B

FIRST REVISION TEST - 2024

Standard XI

Reg.No. | | | | | | |

MATHEMATICS

Time : 3.00 hrs *Marks : 90*

Part - I

I. Choose the correct answer: 20 x 1 = 20

1. Let $f:R \to R$ be defined by $f(x) = 1 - |x|$. Then the range of f is
   a) R b) $(1, \infty)$ c) $(-1, \infty)$ d) $(-\infty, 1]$
2. The solution set of $|x - 1| \geq |x - 3|$ is
   a) $[0, 2]$ b) $(0, 2)$ c) $[2, \infty]$ d) $(-\infty, 2)$
3. If $\sin\alpha + \cos\alpha = b$, then $\sin 2\alpha$ is equal to
   a) $b^2 - 1$, if $b > \sqrt{2}$ b) $b^2 - 1$, if $b \geq 1$
   c) $b^2 - 1$, if $b \geq \sqrt{2}$ d) $b^2 - 1$, if $b \leq \sqrt{2}$
4. $1 + 3 + 5 + 7 + \ldots\ldots + 17$ is equal to
   a) 81 b) 101 c) 61 d) 71
5. The sequence $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}+\sqrt{2}}, \frac{1}{\sqrt{3}+2\sqrt{2}}, \ldots\ldots$ form an
   a) A.P b) G.P c) H.P d) AGP
6. The line $(p + 2q)x + (p - 3q)y = p - q$ for different values of p and q passes through the point
   a) $\left(\frac{3}{2}, \frac{5}{2}\right)$ b) $\left(\frac{2}{5}, \frac{2}{5}\right)$ c) $\left(\frac{3}{5}, \frac{3}{5}\right)$ d) $\left(\frac{2}{5}, \frac{3}{5}\right)$
7. The value of the determinant of $A = \begin{bmatrix} 0 & a & -b \\ -a & 0 & c \\ b & -c & 0 \end{bmatrix}$ is
   a) $-2abc$ b) abc c) 0 d) $a^2 + b^2 + c^2$
8. Vectors $\vec{a}$ and $\vec{b}$ are inclined at an angle $\theta = 120^\circ$. If $|\vec{a}| = 1$, $|\vec{b}| = 2$, then $\left[(\vec{a} + 3\vec{b}) \times (3\vec{a} - \vec{b})\right]^2$ a) 225 b) 275 c) 325 d) 300
9. $\lim_{x \to \infty} \frac{a^x - b^x}{x} =$
   a) $\log ab$ b) $\log\left(\frac{a}{b}\right)$ c) $\log\left(\frac{b}{a}\right)$ d) $\frac{a}{b}$
10. If $f(x) = x + 2$, then $f'(f(x))$ at $x = 4$ is
    a) 8 b) 4 c) 1 d) 5
11. $\int e^{\sqrt{x}}\, dx =$
    a) $2e^{\sqrt{x}}(\sqrt{x} - 1) + c$ b) $2e^{\sqrt{x}}(1 - \sqrt{x}) + c$
    c) $2\sqrt{x}\left(1 - e^{\sqrt{x}}\right) + c$ d) $2\sqrt{x}\left(e^{\sqrt{x}} - 1\right) + c$

12. Ten coins are tossed. The probability of getting at least 8 heads is

a) $\frac{7}{64}$ b) $\frac{7}{128}$ c) $\frac{7}{16}$ d) $\frac{7}{32}$

13. If R is a relation from a set A to a set B, then

a) $R = A \cup B$ b) $R = A \cap B$ c) $R \subseteq A \times B$ d) $R \subseteq B \times A$

14. The coefficient of x^{-3} in the expansion of $\left(x - \frac{m}{x}\right)^{11}$ is

a) $-924\, m^7$ b) $-792\, m^5$ c) $-792\, m^6$ d) $-330\, m^7$

15. The equation of the straight line passing through the point (3,2) and perpendicular to the line $y = x$ is

a) $x - y = 5$ b) $x + y = 5$ c) $x + y = 1$ d) $x - y = 1$

16. The value of $2^{1/4}.4^{1/8}.8^{1/16} \ldots\ldots\infty$ is

a) 1 b) 2 c) $3/2$ d) 4

17. If $y = \sec(\tan^{-1} x)$ then $\frac{dy}{dx}$ at $x = 1$ is

a) $1/\sqrt{2}$ b) $1/2$ c) 1 d) $\sqrt{2}$

18. $\int e^x (\sin x + \cos x)dx =$

a) $e^x \cos x + c$ b) $e^x \sin x + c$
c) $e^x(-\cos x + \sin x) + c$ d) $-e^x \sin x + c$

19. The remainder when 38^{15} is divided by 13 is

a) 1 b) 11 c) 5 d) 12

20. $\int \frac{(\log x)^3}{x} dx =$

a) $\frac{(\log x)^4}{4} + c$ b) $3(\log x)^2 + c$ c) $(\log x)^4 + c$ d) $\frac{1}{3\log x}$

Part - II

II. Answer any 7 questions. (Q.No.30 is compulsory) **7 x 2 = 14**

21. If $n(A) = 10$ and $n(A \cap B) = 3$, then find $n((A \cap B) \cap A)$
22. Compute $\log_3 5 \log_{25} 27$
23. Prove that $\sin(30^\circ + \theta) + \cos(60^\circ + \theta) = \cos\theta$
24. If the different permutations of all letters of the word BHASKARA are listed as in a dictionary, how many strings are there in this list before the first word starting with B?
25. If θ is a parameter, find the equation of the locus of a moving point, whose coordinates are $X = a\cos^3\theta$, $y = a\cos^3\theta$.
26. If $A_\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then find all possible real values of α satisfying the condition $A_\alpha + A_\alpha^T = I$

27. Find λ, when the projection of $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$ on $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ is 4 units.

28. Calculate : $\lim_{x \to \infty} \frac{1 - x^3}{3x + 2}$

29. Integrate with respect to x : $x^2 \cos x$

30. If $y = a^{(\sin^{-1} x)^2}$, find $\frac{dy}{dx}$

Part - III

III. Answer any 7 questions. (Q.No.40 is compulsory) **7 x 3 = 21**

31. Prove that $ap + q = 0$ if $f(x) = x^3 - 3px + 2q$ is divisible by $g(x) = x^2 + 2ax + a^2$

32. Solve : $\sqrt{3}\tan^2\theta + (\sqrt{3} - 1)\tan\theta - 1 = 0$

33. Prove that $^{24}C_4 + \sum_{r=0}^{4} (28 - r)C_3 = {}^{29}C_4$

34. Show that the straight lines joining the origin to the points of intersection of $3x - 2y + 2 = 0$ and $3x^2 + 5xy - 2y^2 + 4x + 5y = 0$ are at right angles.

35. Find the area of the triangle whose vertices are A(3, –1, 2), B(1, –1, –3) and (4, –3, 1)

36. Using Factor Theorem, show that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (x - y)(y - z)(z - x)$

37. Find the derivative $\frac{dy}{dx}$ if $x = a(\cos t + t \sin t)$; $y = a(\sin t - t \cos t)$

38. Let the matrix $M = \begin{bmatrix} x & y \\ z & 1 \end{bmatrix}$. If x, y and z are chosen at random from the set {1,2,3} and repetition is allowed (ie., x = y = z). What is the probability that the given matrix M is a singular matrix?

39. Using Binomial Theorem, prove that $6^n - 5n$ always leaves remainder 1 when divided by 25 for all positive integer n.

40. If $f(x) = \frac{4x + 3}{6x - 4}$, $x \neq \frac{2}{3}$, show that (fof) (x) = x, what is the inverse of f.

Part - IV

IV. Answer all the questions. **7 x 5 = 35**

41. a) Write the values of f at –3, 5, 2, –1, 0 if $f(x) = \begin{cases} x^2 + x - 5 & \text{if } x \in (-\infty, 0) \\ x^2 + 3x - 2 & \text{if } x \in (3, \infty) \\ x^2 & \text{if } x \in (0,2) \\ x^2 - 3 & \text{otherwise} \end{cases}$

(OR)

b) In a triangle ABC, prove that $\frac{a^2+b^2}{a^2+c^2} = \frac{1+\cos(A-B)\cos C}{1+\cos(A-c)\cos B}$ 1M1

42. a) Resolve into partial fractions : $\frac{6x^2-x+1}{x^3+x^2+x+1}$ **(OR)** V-1 Pg : 71

b) Find the number of strings of 5 letters that can be formed with the letters of the word PROPOSITION. 185

43. a) Find the sum up to the 17^{th} term of the series $\frac{1^3}{1} + \frac{1^3+2^3}{1+3} = \frac{1^3+2^3+3^3}{1+3+5} + \ldots\ldots$ V-1 220

(OR)

b) In a bolt factory, machines A, B and C manufacture respectively 25%, 35% and 40% of the total bolts of their outputs 5%, 4% and 2% are respectively defective bolts. A bolt is drawn at random from the product. If the bolt drawn is found to be defective, what is the probability that it is manufactured by the machine B?

44. a) Show that the medians of a triangle are concurrent. **(OR)** V-2 58

b) Describe the interval(s) on which each function is continuous.

i) $f(x) = \tan x$ ii) $g(x) = \begin{cases} \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ iii) $h(x) = \begin{cases} x\sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ V-2 124

45. a) A straight line L with negative slope passes through the point (9, 4) cuts the positive coordinate axes at the point P and Q. As L varies, find the minimum value of |OP| + |OQ|, where O is the origin. **(OR)** V-1 255

b) If $A + B + C = \pi$, then prove that

$$\sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2} = 1 + 4\sin\left(\frac{\pi-A}{4}\right)\sin\left(\frac{\pi-B}{4}\right)\sin\left(\frac{\pi-C}{4}\right)$$ V-1 123

46. a) Integrate with respect to x : $\frac{2x+1}{\sqrt{9+4x-x^2}}$ **(OR)** V-2 Pg 222

b) Three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are such that $|\vec{a}| = 2$, $|\vec{b}| = 3$, $|\vec{c}| = 4$ and $\vec{a}+\vec{b}+\vec{c} = \vec{0}$, find $4\vec{a}.\vec{b} + 3\vec{b}.\vec{c} + 3\vec{c}.\vec{a}$ V-2 74

47. a) If $y = \left(x + \sqrt{1+x^n}\right)^n$, then prove that $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = n^2y$ **(OR)**

b) If $\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} = Ax^3 + Bx^2 + Cx + D$, then find B + C

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XI Maths (Kancheepuram Dt) Key

1st Revision Test - 2024

Choose :

1) d) $(-\infty, 1]$

2) c) $[2, \infty]$

3) d) $b^2 - 1$, if $b \leq \sqrt{2}$

4) a) 81

5) c) H.P

6) d) $\left[\frac{2}{5}, \frac{3}{5}\right]$

7) c) 0

8) d) 300

9) b) $\log\left(\frac{a}{b}\right)$

10) c) 1

11) a) $2e^{\sqrt{x}}(\sqrt{x}-1)+c$

12) c) $7/16$

13) c) $R \subseteq A \times B$

14) d) $-330 m^7$

15) b) $x+y=5$

16) b) 2

17) a) $1/\sqrt{2}$

18) b) $e^x \sin x + c$

19) d) 12

20) a) $\frac{(\log x)^4}{4} + c$

II

21) $n((A \cap B') \cap A)$

$= n(A - B) = n(A) - n(A \cap B)$

$= 10 - 3 = 7$

22) $\log_3 5 \times \frac{3}{2} \log_5 3 = \frac{3}{2}$

23) $\sin 30 \cos\theta + \cos 30 \sin\theta +$
$\cos 60 \cos\theta - \sin 60 \sin\theta$

$= \frac{\cos\theta}{2} + \frac{\cos\theta}{2} = \cos\theta.$

24) A, A, B, H, K, R, S $= \frac{7!}{2!} = 2520$

25) Mistake in question

26) $\alpha = 2n\pi \pm \pi/3$, $n \in Z$ $(\cos\alpha = 1/2)$

27) $\frac{2\lambda + 6 \div 12}{\sqrt{49}} = 4$; $\lambda = 5$

28) $\frac{1/x - x^2}{3 \div 2/x} \to -\infty$ as $x \to \infty$.

29) $x^2 \sin x + 2x \cos x - 2\sin x + c$

30) $\boxed{\frac{d}{dx} a^x = a^x \log a}$

$\frac{dy}{dx} = a^{(\sin^{-1}x)^2} \log a.$

III

$g(x) = bx + c$

31.) $F(x) = g(x)\,g(x)$

$= bx^3 + cx^2 + 2abx^2 + 2acx + a^2bx + a^2c$

by comparing

$b = 1$, $c = -2a$, $P = -a^3$

$2ac + a^2b = -3P$

$-3a^2 = -3P \Rightarrow P = a^2$.

$\therefore aP + P = a(a^2) + (-a^3) = 0$

32.) $(\sqrt{3}\tan - 1)(\tan + 1) = 0$

$\tan\theta = \frac{1}{\sqrt{3}}$ (or) -1

$\theta = \pi/6$, $-\pi/4$

$\theta = n\pi + \pi/6$, $n\pi - \pi/4$, $n \in Z$.

33.) ${}^{24}C_4 + {}^{28}C_3 + {}^{27}C_3 + {}^{26}C_3 + {}^{25}C_3 + {}^{24}C_3$

$= {}^{29}C_4$. by ${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$

34.) $\frac{3x-2y}{-2} = 1$

$3x^2 + 5xy - 2y^2 + (4x + 5y)\left(\frac{3x-2y}{-2}\right) = 0$

$a = 2$, $b = -2$, $a + b = 0$

35.) $\frac{1}{2}|AB \times AC| = \frac{1}{2}\sqrt{165}$ sq units

36.) $x = y$ is factor

$\therefore |A| = 0$ Similar way

$(y - z)(z - x)$ are factors

$m = 3 - 3 = 0$

Remaining factor $k = 1$

$\therefore |A| = (x - y)(y - z)(z - x)$

37.) $\frac{dy}{dx} = \frac{d(\cos t + t\sin t - \cos t)}{d(-\sin t + t\cos t + \sin t)}$

$= \frac{t\sin t}{t\cos t} = \tan t$.

38.) $x - yz = 0$

$A = \{(1,1,2), (2,1,2), (2,2,1), (3,1,3), (3,3,1)\} = A$

$n(A) = 5$

$n(S) = 3^3 = 27$.

$P(A) = 5/27$ //

39.) $6^n - 5n = 25k + 1$

$(1+5)^n = {}^{n}C_0 + \cdots + {}^{n}C_{n-1}5^{n-1} + {}^{n}C_n 5^n$

$6^n = 1 + 5n + 25({}^{n}C_2 + \cdots {}^{n}C_n 5^{n-2})$

$6^n - 5n = 1 + 25k$

H.P

40.) $Fof(x) = \frac{4\left(\frac{4x+3}{6x-4}\right) + 3}{6\left(\frac{4x+3}{6x-4}\right) - 4} = x$

Inverse

$y = \frac{4x+3}{6x-4} \Rightarrow x = \frac{4y+3}{6y-4}$

41) a) $F(-3)=1$, $F(5)=38$

$F(2)=1$, $F(-1)=-5$, $F(0)=-3$

b) $\frac{a^2+b^2}{a^2+c^2} = \frac{(2R\sin A)^2+(2R\sin B)^2}{(2R\sin A)^2+(2R\sin C)^2}$

$= \frac{1-(\cos^2 A-\sin^2 B)}{1-(\cos^2 A-\sin^2 C)}$

$= \frac{1+\cos(A-B)\cos C}{1+\cos(A-C)\cos B}$.

42) a)

$\frac{6x^2-x+1}{x^3+x^2+x+1} = \frac{A}{x+1}+\frac{Bx+C}{x^2+1}$

$= \frac{4}{x+1}+\frac{2x-3}{x^2+1}$

b) $R,S,T,N,P,I,O \quad (^7C_5 \times 5!) = 2520$

DDD
PP,II $\quad (^1C_1 \times {}^2C_1 \times \frac{5!}{3!\times 2!}) = 20$

PDD
(2) $\quad {}^1C_1 \times {}^6C_2 \times \frac{5!}{3!} = 300$

PP,II,OO $\quad {}^3C_2 \times {}^5C_1 \times \frac{5!}{2!\times 2!} = 450$

(1) $\quad {}^3C_1 \times {}^6C_3 \times \frac{5!}{2!} = 3600$

(2),(3)

6890 //

43) a) $t_n = \frac{1^3+2^3+\cdots+n^3}{1+3+\cdots n\text{ terms}}$

$= \frac{\sum n^3}{n^2}\left(\frac{2n-1+1}{2}\right)^2$

$= \frac{n^2+2n+1}{4}$

$S_{17} = \frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6}+\frac{2(n)(n+1)}{2}+n\right]$

$= \frac{1}{4}\left[17\times 105 + 17\times 18 + 17\right] = 527$

44) b) $P(D) = P(A)P\left(\frac{D}{B}\right)+P(B)P\left(\frac{D}{B}\right)+P(C)P\left(\frac{D}{C}\right)$

$P\left(\frac{B}{D}\right) = \frac{\frac{35}{100}\left(\frac{4}{100}\right)}{\frac{25}{100}\left(\frac{5}{100}\right)+\frac{35}{100}\left(\frac{4}{100}\right)+\frac{40}{100}\left(\frac{2}{100}\right)}$

$= \frac{28}{69}$

45) a) $y-4 = m(x-9)$

$|OP|+|OQ| = \left|9-\frac{4}{m}\right|+|4-9m|$

$\geq 13+2\sqrt{\frac{4}{k}\times 9k}$

≥ 25 //

b) $\cos\left(\frac{\pi}{2}-\frac{A}{2}\right)+\cos\left(\frac{\pi}{2}-\frac{B}{2}\right)+\cos\left(\frac{\pi}{2}-\frac{C}{2}\right)$

$= 1+2\sin\left(\frac{\pi-C}{4}\right)\left(\cos\frac{B-A}{4}-\sin\frac{A+B}{4}\right)$

$= 1+4\sin\left(\frac{\pi-A}{4}\right)\sin\left(\frac{\pi-B}{4}\right)\sin\left(\frac{\pi-C}{4}\right)$

46) a) Medians of Δ concurrent

$OG = \frac{\bar{a}+\bar{b}+\bar{c}}{3}$

b) i) $x=(2n+1)\frac{\pi}{2}$, (continues in other Int)

ii) continuous in $(-\infty,0),(0,\infty)$

iii) Continuous in R.

46) a) $2x+1 = A(4-2x)+B$

$$I = \int \frac{-(4-2x)}{\sqrt{9+4x-x^2}}dx + 5\int \frac{dx}{\sqrt{9+4x-x^2}}$$

$$I = -2\sqrt{9+4x-x^2} + 5\sin^{-1}\frac{x-2}{\sqrt{13}} + C$$

b) $(\bar{a}+\bar{b})^2 = (-\bar{c})^2$

$= 4\,\bar{a}\cdot\bar{b} + 3\bar{b}\cdot\bar{c} + 3\bar{c}\cdot\bar{a}$

$= 4\left(\frac{3}{2}\right) + 3\left(\frac{-21}{2}\right) + 3\left(\frac{-11}{2}\right)$

$= -42.$

47.) diff 2 times

a)

$$\sqrt{1+x^2}\,y'' + \frac{2x}{2\sqrt{1+x^2}}y' = n\,y'$$

$$(1+x^2)y'' + xy' = n^2 y$$

b) $(x-1)(x-2)\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 1 & 1 & 1 \\ 1 & 2 & 6 \end{vmatrix}$

$R_2 \to R_2 - R_1 \cdot R_3 \to R_3 - R_2.$

$= -3x^3 + 12x^2 - 15x + 6$

$B + C = 12 - 15 = -3$