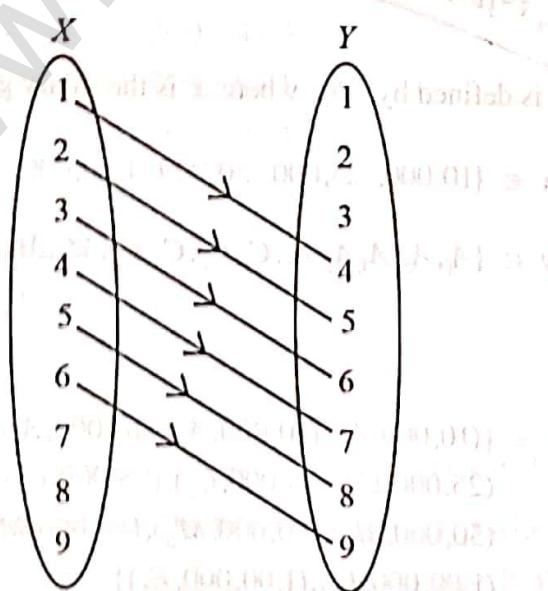
$$x = 7, y = 7+3=10$$

$$x = 8, y = 8+3=11$$

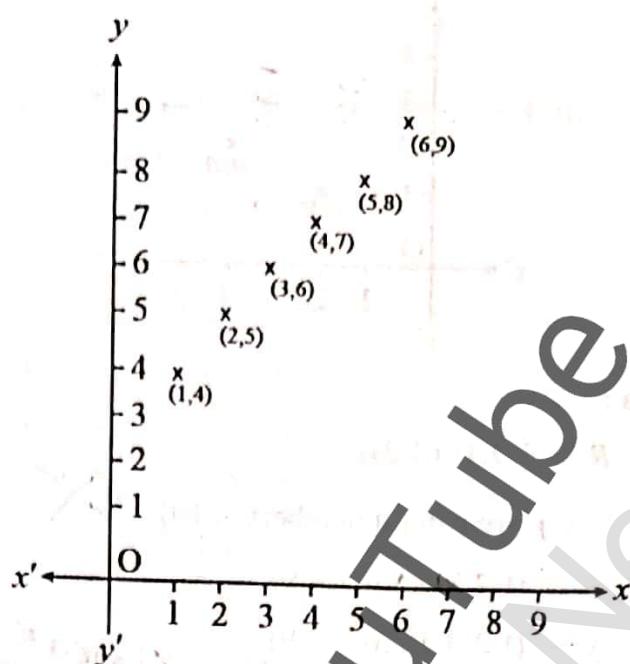
$$x = 9, y = 9+3=12$$

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(a) Arrow diagram :



(b) Graph :



(c) Roster form :

$$R = \{(1,4), (2,5), (3,6), (4,7), (5,8), (6,9)\}$$

5. A company has four categories of employees, namely Assistants ( $A$ ), Clerks ( $C$ ), Managers ( $M$ ) and an Executive Officer ( $E$ ). The company provide ₹ 10,000, ₹ 25,000, ₹ 50,000 and ₹ 1,00,000 as salaries to the people who work in the categories  $A$ ,  $C$ ,  $M$  and  $E$  respectively. The employees  $A_1, A_2, A_3, A_4$  and  $A_5$  are Assistants ;  $C_1, C_2, C_3, C_4$  are Clerks;  $M_1, M_2, M_3$  are managers and  $E_1, E_2$  were Executive officers. If the relation  $R$  is defined by  $xRy$ , where  $x$  is salary given to person  $y$ , express the relation  $R$  through a set of ordered pairs and by an arrow diagram.

**Solution :**

The relation ( $R$ ) is defined by  $xRy$  where  $x$  is the salary given to person  $y$ .

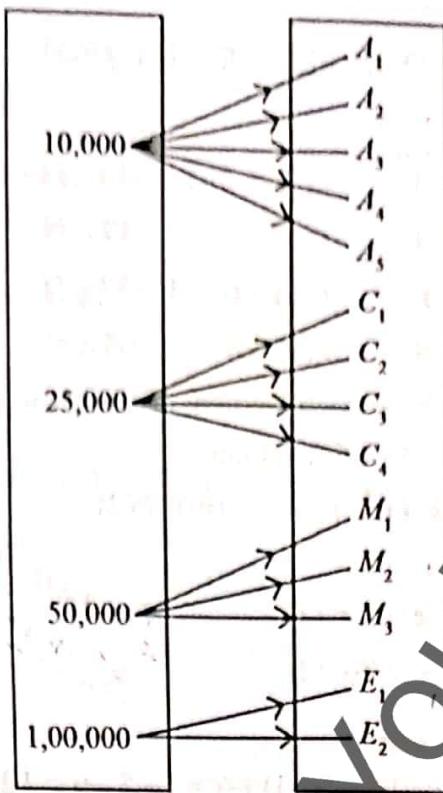
$$x \in \{10,000, 25,000, 50,000, 1,00,000\}$$

$$y \in \{A_1, A_2, A_3, A_4, A_5, C_1, C_2, C_3, C_4, M_1, M_2, M_3, E_1, E_2\}$$

**Ordered pair**

$$\begin{aligned} R = & \{(10,000, A_1), (10,000, A_2), (10,000, A_3), (10,000, A_4), (10,000, A_5) \\ & (25,000, C_1), (25,000, C_2), (25,000, C_3), (25,000, C_4) \\ & (50,000, M_1), (50,000, M_2), (50,000, M_3) \\ & (1,00,000, E_1), (1,00,000, E_2)\} \end{aligned}$$

**Arrow diagram :**



### EXERCISE 1.3

1. Let  $f = \{(x, y) | x, y \in \mathbb{N} \text{ and } y = 2x\}$  be a relation on  $\mathbb{N}$ . Find the domain, co-domain and range. Is this relation a function?

**Solution :**

$$f = \{(x, y) | x, y \in \mathbb{N} \text{ and } y = 2x\}$$

$$x \in X = \{1, 2, 3, 4, \dots\}$$

$$\text{Given } y = 2x$$

$$y \in Y = \{1, 2, 3, 4, \dots\}$$

$$\text{Put } x = 1, \quad y = 2(1) = 2 \in Y$$

$$x = 2, \quad y = 2(2) = 4 \in Y$$

$$x = 3, \quad y = 2(3) = 6 \in Y$$

$$x = 4, \quad y = 2(4) = 8 \in Y$$

$$f = \{(1, 2), (2, 4), (3, 6), (4, 8), \dots\}$$

$$\text{Domain} = \{1, 2, 3, 4, \dots\}$$

$$\text{Co-domain} = \{1, 2, 3, 4, \dots\}$$

$$\text{Range} = \{2, 4, 6, 8, \dots\}$$

$f$  is a function on  $\mathbb{N}$ .

$f : X \rightarrow Y$  is a function as each element of  $X$  has unique image in  $Y$ .

important Examples:

$$1.6, 1.7, 1.8 \rightarrow 2^M$$

2. Let  $X = \{3, 4, 6, 8\}$ . Determine whether the relation  $R = \{(x, f(x)) | x \in X, f(x) = x^2 + 1\}$  is a function from  $X$  to  $\mathbb{N}$  ?

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**Solution :**

Given,  $X = \{3, 4, 6, 8\}$ ,  $R = \{(x, f(x)), x \in X, f(x) = x^2 + 1\}$   
 Now  $f(x) = x^2 + 1$

$$\text{put } x = 3, f(3) = 3^2 + 1 = 10 \in \mathbb{N}$$

$$x = 4, f(4) = 4^2 + 1 = 17 \in \mathbb{N}$$

$$x = 6, f(6) = 6^2 + 1 = 37 \in \mathbb{N}$$

$$x = 8, f(8) = 8^2 + 1 = 65 \in \mathbb{N}$$

$R$  is a function from  $X$  to  $\mathbb{N}$  as each element of  $X$  has unique image in  $\mathbb{N}$ .

(3) For the function  $f : x \rightarrow x^2 - 5x + 6$ , evaluate

$$(i) f(-1)$$

$$(ii) f(2a)$$

$$(iii) f(2)$$

$$(iv) f(x-1)$$

**Solution :**

Given  $f : x \rightarrow x^2 - 5x + 6$

$$f(x) = x^2 - 5x + 6$$

$$(i) f(-1)$$

$$f(-1) = (-1)^2 - 5(-1) + 6 = 1 + 5 + 6 = 12.$$

$$(ii) f(2a)$$

$$f(2a) = (2a)^2 - 5(2a) + 6 = 4a^2 - 10a + 6$$

$$(iii) f(2)$$

$$f(2) = (2)^2 - 5(2) + 6 = 4 - 10 + 6 = 0.$$

$$(iv) f(x-1)$$

$$f(x-1) = (x-1)^2 - 5(x-1) + 6$$

$$f(x-1) = x^2 - 7x + 12$$

4. A graph representing a function  $f(x)$  is given in the diagram.

(i) Find the following :

$$f(0), f(7), f(2), f(10)$$

(ii) For what value of  $x$ ,  $f(x) = 1$ ?

(iii) Find the domain and the range of  $f(x)$

(iv) What is the image of 6 under  $f$ ?

**Solution :**

(i) From the given graph it is clear that

$$f(0) = 9$$

$$f(7) = 6$$

$$f(2) = 6$$

$$f(10) = 0$$

(ii) It is clear that

$$f(9.5) = 1$$

when  $x = 9.5$ ,  $f(x) = 1$

$$\text{Domain} = \{x / 0 \leq x \leq 10, x \in \mathbb{R}\}$$

$$\text{Range} = \{y / 0 \leq y \leq 9, y \in \mathbb{R}\}$$

(iv) From the graph it is clear that

$$f(6) = 5$$

∴ The image of 6 under  $f$  is 5.

Q5. Let  $f(x) = 2x + 5$ , If  $x \neq 0$  then find  $\frac{f(x+2)-f(2)}{x}$ .

Solution :

$$f(x) = 2x + 5$$

$$f(x+2) = 2x + 9$$

$$f(2) = 9$$

$$\frac{f(x+2)-f(2)}{x} = \frac{(2x+9)-9}{x} = \frac{2x}{x} = 2$$

Q6. A function  $f$  is defined by  $f(x) = 2x - 3$ . Find

$$(i) \frac{f(0)+f(1)}{2}$$

$$(ii) x \text{ such that } f(x) = 0.$$

$$(iii) x \text{ such that } f(x) = x.$$

$$(iv) x \text{ such that } f(x) = f(1-x).$$

Solution :

$$\text{Given } f(x) = 2x - 3$$

$$(i) \frac{f(0)+f(1)}{2}$$

$$f(0) = 2(0) - 3 = -3$$

$$f(1) = 2(1) - 3 = -1$$

$$\frac{f(0)+f(1)}{2} = \frac{-3+(-1)}{2} = -2$$

$$(ii) f(x) = 0$$

$$\text{i.e., } 2x - 3 = 0$$

$$2x = 3$$

$$x = \frac{3}{2}$$

$$(iii) f(x) = x$$

$$\text{i.e., } 2x - 3 = x$$

$$2x - x = 3$$

$$x = 3$$

$$(iv) f(x) = f(1-x)$$

$$\text{i.e., } 2x - 3 = 2(1-x) - 3$$

$$2x - 3 = 2 - 2x - 3$$

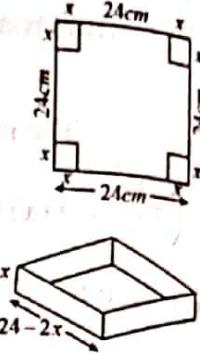
$$4x = 2$$

$$x = \frac{1}{2}$$

7. An open box is to be made from a square piece of material, whose length of a side is 24 cm. By cutting equal squares from the corners and turning up the sides as shown in the figure. Express the volume V of the box as a function of  $x$ .

**Solution**

The side of the square material = 24 cm.



Let  $x$  be the length of the side of the square to be removed.

$$\begin{aligned}\text{Length of the box} &= 24 - x - x \\ &= (24 - 2x)\end{aligned}$$

$$\text{Breadth of the box} = (24 - 2x)$$

$$\text{Height of the box} = x$$

$$\text{Volume of cuboid} = lbh$$

$$\begin{aligned}\text{Volume of box} &= (24 - 2x)(24 - 2x)x \\ &= (24 - 2x)(24x - 2x^2) \\ &= 576x - 48x^2 - 48x^2 + 4x^3 \\ &= 4x^3 - 96x^2 + 576x\end{aligned}$$

Volume of box is  $(4x^3 - 96x^2 + 576x)$  cu.cm. i.e.,  $V(x) = 4x^3 - 96x^2 + 576x$ .

8. A function  $f$  is defined by  $f(x) = 3 - 2x$ . Find  $x$  such that  $f(x^2) = (f(x))^2$ .

**Solution :**

$$\text{Given, } f(x^2) = [f(x)]^2$$

$$3 - 2x^2 = (3 - 2x)^2$$

$$\text{i.e., } 3 - 2x^2 = 9 - 12x + 4x^2$$

$$6x^2 - 12x + 6 = 0$$

$$\text{i.e., } x^2 - 2x + 1 = 0$$

$$(x-1)(x-1) = 0$$

$$x = 1, 1$$

$\therefore$  The value of  $x$  is 1.

9. A plane is flying at a speed of 500 km per hour. Express the distance  $d$  travelled by the plane as function of time  $t$  in hours.

**Solution**

$$\text{speed of the plane} = 500 \text{ km/hr}$$

$$\text{time} = t \text{ hrs}$$

$$\text{distance} = d \text{ km}$$

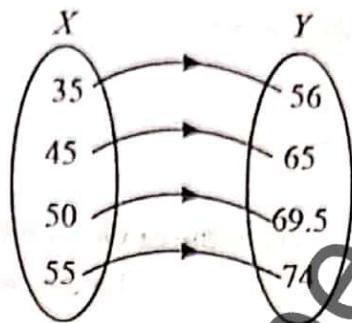
$$\text{distance} = \text{speed} \times \text{time}$$

$$= 500 \times t$$

$$d = 500t \quad \text{i.e., } d(t) = 500t$$

10. The data given in the table depicts the length of a person's forehand and their corresponding height. Based on this data, a student finds a relationship between the height ( $y$ ) and the forehand length ( $x$ ) as  $y = ax + b$ , where  $a, b$  are constants.

Length ' $x$ ' of a forehand (in cm)	Height ' $y$ ' (in inches)
35	56
45	65
50	69.5
55	74



- (i) Check if this relation is a function.
- (ii) Find  $a$  and  $b$ .
- (iii) Find the height of a person whose forehand length is 40 cm.
- (iv) Find the length of forehand of a person if the height is 60.54 inches.

### Solution

(i) Let  $X$  be the set of lengths of forehands and  $Y$  be the set of their height.  
 $\therefore x \in X, y \in Y$ .

From the table,

For each person's forehand length ( $x$ ), their corresponds to a unique person's height ( $y$ ).

$\therefore$  this relation is a function.

(ii)

$$y = ax + b$$

$$\text{Put } x = 35, y = 56$$

$$56 = a(35) + b$$

$$35a + b = 56 \quad \dots (1)$$

$$\text{Again put } x = 45, y = 65$$

$$65 = a(45) + b$$

$$45a + b = 65 \quad \dots (2)$$

$$(1) - (2) \Rightarrow -10a = -9$$

$$a = \frac{9}{10}$$

$$\text{Put } a = \frac{9}{10} \text{ in (1)}$$

$$35\left(\frac{9}{10}\right) + b = 56$$

$$b = 56 - \frac{315}{10}$$

$$b = 56 - 31.5$$

$$= 24.5$$

$$\text{Thus } a = 0.9, b = 24.5$$

(iii) Length of forehand = 40 cm

Find  $y$  when  $x = 40$ We know that  $y = 0.9x + 24.5$ Put  $x = 40$ 

$$y = (0.9)(40) + 24.5$$

$$y = 36.0 + 24.5$$

$$y = 60.5$$

Height of the person is 60.5 inches.

(iv) Find  $x$  when  $y = 53.3$ 

$$y = 0.9x + 24.5$$

Put  $y = 53.3$ 

$$53.3 = (0.9)x + 24.5$$

$$53.3 - 24.5 = 0.9x$$

$$0.9x = 28.8$$

$$x = 32$$

Length of forehand is 32 cm.

**EXERCISE 1.4**

- I. Determine whether the graph given below represent functions. Give reason.

**Solution**

(i) The given graph does not represent a function as the vertical line meets the curve at more than one point i.e., at two points.

(ii)

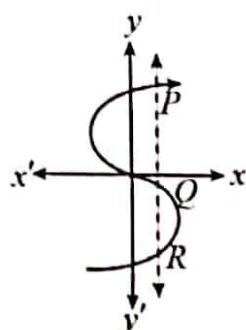
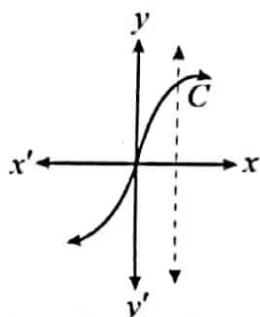
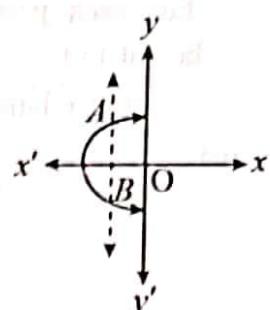
(ii) The given graph represents a function as any vertical line meets the curve at only one point.

**Important Example Summary** $1.11 \rightarrow 5M, 1.18 \rightarrow 5M,$ 

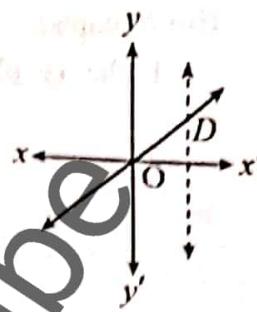
Types of functions - definition - 2M.

 $1.13, 1.14, 1.15 \rightarrow 2M,$ 

1.19 (iii) The given graph does not represent a function as the vertical line meets the curve at more than one point i.e., at three points.



- (iv) The given graph represents a function as any vertical line meets the curve at only one point.



2. Let  $f : A \rightarrow B$  be a function defined by  $f(x) = \frac{x}{2} - 1$ , where  $A = \{2, 4, 6, 10, 12\}$ ,  $B = \{0, 1, 2, 4, 5, 9\}$ . Represent  $f$  by

- (i) a set of ordered pairs   (ii) a table   (iii) an arrow diagram   (iv) a graph

Solution

$$A = \{2, 4, 6, 10, 12\}, \quad B = \{0, 1, 2, 4, 5, 9\}$$

$$f(x) = \frac{x}{2} - 1$$

$$\text{Put } x = 2, \quad f(2) = \frac{2}{2} - 1 = 1 - 1 = 0$$

$$x = 4, \quad f(4) = \frac{4}{2} - 1 = 2 - 1 = 1$$

$$x = 6, \quad f(6) = \frac{6}{2} - 1 = 3 - 1 = 2$$

$$x = 10, \quad f(10) = \frac{10}{2} - 1 = 5 - 1 = 4$$

$$x = 12, \quad f(12) = \frac{12}{2} - 1 = 6 - 1 = 5$$

- (i) Set of ordered pairs :

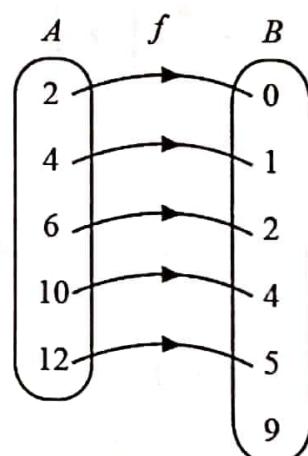
$$f = \{(2, 0), (4, 1), (6, 2), (10, 4), (12, 5)\}$$

- (ii) A table :

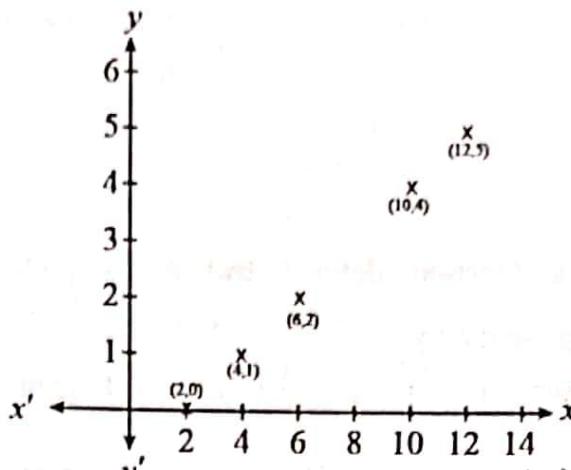
$x$	2	4	6	10	12
$f(x)$	0	1	2	4	5

- (iii) An arrow diagram :

- The function  $f : A \rightarrow B$  can be represented as an arrow diagram as in the figure



(iv) A graph :

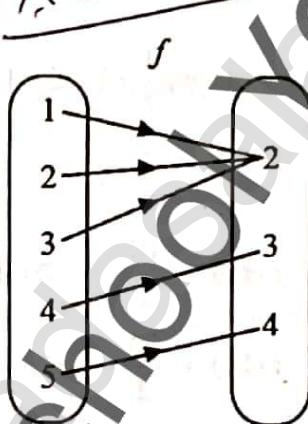
In the  $xy$ -plane plot the points  $(2,0), (4,1), (6,2), (10,4), (12,5)$ .

3. Represent the function  $f = \{(1,2), (2,2), (3,2), (4,3), (5,4)\}$  through

- (i) an arrow diagram      (ii) a table form      (iii) a graph

**Solution**

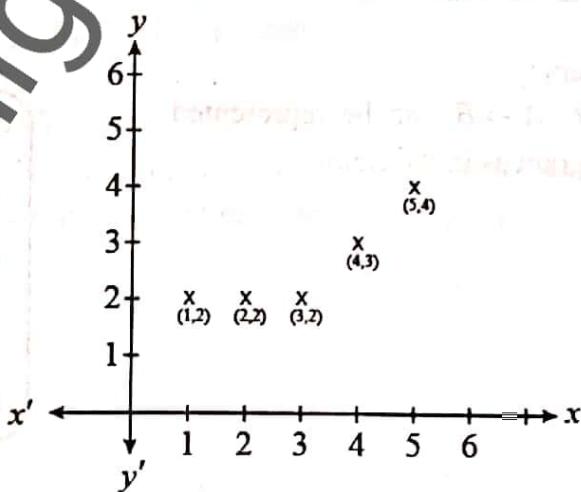
(i) Arrow diagram

~~SMARK Qn~~

(ii) A table form :

$x$	1	2	3	4	5
$f(x)$	2	2	2	3	4

(iii) A graph :

In the  $xy$ -plane, plot the points  $(1,2), (2,2), (3,2), (4,3), (5,4)$ .

4) Show that the function  $f : \mathbb{N} \rightarrow \mathbb{N}$  defined by  $f(x) = 2x - 1$  is one-one but not onto.

**Solution**

$f : \mathbb{N} \rightarrow \mathbb{N}$  is defined by  $f(x) = 2x - 1$

$$\mathbb{N} = \{1, 2, 3, 4, \dots\}$$

$$\text{Put } x = 1, f(1) = 2(1) - 1 = 1$$

$$x = 2, f(2) = 2(2) - 1 = 3$$

$$x = 3, f(3) = 2(3) - 1 = 5$$

$$x = 4, f(4) = 2(4) - 1 = 7$$

and so on.

Different elements in the domain  $\mathbb{N}$  have different images in the co-domain.

$\therefore f : \mathbb{N} \rightarrow \mathbb{N}$  is one-one function.

$$\text{Range} = \{1, 3, 5, 7, \dots\}$$

$$\text{Range} \neq \text{co-domain}$$

$f$  is not onto function.

$f : \mathbb{N} \rightarrow \mathbb{N}$  is one-one but not onto function.

5) Show that the function  $f : \mathbb{N} \rightarrow \mathbb{N}$  defined by  $f(m) = m^2 + m + 3$  is one-one function.

**Solution**

$f : \mathbb{N} \rightarrow \mathbb{N}$  is defined by

$$f(m) = m^2 + m + 3$$

$$\mathbb{N} = \{1, 2, 3, 4, \dots\}$$

$$\text{Put } m = 1, f(1) = 1^2 + 1 + 3 = 5 \in \mathbb{N}$$

$$m = 2, f(2) = 2^2 + 2 + 3 = 9 \in \mathbb{N}$$

$$m = 3, f(3) = 3^2 + 3 + 3 = 15 \in \mathbb{N}$$

$$m = 4, f(4) = 4^2 + 4 + 3 = 23 \in \mathbb{N}$$

and so on.

Different elements in the domain  $\mathbb{N}$  has different images in the co-domain  $\mathbb{N}$ .

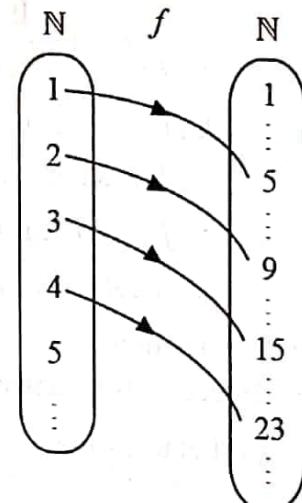
$\therefore f : \mathbb{N} \rightarrow \mathbb{N}$  is a one-one function.

2 Marks

6) Let  $A = \{1, 2, 3, 4\}$  and  $B = \mathbb{N}$ . Let  $f : A \rightarrow B$  be defined by  $f(x) = x^3$ . Then,

(i) find the range of  $f$

(ii) identify the type of function



**Solution**

(i)  $A = \{1, 2, 3, 4\}$

$B = \mathbb{N} = \{1, 2, 3, 4, \dots\}$

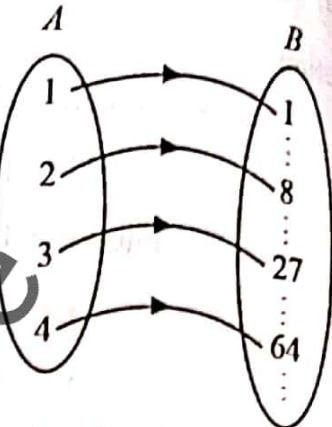
Put  $x = 1, f(1) = 1^3 = 1 \in \mathbb{N}$

$x = 2, f(2) = 2^3 = 8 \in \mathbb{N}$

$x = 3, f(3) = 3^3 = 27 \in \mathbb{N}$

$x = 4, f(4) = 4^3 = 64 \in \mathbb{N}$

Range = {1, 8, 27, 64}



$f: A \rightarrow B$  is one-one function since different elements in the domain  $A$  have different images in co-domain  $B$ .

Range = {1, 8, 27, 64}

Range  $\neq$  co-domain  $B(\mathbb{N})$

So  $f$  is not onto function, that is into function.

$\therefore f: A \rightarrow B$  is one-one and into function.

7. In each of the following cases, state whether the given functions are bijective or not.

- (i)  $f: \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = 2x + 1$  (ii)  $f: \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = 3 - 4x^2$

**Solution**

(i) Let us assume

$f(x_1) = f(x_2)$

$2x_1 + 1 = 2x_2 + 1$

$2x_1 = 2x_2$

$x_1 = x_2$

If  $f(x_1) = f(x_2)$  then  $x_1 = x_2$

$\therefore f: \mathbb{R} \rightarrow \mathbb{R}$  is one-one function.

All the elements in the co-domain  $\mathbb{R}$  have pre-images in the domain  $\mathbb{R}$

$\therefore f$  is an onto function.

$\therefore f: \mathbb{R} \rightarrow \mathbb{R}$  is both one-one and onto function.

$\Rightarrow f: \mathbb{R} \rightarrow \mathbb{R}$  is a bijective function.

**Note :** The function  $f(x) = 2x + 1$  is linear and trivially  $f$  is one-one on  $\mathbb{R}$ . Further the range is also  $\mathbb{R}$ . Hence  $f$  is bijective.

(ii) Let us assume

$f(x_1) = f(x_2)$

$3 - 4x_1^2 = 3 - 4x_2^2$

$4x_1^2 = 4x_2^2$

$x_1^2 = x_2^2$

Taking square root on both sides

$x_1 = \pm x_2$

i.e.,  $x_1 = x_2 \Rightarrow x_1 = -x_2$   $\therefore x_1 \neq x_2$

$\therefore x_1 \neq x_2$

If  $f(x_1) = f(x_2)$  then  $x_1 \neq x_2$

$f$  is not one-one function and hence not bijective function.

**Note :** The given function is non-linear of degree 2 and hence not bijective on  $\mathbb{R}$ .

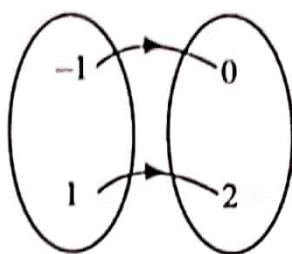
8. Let  $A = \{-1, 1\}$  and  $B = \{0, 2\}$ . If the function  $f : A \rightarrow B$  defined by  $f(x) = ax + b$  is an onto function, find  $a$  and  $b$ .

**Solution**

Given,  $A = \{-1, 1\}$ ,  $B = \{0, 2\}$

Since  $f$  is an onto function, there are two possibilities

(i)



$$f(-1) = 0 \text{ and}$$

$$f(1) = 2$$

$$f(x) = ax + b$$

$$f(-1) = a(-1) + b$$

$$0 = -a + b$$

$$\therefore a = b$$

$$f(1) = a(1) + b$$

$$2 = a + b$$

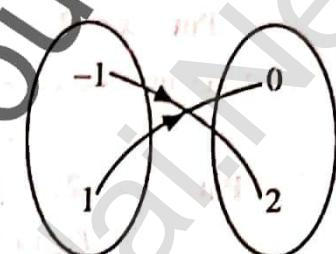
$$2 = a + a (\because a = b)$$

$$2 = 2a$$

$$a = 1$$

$$a = 1, b = 1 \text{ (or) } a = -1, b = 1$$

(ii)



$$f(-1) = 2 \text{ and}$$

$$f(1) = 0$$

$$f(x) = a(1) + b$$

$$0 = a + b$$

$$-a = b \quad (1)$$

$$f(-1) = a(-1) + b$$

$$2 = -a + b$$

$$2 = b + b \quad (\because -a = b)$$

$$2b = 2 \Rightarrow b = 1$$

$$\text{Now, } -a = b \Rightarrow -a = 1$$

$$\therefore a = -1$$

9. If the function  $f$  is defined by  $f(x) = \begin{cases} x+2 & \text{if } x > 1 \\ 2 & \text{if } -1 \leq x \leq 1 \\ x-1 & \text{if } -3 < x < -1 \end{cases}$ ; find the values of

5 Mark Qns.

(i)  $f(3)$

(ii)  $f(0)$

(iii)  $f(-1.5)$

(iv)  $f(2) + f(-2)$

**Solution**

$$f(x) = \begin{cases} x+2 & ; x > 1 \\ 2 & ; -1 \leq x \leq 1 \\ x-1 & ; -3 < x < -1 \end{cases}$$

(i)  $x = 3$  lies in the interval  $x > 1$

$$\therefore f(x) = x+2$$

$$\text{Put } x = 3, \quad f(3) = 3+2 = 5$$

(ii)  $x = 0$  lies in interval  $-1 \leq x \leq 1$

$$\therefore f(x) = 2$$

$$\text{Put } x = 0, \quad f(0) = 2$$

(iii)  $x = -1.5$  lies in the interval  $-3 < x < -1$

$$\therefore f(x) = x-1$$

$$\text{Put } x = -1.5, \quad f(-1.5) = -1.5-1 = -2.5$$

(iv)  $x = 2$  lies in  $x > 1$

$$f(x) = x+2$$

$$\text{Put } x = 2, \quad f(2) = 2+2 = 4$$

$x = -2$  lies in  $-3 < x < -1$

$$\therefore f(x) = x-1$$

$$\text{Put } x = -2, \quad f(-2) = -2-1 = -3$$

$$f(2)+f(-2) = 4-3 = 1$$

(10) A function  $f : [-5, 9] \rightarrow \mathbb{R}$  is defined as follows:

$$f(x) = \begin{cases} 6x+1 & \text{if } -5 \leq x \leq 2 \\ 5x^2-1 & \text{if } 2 \leq x \leq 6 \\ 3x-4 & \text{if } 6 \leq x \leq 9 \end{cases}$$

X Smart Qns:

Find (i)  $f(-3)+f(2)$  (ii)  $f(7)=f(1)$  (iii)  $2f(4)+f(8)$  (iv)  $\frac{2f(-2)-f(6)}{f(4)+f(-2)}$

**Remark :** Closed interval is not defined.

**Solution**

$$(i) f(-3)+f(2)$$

$x = -3$  lies in  $-5 \leq x \leq 2$ ,

$$\therefore f(x) = 6x+1$$

$$\text{Put } x = -3, \quad f(-3) = 6(-3)+1 = -18+1 = -17$$

$x = 2$  lies in the interval  $2 \leq x \leq 6$

$$\therefore f(x) = 5x^2-1$$

$$\text{Put } x = 2, \quad f(2) = 5(2)^2-1 = 5(4)-1 = 20-1 = 19$$

$$f(-3)+f(2) = -17+19 = 2$$

$$(ii) f(7)=f(1)$$

$x = 7$  lies in the interval  $6 \leq x \leq 9$ ,

$$\therefore f(x) = 3x-4$$

$$\text{Put } x = 7, \quad f(7) = 3(7)-4 = 21-4 = 17$$

$x = 1$  lies in the interval  $-5 \leq x < 2$

$$\therefore f(x) = 6x + 1$$

$$\text{Put } x = 1, \quad f(1) = 6(1) + 1$$

$$f(1) = 7$$

$$\therefore f(7) - f(1) = 17 - 7$$

$$f(7) - f(1) = 10$$

$$\text{(iii)} \quad 2f(4) + f(8)$$

$x = 4$  lies in the interval  $2 \leq x < 6$

$$\therefore f(x) = 5x^2 - 1$$

$$\text{Put } x = 4, \quad f(4) = 5(4)^2 - 1$$

$$= 5(16) - 1$$

$$= 80 - 1$$

$$f(4) = 79$$

$x = 8$  lies in the interval  $6 \leq x \leq 9$

$$\therefore f(x) = 3x - 4$$

$$\text{Put } x = 8, \quad f(8) = 3(8) - 4$$

$$= 24 - 4$$

$$f(8) = 20$$

$$2f(4) + f(8) = 2(79) + 20$$

$$= 158 + 20$$

$$2f(4) + f(8) = 178$$

$$\text{(iv)} \quad \frac{2f(-2) - f(6)}{f(4) + f(-2)}$$

$x = -2$  lies in the interval  $-5 \leq x < 2$

$$\therefore f(x) = 6x + 1$$

$$\text{Put } x = -2, \quad f(-2) = 6(-2) + 1$$

$$= -12 + 1$$

$$f(-2) = -11$$

$x = 6$  lies in interval  $6 \leq x \leq 9$

$$\therefore f(x) = 3x - 4$$

$$\text{Put } x = 6, \quad f(6) = 3(6) - 4$$

$$= 18 - 4$$

$$f(6) = 14$$

$$f(4) = 79 \quad (\text{by (iii)})$$

$$\begin{aligned}\frac{2f(-2)-f(6)}{f(4)+f(-2)} &= \frac{2(-11)-14}{79+(-11)} \\ &= \frac{-22-14}{79-11} = \frac{-36}{68} \\ \frac{2f(-2)-f(6)}{f(4)+f(-2)} &= \frac{-9}{17}\end{aligned}$$

11. An object travels a distance  $s$  under the influence of gravity in time  $t$  seconds is given by

$$s(t) = \frac{1}{2}gt^2 + at + b, \text{ where } (g \text{ is the acceleration due to gravity), } a \text{ and } b \text{ are constants.}$$

Check if the function  $s(t)$  is one-one.

**Solution**

Let us assume

$$s(t_1) = s(t_2)$$

$$\frac{1}{2}gt_1^2 + at_1 + b = \frac{1}{2}gt_2^2 + at_2 + b$$

$$\frac{1}{2}g(t_1^2 - t_2^2) + a(t_1 - t_2) = 0$$

$$\frac{1}{2}g(t_1 + t_2)(t_1 - t_2) + a(t_1 - t_2) = 0$$

$$(t_1 - t_2) \left[ \frac{1}{2}g(t_1 + t_2) + a \right] = 0$$

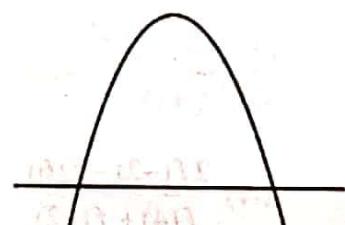
$$t_1 - t_2 = 0 \quad (\text{or}) \quad \frac{1}{2}g(t_1 + t_2) + a = 0$$

$$t_1 = t_2 \quad (\text{or}) \quad \frac{1}{2}g(t_1 + t_2) = -a$$

$$t_1 + t_2 = \frac{-2a}{g}$$

$$t_1 = \frac{-2a}{g} - t_2$$

$$t_1 \neq t_2$$



If  $s(t_1) = s(t_2)$  then  $t_1$  need not be equal to  $t_2$ .

Horizontal line test

$\therefore s(t)$  is not one-one.

**Note :** The graph of the function is open downward parabola as given in the diagram. By horizontal line test, the function is not one-one.

12. The function ' $t$ ' which maps temperature in Celsius ( $C$ ) into in Fahrenheit ( $F$ ) is defined by

$$t(C) = F \text{ where } F = \frac{9}{5}C + 32. \text{ Find,}$$

$$(i) t(0) \quad (ii) t(28) \quad (iii) t(-10)$$

$$(iv) \text{ the value of } C \text{ when } F = 212$$

$$(v) \text{ the temperature when the Celsius value is equal to the Fahrenheit value}$$

**Solution**

$$(i) t(0)$$

$$t(C) = F \text{ where } F = \frac{9}{5}C + 32$$

$$\text{i.e., } t(C) = \frac{9}{5}C + 32 \quad \dots (1)$$

Put  $C = 0$  in (1)

$$\begin{aligned} t(0) &= \frac{9}{5}(0) + 32 \\ &= 0 + 32 \end{aligned}$$

$$t(0) = 32^\circ F$$

(ii)  $t(28)$

Put  $C = 28$  in (1)

$$\begin{aligned} t(28) &= \frac{9}{5}(28) + 32 \\ &= \frac{252}{5} + 32 \\ &= 50.4 + 32 \end{aligned}$$

$$t(28) = 82.4^\circ F$$

(iii)  $t(-10)$

Put  $C = -10$  in (1)

$$\begin{aligned} t(-10) &= \frac{9}{5}(-10) + 32 \\ &= -18 + 32 \end{aligned}$$

$$t(-10) = 14^\circ F$$

(iv) the value of  $C$  when  $F = 212$

$$\frac{9}{5}C + 32 = 212$$

$$\frac{9}{5}C = 212 - 32$$

$$\frac{9}{5}C = 180$$

$$C = 180 \times \frac{5}{9}$$

$$C = 100^\circ$$

(v) Since the temperature is same for Celsius value and Fahrenheit value,

$$F = C$$

$$\frac{9}{5}C + 32 = C$$

$$\frac{9C - 5C}{5} = -32$$

$$4C = -32 \times 5$$

$$4C = -160$$

$$C = \frac{-160}{4}$$

$$C = -40^\circ$$

Celsius value is equal to Fahrenheit value at  $-40^\circ$ , i.e.,  $-40^\circ F = -40^\circ C$ .

### EXERCISE 1.5

1. Using the functions  $f$  and  $g$  defined given below, find  $f \circ g$  and  $g \circ f$ . Check whether  $f \circ g = g \circ f$ .

(i)  $f(x) = x - 6, g(x) = x^2$

~~X~~ 5mark Ans

(ii)  $f(x) = \frac{2}{x}, g(x) = 2x^2 - 1$

(iii)  $f(x) = \frac{x+6}{3}, g(x) = 3-x$

(iv)  $f(x) = 3+x, g(x) = x-4$

(v)  $f(x) = 4x^2 - 1, g(x) = 1+x$

**Solution**

(i)  $f(x) = x - 6, g(x) = x^2$

$(f \circ g)(x) = f(g(x))$

$= f(x^2)$

$(f \circ g)(x) = x^2 - 6$

1.24, 1.25 - 5mark ... (1)

$(g \circ f)x = g(f(x))$

$= g(x-6)$

$= (x-6)^2$

$= x^2 - 2(x)(6) + 6^2$

$(g \circ f)x = x^2 - 12x + 36$

... (2)

From (1) and (2)

$(f \circ g)(x) \neq (g \circ f)(x)$

(ii)  $f(x) = \frac{2}{x}, g(x) = 2x^2 - 1$

$(f \circ g)(x) = f(g(x))$

$= f(2x^2 - 1)$

$= \frac{2}{2x^2 - 1}$

... (1)

$(g \circ f)x = g(f(x))$

$= g\left(\frac{2}{x}\right)$

$= 2\left(\frac{2}{x}\right)^2 - 1$

$= 2\left(\frac{4}{x^2}\right) - 1$

$$(g \circ f)x = \frac{8}{x^2} - 1$$

$$(f \circ g)(x) \neq (g \circ f)x$$

... (2)

(iii)  $f(x) = \frac{x+6}{3}, g(x) = 3-x$

$$(f \circ g)(x) = f(g(x))$$

$$= f(3-x)$$

$$= \frac{(3-x)+6}{3}$$

$$(f \circ g)(x) = \frac{9-x}{3}$$

... (1)

$$(g \circ f)x = g(f(x))$$

$$= g\left(\frac{x+6}{3}\right)$$

$$= 3 - \left(\frac{x+6}{3}\right)$$

$$= \frac{9-(x+6)}{3}$$

$$= \frac{9-x-6}{3}$$

$$(g \circ f)x = \frac{3-x}{3}$$

.. (2)

From (1) and (2)

(iv)  $f(x) = 3+x, g(x) = x-4$

$$(f \circ g)(x) \neq (g \circ f)x$$

$$(f \circ g)(x) = f(g(x))$$

$$= f(x-4)$$

$$= 3+(x-4)$$

$$(f \circ g)(x) = x-1$$

... (1)

$$(g \circ f)x = g(f(x))$$

$$= g(3+x)$$

$$= (3+x)-4$$

$$= x-1$$

... (2)

From (1) and (2)

$$(f \circ g)(x) = (g \circ f)x$$

(v)  $f(x) = 4x^2 - 1, g(x) = 1+x$

$$(f \circ g)(x) = f(g(x))$$

$$= f(1+x)$$

$$\begin{aligned}
 &= 4(1+x)^2 - 1 \\
 &= 4(1+2x+x^2) - 1 \\
 (f \circ g)(x) &= 4x^2 + 8x + 3 \quad \dots (1) \\
 (g \circ f)x &= g(f(x)) \\
 &= g(4x^2 - 1) \\
 &= 1 + (4x^2 - 1) \\
 &= 4x^2 \\
 (g \circ f)x &= 4x^2 \quad \dots (2)
 \end{aligned}$$

From (1) and (2)

Q. Find the values of  $k$  for the given functions if  $f \circ g = g \circ f$ .

- (i)  $f(x) = 3x + 2, g(x) = 6x - k$       (ii)  $f(x) = 2x - k, g(x) = 4x + 5$

Solution

$$\begin{aligned}
 f(x) &= 3x + 2 \\
 g(x) &= 6x - k \quad \text{X. Same k Sol} \\
 (f \circ g)(x) &= f(g(x)) \\
 &= f(6x - k) \\
 &= 3(6x - k) + 2 \\
 (f \circ g)(x) &= 18x - 3k + 2 \quad \dots (1) \\
 (g \circ f)x &= g(f(x)) \\
 &= g(3x + 2) \\
 &= 6(3x + 2) - k \\
 (g \circ f)x &= 18x + 12 - k \quad \dots (2)
 \end{aligned}$$

Given,  $f \circ g = g \circ f$

From (1) and (2)

$$\begin{aligned}
 18x - 3k + 2 &= 18x + 12 - k \\
 -3k + k &= 12 - 2 \\
 -2k &= 10 \\
 k &= -5
 \end{aligned}$$

- (i)  $f(x) = 2x - k, g(x) = 4x + 5$

$$\begin{aligned}
 (f \circ g)(x) &= f(g(x)) \\
 &= f(4x + 5) \\
 &= 2(4x + 5) - k
 \end{aligned}$$

$$(f \circ g)(x) = 8x + 10 - k \quad \dots (1)$$

$$\begin{aligned}
 (g \circ f)x &= g(f(x)) \\
 &= g(2x - k) \\
 &= 4(2x - k) + 5
 \end{aligned}$$

$$(g \circ f)x = 8x - 4k + 5 \quad \dots (2)$$

Given,  $f \circ g = g \circ f$

From (1) and (2)

$$\begin{aligned} 8x + 10 - k &= 8x - 4k + 5 \\ 10 - 5 &= -4k + k \\ 5 &= -3k \\ k &= \frac{-5}{3} \end{aligned}$$

3. If  $f(x) = 2x - 1$ ,  $g(x) = \frac{x+1}{2}$ , show that  $(f \circ g)(x) = (g \circ f)(x) = x$

Solution

~~Smart Qns~~

$$\begin{aligned} f(x) &= 2x - 1 \\ g(x) &= \frac{x+1}{2} \\ (f \circ g)(x) &= f(g(x)) \\ &= f\left(\frac{x+1}{2}\right) \\ &= 2\left(\frac{x+1}{2}\right) - 1 \\ &= x + 1 - 1 \\ (f \circ g)(x) &= x \quad \dots (1) \\ (g \circ f)x &= g(f(x)) \\ &= g(2x - 1) \\ &= \frac{2x - 1 + 1}{2} \\ &= \frac{2x}{2} \\ (g \circ f)x &= x \quad \dots (2) \end{aligned}$$

From (1) and (2)

$$(f \circ g)(x) = (g \circ f)(x) = x$$

4. If  $f(x) = x^2 - 1$ ,  $g(x) = x - 2$  and  $(g \circ f)(a) = 1$ , find the values of  $a$ . ~~2 Mark~~

Solution

$$\begin{aligned} f(x) &= x^2 - 1 \\ g(x) &= x - 2 \\ g(f(a)) &= 1 \\ g(a^2 - 1) &= 1 \\ (a^2 - 1) - 2 &= 1 \\ a^2 - 3 &= 1 \\ a^2 &= 4 \\ a &= \pm 2 \end{aligned}$$

5. Let  $A, B, C \subseteq \mathbb{N}$ . Let the functions  $f: A \rightarrow B$  be defined by  $f(x) = 2x + 1$ , and  $g: B \rightarrow C$  be defined by  $g(x) = x^2$ . Find the range of  $f \circ g$  and  $g \circ f$ .

**Solution**

$$(f \circ g)(x) = f(g(x))$$

$$= f(x^2)$$

$$(f \circ g)(x) = 2(x^2) + 1 = 2x^2 + 1$$

Range of  $f \circ g = \{y / y = 2x^2 + 1, x \in \mathbb{N}\}$  or  $\{2x^2 + 1 / x \in \mathbb{N}\}$

$$(g \circ f)x = g(f(x))$$

$$= g(2x + 1)$$

$$= (2x + 1)^2$$

Range of  $g \circ f = \{y / y = (2x + 1)^2, x \in \mathbb{N}\}$

6. If  $f(x) = x^2 - 1$ , find (i)  $f \circ f$  (ii)  $f \circ f \circ f$

**Solution**

(i)

$$f(x) = x^2 - 1$$

$$(f \circ f)(x) = f(f(x))$$

$$= f(x^2 - 1)$$

$$= (x^2 - 1)^2 - 1$$

$$= (x^2)^2 - 2(x^2)(1) + 1^2 - 1$$

$$= x^4 - 2x^2 + 1 - 1$$

$$(f \circ f)(x) = x^4 - 2x^2$$

(ii)

$$(f \circ f \circ f)(x) = f((f \circ f)(x))$$

$$= f(x^4 - 2x^2)$$

$$(f \circ f \circ f)(x) = (x^4 - 2x^2)^2 - 1$$

7. Let  $f$  and  $g$  be two functions from  $\mathbb{R}$  to  $\mathbb{R}$  and defined by

$f(x) = x^5$  and  $g(x) = x^4$ . Check whether  $f$ ,  $g$  and  $f \circ g$  are one-one?

**Solution**

$f : \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = x^2$

$g : \mathbb{R} \rightarrow \mathbb{R}$  defined by  $g(x) = x^4$

(i) Let us assume  $f(x_1) = f(x_2)$

$$(x_1)^5 = (x_2)^5$$

Since the power is 5 which is odd,  $x_1 = x_2$

If  $f(x_1) = f(x_2)$  then  $x_1 = x_2$

$f$  is one-one.

(ii) Let us assume  $g(x_1) = g(x_2)$

$$(x_1)^4 = (x_2)^4$$

Since the power is even,  $x_1$  need not be equal to  $x_2$

$\therefore g$  is not one-one.

(iii)

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\&= f(x^4) \\&= (x^4)^5 \\(f \circ g)(x) &= x^{20}\end{aligned}$$

Let us assume  $(f \circ g)(x_1) = (f \circ g)(x_2)$ 

$$(x_1)^{20} = (x_2)^{20}$$

Since the power is 20 which is even and hence  $x_1$  need not be equal to  $x_2$ 

$$x_1 \neq x_2$$

 $\therefore f \circ g$  is not one-one.8. Consider the functions  $f(x), g(x)$  and  $h(x)$  as defined below.Show that  $(f \circ g) \circ h = f \circ (g \circ h)$  in each case.

- (i)  $f(x) = x - 1, g(x) = 3x + 1$  and  $h(x) = x^2$
- (ii)  $f(x) = x^2, g(x) = 2x$  and  $h(x) = x + 4$
- (iii)  $f(x) = x - 4, g(x) = x^2$  and  $h(x) = 3x - 5$

X, SMark Qns

Solution

- (i)  $f(x) = x - 1, g(x) = 3x + 1$  and  $h(x) = x^2$

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\&= f(3x + 1) \\&= 3x + 1 - 1 \\(f \circ g)(x) &= 3x \\LHS &= ((f \circ g) \circ h)(x) = (f \circ g)(h(x)) \\&= (f \circ g)(x^2) \\&= 3(x^2) \\((f \circ g) \circ h)(x) &= 3x^2\end{aligned} \quad \dots(1)$$

$$\begin{aligned}(g \circ h)x &= g(h(x)) \\&= g(x^2) \\(g \circ h)x &= 3x^2 + 1 \\RHS &= (f \circ (g \circ h))x = f(3x^2 + 1) \\&= 3x^2 + 1 - 1 \\&= 3x^2\end{aligned} \quad \dots(2)$$

From (1) and (2) we get

$$(f \circ g) \circ h = f \circ (g \circ h)$$

Hence proved.

- (ii)  $f(x) = x^2, g(x) = 2x$  and  $h(x) = x + 4$

$$LHS = (f \circ g) \circ h$$

$$\begin{aligned}
 (f \circ g)(x) &= f(g(x)) \\
 &= f(2x) \\
 &= (2x)^2 \\
 (f \circ g)(x) &= 4x^2 \\
 ((f \circ g) \circ h)(x) &= (f \circ g)(h(x)) \\
 &= (f \circ g)(x+4) \\
 &= 4(x+4)^2 \\
 ((f \circ g) \circ h)(x) &= 4(x+4)^2
 \end{aligned} \tag{1}$$

$$(g \circ h)(x) = g(h(x))$$

$$= g(x+4)$$

$$(g \circ h)(x) = 2(x+4)$$

$$\begin{aligned}
 RHS &= (f \circ (g \circ h))(x) = f(2(x+4)) \\
 &= [2(x+4)]^2 \\
 &= 4(x+4)^2
 \end{aligned}$$

$$(f \circ (g \circ h))(x) = 4(x+4)^2 \tag{2}$$

From (1) and (2) we get

$$(f \circ g) \circ h = f \circ (g \circ h)$$

Hence proved.

(iii)  $f(x) = x - 4$ ,  $g(x) = x^2$  and  $h(x) = 3x - 5$

$$\begin{aligned}
 (f \circ g)(x) &= f(g(x)) \\
 &= f(x^2) \\
 &= x^2 - 4
 \end{aligned}$$

$$(f \circ g)(x) = x^2 - 4$$

$$\begin{aligned}
 LHS &= ((f \circ g) \circ h)(x) = (f \circ g)(h(x)) \\
 &= (f \circ g)(3x - 5) \\
 &= (3x - 5)^2 - 4
 \end{aligned} \tag{1}$$

$$(g \circ h)(x) = g(h(x))$$

$$= g(3x - 5)$$

$$(g \circ h)(x) = (3x - 5)^2$$

$$\begin{aligned}
 RHS &= (f \circ (g \circ h))(x) = f[(3x - 5)^2] \\
 &= (3x - 5)^2 - 4
 \end{aligned} \tag{2}$$

From (1) and (2) we get

$$(f \circ g) \circ h = f \circ (g \circ h)$$

Hence proved.

9. Let  $f(x) = mx + c$  be a linear function from  $\mathbb{R} \rightarrow \mathbb{R}$  and  $f(-1) = 3$ ,  $f(0) = -1$  and  $f(2) = -9$ . Find the linear function.

**Solution**

$$f(x) = mx + c$$

$$\therefore f(-1) = m(-1) + c$$

$$3 = -m + c$$

$$m = c - 3$$

$$\text{i.e., } f(0) = m(0) + c$$

$$-1 = c$$

$$c = -1$$

Put  $c = -1$  in (1)

$$m = -1 - 3$$

$$m = -4$$

$$\therefore f(x) = (-4)x + (-1)$$

$$f(x) = -4x - 1$$

10. In electrical circuit theory, a circuit  $C(t)$  is called a linear circuit if it satisfies the superposition principle given by  $C(at_1 + bt_2) = a C(t_1) + b C(t_2)$ , where  $a, b$  are constants. Show that the circuit  $C(t) = 3t$  is linear.

**Solution**

To prove  $C(t) = 3t$  is linear,

We have to prove  $C(at_1 + bt_2) = aC(t_1) + bC(t_2)$

2 Marks

$$C(t_1) = 3t_1$$

$$C(t_2) = 3t_2$$

$$C(at_1 + bt_2) = 3(at_1 + bt_2)$$

$$= a \cdot 3t_1 + b \cdot 3t_2$$

$$= aC(t_1) + bC(t_2)$$

$C(t) = 3t$  is a linear circuit.



7. Let  $n(A) = m$  and  $n(B) = n$ . Then the total number of non-empty relations that can be defined from  $A$  to  $B$  is

- (A)  $m^n$       (B)  $n^m$       (C)  $2^{mn} - 1$       (D)  $2^{mn}$

### Solution

Total number of relations from  $A$  to  $B$  =  $2^{mn}$

Total number of non-empty relations from  $A$  to  $B$  =  $2^{mn} - 1$

[Option : (C)]

8. If  $\{(a,8),(6,b)\}$  represents an identity function, then the values of  $a$  and  $b$  are respectively

- (A) (8, 6)      (B) (8, 8)      (C) (6, 8)      (D) (6, 6)

### Solution

If  $f$  is an identity function, then  $f(x) = x$ .

$$a = 8, b = 6$$

[Option : (A)]

9. Let  $A = \{1, 2, 3, 4\}$  and  $B = \{4, 8, 9, 10\}$ .

The function  $f : A \rightarrow B$  given by  $f = \{(1,4), (2,8), (3,9), (4,10)\}$  is

- (A) a many-one function      (B) an identity function  
 (C) an one-to-one function      (D) an into function

### Solution

Distinct elements of  $A$  have distinct images in  $B$ .

So,  $f$  is a one-to-one function.

[Option : (C)]

10. If  $f(x) = 2x^2$  and  $g(x) = \frac{1}{3x}$ , then  $f \circ g$  is

- (A)  $\frac{3}{2x^2}$       (B)  $\frac{2}{3x^2}$       (C)  $\frac{2}{9x^2}$       (D)  $\frac{1}{6x^2}$

### Solution

$$(f \circ g)(x) = f[g(x)] = f\left(\frac{1}{3x}\right) = 2\left(\frac{1}{3x}\right)^2 = \frac{2}{9x^2}$$

[Option : (C)]

11. If  $f : A \rightarrow B$  is a bijective function and  $n(B) = 7$ , then  $n(A)$  is

- (A) 7      (B) 49      (C) 1      (D) 14

### Solution

If  $f : A \rightarrow B$  is a bijective function, then  $n(A) = n(B) \Rightarrow n(A) = 7$  [Option : (A)]

12. Let  $f$  and  $g$  be two functions defined by  $f = \{(0,1), (2,0), (3,-4), (4,2), (5,7)\}$ ,

$g = \{(0,2), (1,0), (2,4), (-4,2), (7,0)\}$ . Then the range of  $f \circ g$  is

- (A) {0, 2, 3, 4, 5}      (B) {-4, 1, 0, 2, 7}      (C) {1, 2, 3, 4, 5}      (D) {0, 1, 2}

### Solution

$$(f \circ g)(0) = f[g(0)] = f(2) = 0, (f \circ g)(1) = f[g(1)] = f(0) = 1,$$

$$(f \circ g)(2) = f[g(2)] = f(4) = 2, (f \circ g)(-4) = f[g(-4)] = f(2) = 0$$

$$(f \circ g)(7) = f[g(7)] = f(0) = 1.$$

$$\text{Range of } f \circ g = \{0, 1, 2\}$$

[Option : (D)]

13. If  $f(x) = \sqrt{1+x^2}$  then

- (A)  $f(xy) = f(x) \cdot f(y)$   
 (C)  $f(xy) \leq f(x) \cdot f(y)$

- (B)  $f(xy) \geq f(x) \cdot f(y)$   
 (D) None of these

**Solution**

$$f(xy) = \sqrt{1+(xy)^2} = \sqrt{1+x^2y^2}$$

$$f(x) \cdot f(y) = \sqrt{1+x^2} \sqrt{1+y^2} = \sqrt{1+x^2 + y^2 + x^2y^2}$$

$$(1) \text{ and } (2) \Rightarrow f(xy) \leq f(x) \cdot f(y)$$

[Option : (C)]

14. If  $g = \{(1,1), (2,3), (3,5), (4,7)\}$  is a function given by  $g(x) = \alpha x + \beta$  then the values of  $\alpha$  and  $\beta$  are

(A) -1,2

(B) 2,-1

(C) -1,-2

(D) 1,2

**Solution**

$$\text{Given : } g(x) = \alpha x + \beta,$$

$$g(1) = 1 \Rightarrow \alpha + \beta = 1 \quad \dots (1)$$

$$g(2) = 3 \Rightarrow 2\alpha + \beta = 3 \quad \dots (2)$$

$$\text{Solving (1) and (2), we get } \alpha = 2, \beta = -1$$

[Option : (B)]

15. The function  $f(x) = (x+1)^3 - (x-1)^3$  is

(A) linear

(B) cubic

(C) reciprocal

(D) quadratic

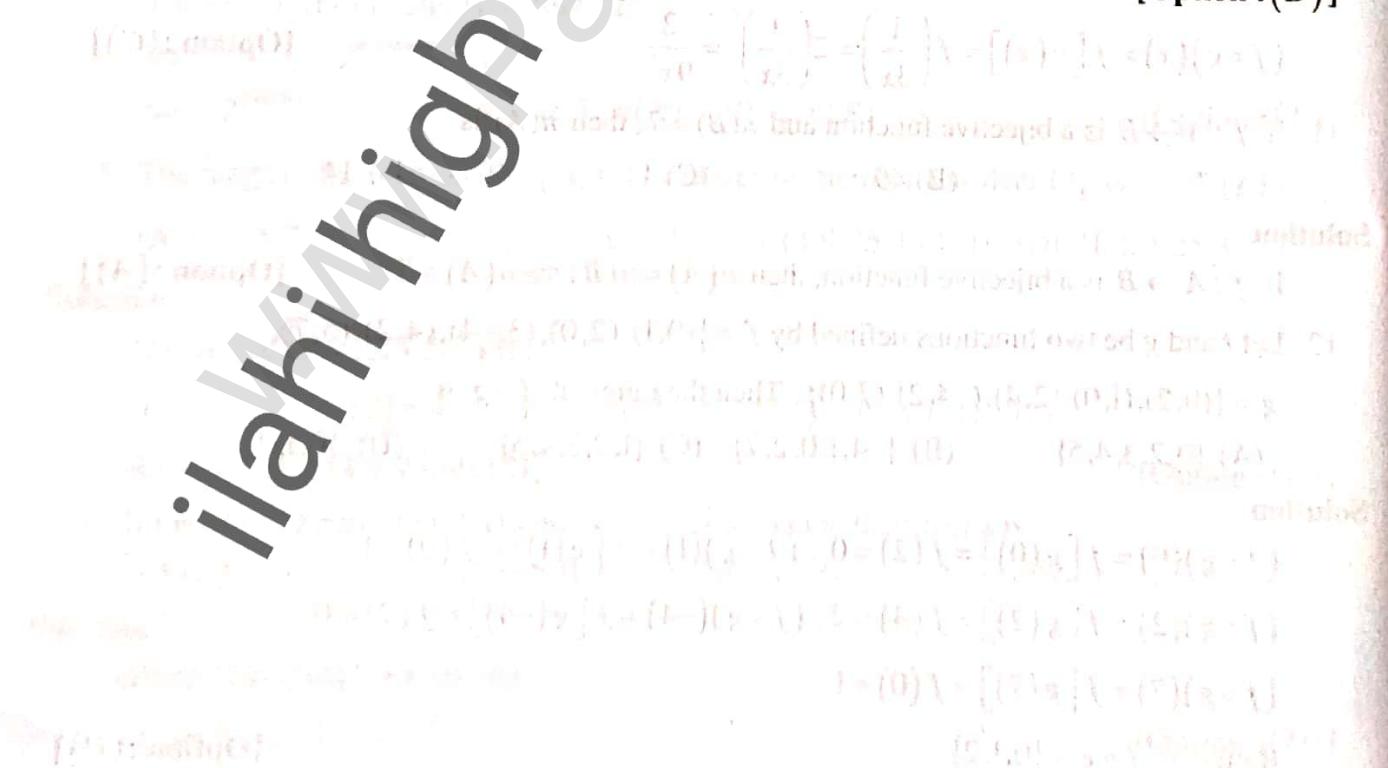
**Solution**

$$f(x) = (x+1)^3 - (x-1)^3$$

$$= (x^3 + 3x^2 + 3x + 1) - (x^3 - 3x^2 + 3x - 1) = 6x^2 + 2$$

$f(x)$  is a quadratic function.

[Option : (D)]



**UNIT EXERCISE - 1**

1. If the ordered pairs  $(x^2 - 3x, y^2 + 4y)$  and  $(-2, 5)$  are equal then find  $x$  and  $y$ .

**Solution**

$$\text{Given } (x^2 - 3x, y^2 + 4y) = (-2, 5)$$

$$x^2 - 3x = -2$$

$$x^2 - 3x + 2 = 0$$

$$(x-1)(x-2) = 0$$

$$x-1 = 0 \text{ and } x-2=0$$

$$x = 1 \text{ and } x=2$$

$$y^2 + 4y = 5$$

$$y^2 + 4y - 5 = 0$$

$$(y-1)(y+5) = 0$$

$$y-1 = 0 \text{ and } y+5=0$$

$$y = 1 \text{ and } y=-5$$

The value of  $x$  are 1 and 2. The value of  $y$  are 1 and -5.

2. The elements  $(-1, 0)$  and  $(0, 1)$  are found in the cartesian product of  $A \times A$ . Find the least possible set  $A$  and the remaining elements of  $A \times A$ .

**Solution**

Since  $(-1, 0)$  and  $(0, 1)$  in  $A \times A$ ,  $A$  contains the minimum number of elements  $-1, 0, 1$ .

$$A = \{-1, 0, 1\}$$

$$A \times A = \{-1, 0, 1\} \times \{-1, 0, 1\}$$

Remaining elements of  $A \times A = \{(-1, -1), (-1, 1), (0, -1), (0, 0), (1, -1), (1, 0), (1, 1)\}$

$$3. \text{ Let } f(x) = \begin{cases} \sqrt{x-1} & x \geq 1 \\ 4 & x < 1 \end{cases}$$

Find (i)  $f(0)$  (ii)  $f(3)$  (iii)  $f(a+1)$  in terms of  $a$  (Given that  $a \geq 0$ )

**Solution**

(i)  $f(0)$

$$x=0 < 1 \therefore f(x)=4$$

$$\text{Put } x = 0$$

$$f(0) = 4$$

(ii)  $f(3)$

$$x=3 > 1 \therefore f(x) = \sqrt{x-1}$$

$$\text{Put } x = 3$$

$$f(3) = \sqrt{3-1}$$

$$f(3) = \sqrt{2}$$

(iii)  $f(a+1)$

Since  $a \geq 0$ ,  $(a+1) \geq 1$  and hence  $f(a+1) = \sqrt{a+1-1}$

$$\text{Put } x = a+1$$

$$f(a+1) = \sqrt{a+1-1}$$

$$f(a+1) = \sqrt{a}$$

4. Let  $A = \{9, 10, 11, 12, 13, 14, 15, 16, 17\}$  and let  $f: A \rightarrow \mathbb{N}$  be defined by  $f(n) =$  the highest prime factor of  $n \in A$ . Write  $f$  as a set of ordered pairs and find the range of  $f$ .

**Solution**

$$A = \{9, 10, 11, 12, 13, 14, 15, 16, 17\}$$

$f(n) =$  highest prime factor of  $n$  where  $n \in A$

$f$  as set of ordered pairs :

$$n = 9, f(9) = 3$$

$$n = 10, f(10) = 5$$

$$n = 11, f(11) = 11$$

$$n = 12, f(12) = 3$$

$$n = 13, f(13) = 13$$

$$n = 14, f(14) = 7$$

$$n = 15, f(15) = 5$$

$$n = 16, f(16) = 2$$

$$n = 17, f(17) = 17$$

Since

$$9 \rightarrow 1, 3, 9$$

$$10 \rightarrow 1, 2, 5, 10$$

$$11 \rightarrow 1, 11$$

$$12 \rightarrow 1, 2, 3, 4, 6, 12$$

$$13 \rightarrow 1, 13$$

$$14 \rightarrow 1, 2, 7, 14$$

$$15 \rightarrow 1, 3, 5, 15$$

$$16 \rightarrow 1, 2, 4, 8, 16$$

$$17 \rightarrow 1, 17$$

$$f = \{(9, 3), (10, 5), (11, 11), (12, 3), (13, 13), (14, 7), (15, 5), (16, 2), (17, 17)\}$$

$$\text{Range of } f = \{2, 3, 5, 7, 11, 13, 17\}$$

5. Find the domain of the function  $f(x) = \sqrt{1 + \sqrt{1 - \sqrt{1 - x^2}}}$

**Solution**

To define  $f(x)$ ,  $(1 - x^2) \geq 0$

$$1 \geq x^2$$

$$x^2 \leq 1$$

$$-1 \leq x \leq 1$$

$\therefore$  Domain of  $f(x)$  is  $-1 \leq x \leq 1$ .

6. If  $f(x) = x^2$ ,  $g(x) = 3x$  and  $h(x) = x - 2$ , prove that  $(f \circ g) \circ h = f \circ (g \circ h)$ .

**Solution**

$$f(x) = x^2$$

$$g(x) = 3x$$

$$h(x) = x - 2$$

$$(f \circ g)(x) = f(g(x))$$

$$= f(3x)$$

$$= (3x)^2$$

$$(f \circ g)(x) = 9x^2$$

$$((f \circ g))(h(x)) = (f \circ g)(x - 2)$$

$$= 9(x-2)^2 \quad \dots (1)$$

Now  $(g \circ h)(x) = g(h(x))$

$$= g(x-2)$$

$$(g \circ h)(x) = 3(x-2)$$

$$(f \circ (g \circ h))(x) = f(3(x-2))$$

$$= [3(x-2)]^2$$

$$(f \circ (g \circ h))(x) = 9(x-2)^2 \quad \dots (2)$$

From (1) and (2) we get

$$(f \circ g) \circ h = f \circ (g \circ h).$$

7. Let  $A = \{1, 2\}$  and  $B = \{1, 2, 3, 4\}, C = \{5, 6\}$  and  $D = \{5, 6, 7, 8\}$ . Verify whether  $A \times C$  is a subset of  $B \times D$ ?

**Solution**

$$\begin{aligned} A \times C &= \{1, 2\} \times \{5, 6\} \\ &= \{(1, 5), (1, 6), (2, 5), (2, 6)\} \end{aligned}$$

$$\begin{aligned} B \times D &= \{1, 2, 3, 4\} \times \{5, 6, 7, 8\} \\ &= \{(1, 5), (1, 6), (1, 7), (1, 8), \\ &\quad (2, 5), (2, 6), (2, 7), (2, 8), \\ &\quad (3, 5), (3, 6), (3, 7), (3, 8), \\ &\quad (4, 5), (4, 6), (4, 7), (4, 8)\} \end{aligned}$$

All the ordered pairs of  $(A \times C)$  are in  $(B \times D)$ .

$\therefore (A \times C)$  is a subset of  $(B \times D)$ .

8. If  $f(x) = \frac{x-1}{x+1}$  where  $x \neq -1$ , show that  $f(f(x)) = \frac{-1}{x}$ , provided  $x \neq 0$ .

**Solution**

$$\text{Given } f(x) = \frac{x-1}{x+1}, \quad x \neq -1$$

$$f(f(x)) = f\left(\frac{x-1}{x+1}\right)$$

$$= \frac{\left(\frac{x-1}{x+1}\right)-1}{\left(\frac{x-1}{x+1}\right)+1}$$

$$= \frac{x-1-x-1}{x-1+x+1}$$

$$= \frac{-2}{2x} = \frac{-1}{x}$$

$$\therefore f(f(x)) = -\frac{1}{x}.$$

9. The functions  $f$  and  $g$  are defined by  $f(x) = 6x + 8$ ,  $g(x) = \frac{x-2}{3}$ .

- (i) Calculate the value of  $g\left(g\left(\frac{1}{2}\right)\right)$  (ii) Write the expression for  $g(f(x))$  in its simplest form.

**Solution**

$$(i) \quad g(x) = \frac{x-2}{3}$$

$$g\left(\frac{1}{2}\right) = \frac{\left(\frac{1}{2}-2\right)}{3}$$

$$= \frac{1-4}{2} \times \frac{1}{3}$$

$$g\left(\frac{1}{2}\right) = \frac{-1}{2}$$

$$\therefore g\left(g\left(\frac{1}{2}\right)\right) = g\left(-\frac{1}{2}\right)$$

$$= \frac{-1-2}{2}$$

$$\left( \because g(x) = \frac{x-2}{3} \right)$$

$$= \frac{-3}{6}$$

$$(ii) \quad g(f(x)) = g(6x+8)$$

$$= \frac{6x+8-2}{3}$$

$$= \frac{6x+6}{3}$$

$$\therefore g(f(x)) = 2(x+1)$$

10. Write the domain of the following functions :

$$(i) \quad f(x) = \frac{2x+1}{x-9} \quad (ii) \quad p(x) = \frac{-5}{4x^2+1} \quad (iii) \quad g(x) = \sqrt{x-2} \quad (iv) \quad h(x) = x+6$$

**Solution**

$$(i) \quad f(x) = \frac{2x+1}{x-9}$$

$f(x)$  is defined only when  $x-9 \neq 0$

i.e.,  $x \neq 9$

Domain of  $f(x)$  is the set of all real numbers except 9

Domain of  $f(x)$  is  $\mathbb{R} - \{9\}$

$$(ii) p(x) = \frac{-5}{4x^2 + 1}$$

$p(x)$  is defined only when  $4x^2 + 1 \neq 0$

But  $4x^2 \neq -1$  for any real number  $x$

( $x^2$  is always positive)

Domain of  $p(x)$  is the set of all real numbers (i.e.)  $\mathbb{R}$ .

$$(iii) g(x) = \sqrt{x-2}$$

$g(x)$  is defined when  $x-2 \geq 0$

$$x \geq 2$$

Domain of  $f(x)$  is the set of all real numbers greater than or equal to 2.

$$(iv) h(x) = x+6$$

Domain of  $h(x)$  is the set of all real numbers (i.e.)  $\mathbb{R}$ .

Prepared By

M.Abbas Manthiri

B.sc,B.ed,M.A.M.phil

B.T.Assistant

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