## SUBJECT:

# MATHEMATICS 

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Prove that $\log _{4} 2-\log _{8} 2+\log _{16} 2-\cdots=1-\log _{e} 2$.
In a $\triangle A B C$, if $\tan \frac{A}{2}=\frac{5}{6}$ and $\tan \frac{C}{2}=\frac{2}{5}$, then show that $a, b, c$ are in A.P.
If $A=\left[\begin{array}{cc}4 & 2 \\ -1 & x\end{array}\right]$ and $(A-2 I)(A-3 I)=O$, find the value of $x$.
Evaluate : $\lim _{x \rightarrow 0} \frac{\sqrt{x+2}-\sqrt{2}}{x}$.
Compute: $9^{7}$
Find the last two digits of the number: $3^{600}$
If $f(x)=y=\frac{a x-b}{c x-a}$, then prove that $f(y)=x$.
Find the value of $\frac{1}{\log _{x}(y z)+1}+\frac{1}{\log _{y}(z x)+1}+\frac{1}{\log _{z}(x y)+1}$.
Find $f^{\prime}(2)$ and $f^{\prime}(4)$ if $f(x)=|x-3|$.
Solve : $\sqrt{3} \sin x+\cos x=2$.
Find $f^{\prime}(x)$, if $f(x)=\sin |x|$, by removing the modulus sign.

Verify the continuity at the point $x=0$ for the function $f(x)=\left\{\begin{array}{rr}\frac{\sin 3 x}{x}+1 \text { if } x \neq 0 \\ 2 & \text { if } x=0\end{array}\right.$
Is it correct to say $\mathrm{A} \times \mathrm{A}=\{(\mathrm{a}, \mathrm{a}): \mathrm{a} \in \mathrm{A}\}$ ? Justify your answer.
Construct a suitable domain X such that $f: \mathrm{X} \rightarrow \mathbf{N}$ defined by $f(\mathrm{n})=\mathrm{n}+3$ to be one to one and onto.

Find $d y / d x$ if $x^{2}+y^{2}=1$.
Evaluate: $\int\left[\frac{12}{(4 x-5)^{3}}+\frac{6}{3 x+2}+16 \mathrm{e}^{4 x+3}\right] d x$.
Differentiate $\mathrm{x}^{\mathrm{x}}$ with respect to x .
If $a \sin ^{2} \theta+b \cos ^{2} \theta=c$, show that $\tan ^{2} \theta=\frac{c-b}{a-c}$.

Evaluate : $\lim _{x \rightarrow 1} \frac{\left(x+x^{2}+x^{3}+\ldots+x^{n}\right)-\mathrm{n}}{x-1}$
If $y=\tan ^{-1}\left(\frac{1-x^{2}}{1+x^{2}}\right)$ find $y^{\prime}$
If $\overrightarrow{\mathrm{a}}=\hat{i}+2 \hat{j}+3 \hat{k}, \overrightarrow{\mathrm{~b}}=-3 \hat{i}+4 \hat{j}-5 \hat{k}$ then find the value of $\overrightarrow{\mathrm{a}} \cdot \overrightarrow{\mathrm{b}}$.
Differentiate $y=\tan ^{2} 4 x$ with respect to $x$.
Find the equation of the line passing through the point $(5,2)$ and perpendicular to the line joining $(2,3)$ and $(3,-1)$.

A committee of 7 has to be formieú froni 9 meit arid 4 womien. In how many ways can this be done when the committee consists exactly 3 women?

Integrate $(x-11)^{7}$ with respect to $x$.
A die is rolled. If it shows an even number, then find the probability of getting 6 .
Integrate $\cos 3 x$ with respect to $x$.

Find the distinct permutation of the letters of the word MATHEMATICS.

Evaluate : $\lim _{n \rightarrow \infty}\left[6^{n}+5^{n}\right]^{\frac{1}{n}}$
If ${ }^{n} C_{r-1}=36,{ }^{n} C_{r}=84$ and ${ }^{n} C_{r+1}=126$ then find the value of $r$.
If $y=e^{\sin x}$, find $d y / d x$.
Find the value of $\tan 165^{\circ}$.
Find the value of: $\operatorname{cosec}\left(-1410^{\circ}\right)$.
Solve $2 x^{2}+x-15 \leq 0$.
Find the number of subsets of $A$ if $A=\{x: x=4 n+1,2 \leqslant n<5, n \in N\}$.
Show that the relation $x y=-2$ is a function for a suitable domain. Find the domain and the range of the function.

If $\mathscr{P} \mid(A)$ denotes the power set of $A$, then find $n(\mathscr{P}(\mathscr{P}(\mathscr{P}(\emptyset))))$.
Write $f(x)=x^{2}+5 x+4$ in completed square form,

If $n(A)=10$ and $n(A \cap B)=3$, find $n\left((A \cap B)^{\prime} \cap A\right)$.
Find the range $: \frac{1}{2 \cos x-1}$.
If $A=30^{\circ}$ then find the value of $2 \sin ^{2} A+\cos 2 A$.

Let $f$ and $g$ be the two functions from $R$ to $R$ defined by $f(x)=3 x-4$ and $g(x)=x^{2}+3$. Find $g$ of and fog.

If $n(A \cap B)=3$ and $n(A \cup B)=10$, then find $n(P(A \Delta B))$
Prove $\log \frac{a^{2}}{b c}+\log \frac{b^{2}}{c a}+\log \frac{c^{2}}{a b}=0$

## Find the number of solutions of $x^{2}+|x-1|=1$

Find the value of $\sin 690^{\circ}$.

Find all values of $x$ that satisfies the inequality $\frac{2 x+3}{(x+2)(x+4)}<0$
If $\left(x^{1 / 2}+x^{-1 / 2}\right)^{2}=9 / 2$, then find the value of $\left(x^{1 / 2}-x^{-1 / 2}\right)$ for $x>1$.
If $x=\sqrt{2}+\sqrt{3}$ find $\frac{x^{2}+1}{x^{2}-2}$.
Compute $\log _{9} 27-\log _{27} 9$.
Let $f$ and $g$ be two functions from $R$ to $R$ denfined by $f(x)=3 x-4$ and $g(x)=x^{2}+3$. find gof, fog.

Solve: $\frac{x+1}{x+3}<3$.
Find the value of $n$ if $\frac{1}{8!}+\frac{1}{9!}=\frac{n}{10!}$
Prove that the equation to the straight lines through the origin, each of which makes an angle $\alpha$ with the straight line $\mathrm{y}=\mathrm{x}$ is $\mathrm{x}^{2}-2 \mathrm{xy} \sec \alpha+\mathrm{y}^{2}=0$.

## Resolve into partial fractions: $\frac{3 x+1}{(x-2)(x+1)}$.

Find the value of $\sin 20$, when $\sin 0=\frac{12}{13}$, 0 lies in the first quadrant.

Find the locus of a point $P$ moves such that its distances from two fixed points $\mathrm{A}(1,0)$ and $\mathrm{B}(5,0)$ are always equal.
Fiñd the equations of a parallel line and peripendicicullar line passing through the point (1, 2) to the line $3 x+4 y=7$

Solve : $|5 x-12|<-2$

$$
\text { If } \frac{1}{7!}+\frac{1}{9!}=\frac{A}{10!} \text {, find } A \text {. }
$$

If in two circles, arcs of the same length subtend angles $60^{\circ}$ and $75^{\circ}$ at the centre, Find the ratio of their radii.

Express the equation $\sqrt{3} x-y+4=0$ in the slope - Intercept form.

Prove that $\cot (A * B)=\frac{\cot A \cot B-1}{\cot A+\cot B}$

Find the general solution of $\sin \theta=\frac{-\sqrt{3}}{2}$.
The slope of one of the lines $a x^{2}+2 h x y+b y^{2}=0$ is three limes the other. Show that $3 \mathrm{~h}^{2}=4 \mathrm{ab} \quad$ IP他 $11: 8!$.

In how many ways the letters of the word PENCIL be arranged so that $N$ is always next to E ?

## Prove that $\cos (A+B) \cos (A-B)=\cos ^{2} B-\sin ^{2} A$.

Find the equation of the straight lines passing through $(8,3)$ and having intercepts whose sum is 1 .

Prove that $\cos (A+B) \cos (A-B)=\cos ^{2} A-\sin ^{2} B=\cos ^{2} B-\sin ^{2} A$.
Find the value of $\tan \pi / 12$.
Prove that $n!+(n+1)!=n!(n+2)$.
Find the equation of the straight line passing through the points
$(1,1)$ and $(5,8)$.
Write the identities of $\cos 2 \mathrm{~A}$.
Find 4 numbers G1,G2,G3,G4 so that the sequence 12 ,G1,G2,G3,G4, 3/8 is in geometric progression.

Find the distinct permutations of the letters of the word MISSISSIPPI.
Find the value of $\cos 15^{\circ}$.

> If $A$ and $B$ are square matrices of order 3 such that $|A|=-1$, $|B|=3$ find the value of $|3 A B|$

Find $(\vec{a}+3 \vec{b}) \cdot(2 \vec{a}-\vec{b})$ if $\vec{a}=\vec{i}+\vec{j}+2 \vec{k}$ and
$\vec{b}=3 \vec{i}+2 \vec{j}-\vec{k}$

## P.T. medians of a triangle concurrent by vector method.

Find the arca of the triangle whose verties are $(-2,-3),(3,2),(-1,-8)$
For any tro vectors $\bar{a}$ and $\bar{b}$ prove that $|\vec{a} X \vec{b}|^{2}+(\vec{a} \bar{b})^{2}=|\vec{a}||\vec{b}|^{2}$
Find the angle between the vectors $5 \hat{i}+3 \hat{j}+4 \hat{k}$ and $6 \hat{i}+8 \hat{j}+\hat{k}$.

$$
\text { Prove that } \lim _{x \rightarrow 0} \frac{x^{n}-a^{n}}{x-a}=n a^{n-1} .
$$

Evaluate $\quad, \ldots \circ \frac{x^{4}-16}{x-2}$

Construct the matrix $A=\left[a_{11}\right]_{1,1}$ where $a_{11}=1$ state whother $A$ is iymmetr: or skew - symmetric.

If $f$ and $g$ are continuous functions with $f(3)=5$ and $\operatorname{Lim}_{x, 3}[2 f(x)-g(x)]=4$ find $g(3)$. (b)


For what value of $\sigma$ in $[0,2 \pi]$ such that matrix $\left\lvert\, \begin{aligned} & 2 \\ & \sin \\ & \cos \end{aligned}\right.$
$\left|\begin{array}{ccc}2 \sin \theta-1 & \sin \theta & \cos \theta \\ \sin (\theta+\pi) & 2 \cos \theta-\sqrt{3} & \tan \theta \\ \cos (\theta-\pi) & \tan (\pi-\theta) & 0\end{array}\right|$ is Skew symmetric.

Also write down the Skew - symmetric matrix.
For any two vector $\vec{a}$ and $\vec{b}$. Prove that i) $|\vec{a}+\vec{b}| \leq|\vec{a}|+|\vec{b}|$ and ii) $|\vec{a}, \vec{b}| \leq|\vec{a}||\vec{b}|$
a) By vector method, prove that internal angle bisectors of a triangle are concurren
b) If $\mathrm{x}, \mathrm{y}, \mathrm{z}$ are different. $\left|\begin{array}{lll}x & x^{2} & 1+x^{\prime} \\ y^{\prime} & y^{2} & 1+y^{\prime} \\ z & z^{2} & 1+z^{\prime}\end{array}\right|=0$ then show that $\mathrm{xyz}+1=0$.

Evaluate $\left|\begin{array}{lll}2014 & 2017 & 0 \\ 2020 & 2023 & 1 \\ 2023 & 2026 & 0\end{array}\right|$
For any vector $\dot{r}$ prove that $\dot{r}=(\dot{f}, i) i+(\dot{i} . j) j+(\dot{j} \cdot \dot{k} \mid \dot{k}$.
$|\vec{a}|=5,|\vec{b}|=6,|\vec{c}|=7$ and $\vec{a}+\vec{b}+\vec{c}=\overrightarrow{0}$ then find the value of $\vec{a} \cdot \vec{b}+\vec{b} \cdot \vec{c}+\vec{c} \cdot \vec{a}$
If $\dot{a}, \bar{b}$ and $\bar{c}$ are three unit vectors satisfying $\vec{a}-\sqrt{3 b}+\vec{c}=\overrightarrow{0}$ then find the angle between $\bar{a}$ and $\bar{c}$.

For any two vectors $\vec{a}$ and $\vec{b}$ prove that
$|\vec{a} \times \vec{b}|^{2}+(\vec{a} \cdot \vec{b})^{2}=|\vec{a}|^{2}|\vec{b}|^{2}$
Find the area of the triangle whose vertices are $A(3,-$
$1,2) B(1,-1,-3)$ and $C(4,-3,1)$
Differentiate: $y=x \log x w r t x$
A die is rolled. If it shows an odd number, find the probability of getting 5 .
Integrate with respect to $x:\left(1+x^{2}\right)^{-1}$
If $y=1+\frac{x}{1!}+\frac{x^{2}}{2!}+\frac{x^{3}}{3!}+\ldots \ldots \ldots$, then prove that $\frac{d y}{d x}=y$.

Given that $P(A)=0.52, P(B)=0.43$ and $P(A \cap B)=0.24$, find $P(A \cap \bar{B})$.
An integer is chosen at random from the first ten positive integers. Find the Probability that it is i) an even number ii) multiple of three.

If $y=\sqrt{\sin \sqrt{x}}$ find $\frac{d y}{d x}$.

Find the general solution of $\tan 4 x=\cot 2 x$.
Prove that $\left(\left(A \cup B^{\prime} \cup C\right) \cap\left(A \cap B^{\prime} \cap C^{\prime}\right)\right) \cup\left(\left(A \cup B \cup C^{\prime}\right) \cap\left(B^{\prime} \cap C^{\prime}\right)\right)=B^{\prime} \cap C$.
Integrate the following functions with respect to $x: \frac{1}{\sqrt{x+3}-\sqrt{x-4}}$
A single card is drawn from a pack of 52 cards. What is the probability that the card is: an Ace are King.
(Playing cards based sums deleted acc. To the 2023-24 academic years' portion.)
If $A$ and $B$ are mutually exclusive events then $P(A)=3 / 8, P(B)=1 / 8$, then find $P(\bar{A} \cup \bar{B})$.
fonsider the function $\mathrm{f}(\mathrm{x})=\sqrt{\mathrm{x}}, \mathrm{x}>0$. DDoes, $\lim _{x \rightarrow 0} \mathrm{f}(\mathrm{f})$ exist?

Prove that the points whase position vectors $2 \vec{i}+4 \vec{j}+3 \vec{k}, 4 \vec{i}+\vec{j}+9 \vec{k}$ and $10 \vec{i}-\vec{j}+6 \vec{k}$ from a right angled triangle.

Find the distance between the parallel lines $3 x-4 y+5=0$ and $6 x-8 y-15=0$.
Differentiate: $x^{y}=y^{x}$

