HALF YEARLY EXAMINATION - 2023

STD - XI

TIME : 3.00 Hrs **MATHS** **MARKS : 90**

PART - I

I. Answer all the questions : **20 x 1 = 20**

1. If the function f : [-3, 3] → S defined by $f(x) = x^2$ is onto, then S is
   a) [-9, 9] b) R c) [-3, 3] d) [0, 9]

2. The number of reflexive relations on a set containing n elements is
   a) $2^{\frac{n^2+n}{2}}$ b) 2^{n^2-n} c) 2^{n^2+n} d) $2^{\frac{n^2-n}{2}}$

3. The value of $\log_{\sqrt{2}} 512$ is
   a) 16 b) 18 c) 9 d) 12

4. If $\frac{1-2x}{3+2x-x^2} = \frac{A}{3-x} + \frac{B}{x+1}$, then the value of A + B is
   a) $\frac{-1}{2}$ b) $\frac{-2}{3}$ c) $\frac{1}{2}$ d) $\frac{2}{3}$

5. $\cos 1^0 + \cos 2^0 + \cos 3^0 + \ldots\ldots\ldots\ldots + \cos 179^0 =$
   a) 0 b) 1 c) -1 d) 89

6. $\sin\theta = \frac{-\sqrt{3}}{2}$, Find the principal solution
   a) $\frac{\pi}{3}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{4}$ d) $\frac{-\pi}{3}$

7. Number of sides of a polygon having 44 diagonals is
   a) 4 b) 4! c) 11 d) 22

8. In $^{2n}C_3 : {}^nC_3 = 11 : 1$ then n is
   a) 5 b) 6 c) 11 d) 7

9. If a is the arithmetic mean and g is the geometric mean of two numbers, then
   a) $a \le g$ b) $a \ge g$ c) $a = g$ d) $a > g$

10. The coefficient of x^5 in the series e^{-2x} is
    a) $\frac{2}{3}$ b) $\frac{3}{2}$ c) $\frac{-4}{15}$ d) $\frac{4}{15}$

11. The Image of the point (2, 3) in the line y = -x is
    a) (-3, -2) b) (-3, 2) c) (-2, -3) d) (3, 2)

12. The area of the triangle formed by the lines $x^2 - 4y^2 = 0$ and x = a is
    a) $2a^2$ b) $\frac{\sqrt{3}}{2}a^2$ c) $\frac{1}{2}a^2$ d) $\frac{2}{\sqrt{3}}a^2$

13. If A is a square matrix, then which of the following is not symmetric?
    a) $A + A^T$ b) AA^T c) A^TA d) $A - A^T$

14. The root of the equation $\begin{vmatrix} 3-x & -6 & 3 \\ -6 & 3-x & 3 \\ 3 & 3 & -6-x \end{vmatrix} = 0$ is

a) 6 b) 3 c) 0 d) -6

15. If ABCD is a parallelogram, then $\overrightarrow{AB}+\overrightarrow{AD}+\overrightarrow{CB}+\overrightarrow{CD}$ is equal to

a) $2(\overrightarrow{AB}+\overrightarrow{AD})$ b) $4\overrightarrow{AC}$ c) $4\overrightarrow{BD}$ d) 0

16. Find $\vec{a}.\vec{b}$ when $\vec{a}=2\vec{i}+\vec{j}-\vec{k}$ and $\vec{b}=-\vec{j}-2\vec{k}$

a) 2 b) 3 c) 1 d) 4

17. $\lim_{x\to 0} \frac{a^x-b^x}{x} =$

a) log ab b) $\log(a/b)$ c) $\log(b/a)$ d) a/b

18. If $y=\frac{1}{a-z}$, then $\frac{dz}{dy}$ is

a) $(a-z)^2$ b) $-(z-a)^2$ c) $(z+a)^2$ d) $-(z+a)^2$

19. $\int \frac{\sec x}{\sqrt{\cos 2x}} dx$ is

a) $\tan^{-1}(\sin x) + C$ b) $2\sin^{-1}(\tan x) + C$ c) $\tan^{-1}(\cos x) + C$ d) $\sin^{-1}(\tan x) + C$

20. If two events A and B are independent such that P(A) = 0.35 and $P(A\cup B) = 0.6$, then P(B) = is

a) $\frac{5}{13}$ b) $\frac{1}{13}$ c) $\frac{4}{13}$ d) $\frac{7}{13}$

PART - II

II. Answer any seven questions. Q.No. 30 is compulsory. **7 x 2 = 14**

21. Solve $|2x-3|=|x-5|$ 56

22. Find the value of $\cos(300^0)$ 104

23. If $nC_{12} = nC_9$ find $21C_n$ 186

24. Find the middle terms in the expansion of $(x+y)^6$ 207

25. Find the distance between the parallel liness. 12x + 5y = 7 and 12x + 5y + 7 = 0 272

26. Determine the value of x+y if $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$ 186

27. If $\frac{1}{2}, \frac{1}{\sqrt{2}}, a$, it are the direction cosines of some vector, then find a. 68

28. $\lim_{x\to 3} \frac{x^n-3^n}{x-3} = 27$ find the value of n. 102

29. If A and B are mutually exclusive events P(A) = $\frac{3}{8}$ and P(B) = $\frac{1}{8}$ then find (i) $p(\bar{A})$ (ii) $p(\bar{A}\cap B)$ 250

30. Evaluate $\int a^x e^x dx$

PART - III

III. Answer any seven questions. Q.No. 40 is compulsory **7 x 3 = 21**

31. If $n(P(A)) = 1024$, $n(A \cup B) = 15$ and $n(P(B)) = 32$ then find $n(A \cap B)$.

32. Simplify and hence find the value of n. $\frac{3^{2n}\, 9^2\, 3^{-n}}{3^{3n}} = 27$

33. Find the distinct permutations of the letters of the word MISSISSIPPI.

34. Write the first 6 terms of the sequences whose n^{th} term a_n is $a_n = \begin{cases} n & \text{if } n \text{ is } 1, 2, 3 \\ a_{n-1} + a_{n-2} + a_{n-3} & \text{if } n > 3 \end{cases}$

35. If $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$, then compute A^4.

36. Find λ, when the projection of $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$ on $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ is 4 units.

37. Evaluate $\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x}$

38. Evaluate $\int e^x (\tan x + \log \sec x)\, dx$

39. Eight coins are tossed once, find the probability of getting (i) exactly two tails, (ii) at least two tails.

40. Find the derivations of $x = a(\cos t + t \sin t)$, $y = a(\sin t - t \cos t)$

PART - IV

IV. Answer all the questions. **7 x 5 = 35**

41. a) If $f : R \to R$ is defined by $f(x) = 2x - 3$. Prove that f is a bijection and find its inverse.

(OR)

b) Evaluate $\lim_{x \to \infty} x\left[3^{1/x} + 1 - \cos\left(\frac{1}{x}\right) - e^{1/x}\right]$

42. a) If $\frac{\log x}{y - z} = \frac{\log y}{z - x} = \frac{\log z}{x - y}$ then prove that $xyz = 1$

(OR)

b) Evaluate $\int \frac{3x + 5}{x^2 + 4x + 7} dx$

43. a) P.T. Napier's formula

(OR)

b) If $y = \frac{\sin^{-1} x}{\sqrt{1 - x^2}}$, show that $(1 - x^2)\, y'' - 3xy' - y = 0$

44. a) By the principle of mathematical induction prove that for all integers $n \geq 1$

$$1^2 + 2^2 + \ldots\ldots\ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

(OR)

b) The slope of one of the straight lines $ax^2+2hxy+by^2 = 0$ is three times the other, show that $3h^2 = 4ab$.

45. a) $8 + 88 + 888 + \ldots\ldots\ldots\ldots$ the sum of first n terms of the series.

(OR)

b) If D and E are the midpoints of the sides AB and AC of a triangle ABC, Prove that $\overrightarrow{BE} + \overrightarrow{DC} = \frac{3}{2}\overrightarrow{BC}$

46. a) Prove that $\begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix}^2 = \begin{vmatrix} 1-2x^2 & -x^2 & -x^2 \\ -x^2 & -1 & x^2-2x \\ -x^2 & x^2-2x & -1 \end{vmatrix}$

(OR)

b) $x^4 + y^4 = 16$ find y''

47. a) Show that the vectors $-\vec{i}-2\vec{j}-6\vec{k}$, $2\vec{i}-\vec{j}+\vec{k}$ and $-\vec{i}+3\vec{j}+5\vec{k}$ form a right angled triangle. (OR)

b) The chances of A, B, C becoming manager of a certain company are 5 : 3 : 2. The probability that the office canteen will be improved of A, B and C become managers are 0.4, 0.5 and 0.3 respectively. If the office canteen has been improved, what is the probability that B was appointed as the manager?

Rajasekar. S (King of Kings Matric Hr. Sec School, Meiyampuli)

HALF YEARLY EXAM – 2023

11th std MATHEMATICS (ANSWER KEY) – RAMANATHAPURAM DISTRICT

I. CHOOSE THE BEST ANSWER

1	d) [0, 9]
2	b) $2^{n2 - n}$
3 (Qtn is Wrong)	b) 18
4	a) -1/2
5	a) 0
6	d) –π/3
7	c) 11
8	b) 6
9	b) a≥g
10	c) -4/15
11	a) (-3, 2)
12	c) ½ a^2
13	d) $A-A^T$
14	c) 0
15	d) 0
16	c) 1
17	b) log(a/b)
18	$(a-z)^2$
19	d) $Sin^{-1}(tanx)+C$
20	a) 5/13

II. 2 MARKS

21	$2x - 3 = x – 5$; $2x - 3 = -x + 5$ $x = –$ **2** ; $3x = 8 \Rightarrow x =$ **8/3**

22	cos(300°) = cos(270° + 30°) = sin 30° = 1/2
23	Given $nC_{12} = nC_9$ We have $nC_x = nC_y \Rightarrow x = y$ or $x + y = n$ $nC_{12} = nC_9$ $\Rightarrow 12 + 9 = n \Rightarrow n = 21$ $\Rightarrow 21C_n = 21C_{21} = 1$
24	Here $n = 6$; which is even. The middle term in the expansion of $(x+y)^6$ is the term containing x^3y^3, that is the term 6C3 x3y3 which is equal to **$20x^3y^3$.**
25	The equation of the given lines are 12x + 5y – 7 = 0 .... (1) 12x + 5y + 7 = 0 .... (2) The distance between the parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is The equation of any line parallel to (1) is $d = \frac{c_1 - c_2}{\sqrt{a^2 + b^2}}$ $\therefore$ The required distance $= \frac{-7 - 7}{\sqrt{12^2 + 5^2}}$ $= \frac{-14}{\sqrt{144 + 25}}$ $= -\frac{14}{\sqrt{169}} = -\frac{14}{13}$ The distance cannot be negative $\therefore$ Required distance = 14/13
26	2x + y = 7 .... (1) 4x = 7y – 13 .... (2) 5x – 7 = y .... (3)

	4x = x + 6 .... (4) from (4) 4x – x = 6 3x = 6 ⇒ x = 6/3 = 2 Substituting x = 2 in (1), we get 2(2) + y = 7 ⇒ 4 + y = 7 ⇒ y = 7 – 4 = 3 So x = 2 and y = 3 ∴ x + y = 2 + 3 = 5
27	$\left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + a^2 = 1$ $\frac{1}{4} + \frac{1}{2} + a^2 = 1$ $\frac{1+2}{4} + a^2 = 1$ $a^2 = 1 - \frac{3}{4}$ $= \frac{4-3}{4} = \frac{1}{4}$ $a = \pm \frac{1}{2}$
28	$\lim_{x \to 3} \frac{x^n - 3^n}{x - 3} = n.3^{n-1} = 27$ That is $n.3^{n-1} = 3 \times 3^2 = 3 \times 3^{3-1} \Rightarrow n = 3$.
29	(i) P(Ā) P(Ā) = 1 – P(A) = 1 – 3/8 P(Ā) = 8–3/8 = 5/8 (ii) P(Ā ∪ B̄) P(Ā ∪ B̄) = 1 – P(A ∩ B)

	Since A and B are mutually exclusive we have A ∩ B = Φ ∴ P(A ∩ B) = 0 $P(\bar{A} \cup \bar{B}) = 1 - 0 = 1$
30	$\int a^x e^x dx = \int (ae)^x dx = \frac{(ae)^x}{\log(ae)} + c$

III.

31	$n(P(A)) = 1024 = 2^{10}$ n(A) = 10 $n(P(B)) = 32 = 2^5 = n(B) = 5$ n(A ∪ B) = n(A) + n(B) – n(A ∩ B) 15 = 10 + 5 – n(A ∩ B) 15 = 15 – n (A ∩ B) n(A ∩ B) = 0
32	Given $\frac{3^{2n} \cdot 9^2 \cdot 3^{-n}}{3^{3n}} = 27$ $\frac{3^{2n-n} \cdot (3^2)^2}{3^{3n}} = 3^3$ $3^n \cdot (3^2)^2 \cdot 3^{-3n} = 3^3$ $3^n \cdot 3^4 \cdot 3^{-3n} = 3^3$ $3^{n+4-3n} = 3^3$ $3^{4-2n} = 3^3$ 4 – 2n = 3 => 2n = 4 – 3 => 2n = 1 => n = 1/2
33	MISSISSIPPI : Number of letters in the word = 11 Number of S's = 4 Number of I's = 4 Number of P's = 2 Number of M's = I Hence the total number of distinct words

	$= \frac{11!}{4! \times 4! \times 2! \times 1!}$ $= \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4!}{4 \times 3 \times 2 \times 1 \times 4! \times 2 \times 1 \times 1}$ $= \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 2}$ $= \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5}{8 \times 6}$ $= 11 \times 10 \times 9 \times 7 \times 5$ $= 34,650$
34	$n = 1, a_n = n, a_1 = 1$ $n = 2, a_n = n, a_2 = 1$ $n = 3, a_n = n, a_3 = 1$ $n = 4, a_n = a_{n-1} + a_{n-2} + a_{n-3}$ $a_4 = a_{4-1} + a_{4-2} + a_{4-3}$ $a_4 = 3 + 2 + 1 = 6$ $n = 5, a_n = a_{n-1} + a_{n-2} + a_{n-3}$ $a_5 = a_{5-1} + a_{5-2} + a_{5-3}$ $a_5 = 6 + 3 + 2 = 11$ $n = 6, a_n = a_{n-1} + a_{n-2} + a_{n-3}$ $a_6 = a_{6-1} + a_{6-2} + a_{6-3}$ $a_6 = 11 + 6 + 3 = 20$ ∴ The first six terms are 1, 2, 3, 6, 11, 20
35	$A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$ $A^2 = A \cdot A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$ $= \begin{bmatrix} 1+0 & a+a \\ 0+0 & 0+1 \end{bmatrix}$ $A^2 = \begin{bmatrix} 1 & 2a \\ 0 & 1 \end{bmatrix}$ $A^4 = A^2 \times A^2$ $= \begin{bmatrix} 1 & 2a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2a \\ 0 & 1 \end{bmatrix}$ $A^4 = \begin{bmatrix} 1+0 & 2a+2a \\ 0+0 & 0+1 \end{bmatrix}$ $A^4 = \begin{bmatrix} 1 & 4a \\ 0 & 1 \end{bmatrix}$

36	$\dfrac{\vec{a}\cdot\vec{b}}{\lvert\vec{b}\rvert} = 4$ $\dfrac{(\lambda\hat{i}+\hat{j}+4\hat{k})\cdot(2\hat{i}+6\hat{j}+3\hat{k})}{\lvert 2\hat{i}+6\hat{j}+3\hat{k}\rvert} = 4$ $\dfrac{(\lambda)(2)+(1)(6)+(4)(3)}{\sqrt{2^2+6^2+3^2}} = 4$ $\dfrac{2\lambda+6+12}{\sqrt{4+36+9}} = 4$ $\dfrac{2\lambda+18}{\sqrt{49}} = 4$ $2\lambda+18 = 4\times 7$ $2\lambda = 28-18$ $2\lambda = 10 \Rightarrow \lambda = 5$
37	$\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-1}{x} = \lim\limits_{x\to 0}\dfrac{(\sqrt{1+x}-1)}{x}\times\dfrac{\sqrt{1+x}+1}{\sqrt{1+x}+1}$ $= \lim\limits_{x\to 0}\left[\dfrac{(1+x)-1}{x(\sqrt{1+x}+1)}\right]$ $= \lim\limits_{x\to 0}\left[\dfrac{x}{x(\sqrt{1+x}+1)}\right]$ $= \lim\limits_{x\to 0}\left[\dfrac{1}{\sqrt{1+x}+1}\right]$ $= \dfrac{1}{\sqrt{1+0}+1} = \dfrac{1}{1+1}$ $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-1}{x} = \dfrac{1}{2}$
38	Let I = $\int e^x(\tan x + \log\sec x)\,dx$ Take f(x) = log sec x f'(x) = 1/secx (sec x tan x) f'(x) = tan x $[\int e^x[f(x)+f(x)]\,dx = e^x f(x) + c]$ ∴ I = $e^x \log\lvert\sec x\rvert + c$

39	$n(S) = 2^8 = 256$ $n(A) = 8C_2$ $= 8\times7/2 = 28$ $n(B) = 8C_2 + 8C_3 + 8C_4 + 8C_5 + 8C_6 + 8C_7 + 8C_8$ $= n(S) - (8C_8 + 8C_1)$ $= n(S) - (1 + 8)$ $= 256 - 9 = 247$ (i) P {getting exactly two tails) = $P(A) = \frac{n(A)}{n(S)}$ $P(A) = \frac{28}{256} = \frac{7}{64}$ **(ii)** P (getting atleast two tails) = $P(B) = \frac{n(B)}{n(S)}$ $P(B) = \frac{247}{256}$
40. Qtn is Wrong	$x = a(\cos t + t \sin t)$, $y = a(\sin t - t\cos t)$ $dx/dt = a[-\sin t + t\cos t + \sin t]$ $dx/dt = at\cos t$ —— (1) $y = a(\sin t - t\cos t)$ $dy/dt = a[\cos t - (t \times -\sin t + \cos t \times 1)]$ $dy/dt = a[\cos t + t\sin t - \cos t]$ $dy/dt = at\sin t$ —— (2) From equations (1) and (2) we get

	$\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{a\,t\ \sin t}{a\,t\ \cos t}$ $\dfrac{dy}{dx} = \tan t$

III.

41. a)	Let $y = 2x - 3$. Then $x = \frac{y+3}{2}$. Let $g(y) = \frac{y+3}{2}$. Now $(g \circ f)(x) = g(f(x)) = g(2x-3) = \frac{(2x-3)+3}{2} = x.$ $(f \circ g)(y) = f(g(y)) = f\left(\frac{y+3}{2}\right) = 2\left(\frac{y+3}{2}\right) - 3 = y.$ Thus, $g \circ f = I_X$ and $f \circ g = I_Y$ This implies that f and g are bijections and inverses to each other. Hence f is a bijection and $f^{-1}(y) = \frac{y+3}{2}$. Replacing y by x we get, $f^{-1}(x) = \frac{x+3}{2}$.
b)	We know $\lim_{x\to 0} \frac{e^x - 1}{x} = 1$, $\lim_{x\to 0} \frac{a^x - 1}{x} = \log a$, $\lim_{x\to 0} \frac{1-\cos x}{x} = 0$ $\lim_{x\to\infty} x\left[3^{\frac{1}{x}} + 1 - \cos\left(\frac{1}{x}\right) - e^{\frac{1}{x}}\right] = \lim_{x\to\infty}\left[\dfrac{3^{\frac{1}{x}} + 1 - \cos\left(\frac{1}{x}\right) - e^{\frac{1}{x}}}{\frac{1}{x}}\right]$ $= \lim_{x\to\infty}\left[\dfrac{3^{\frac{1}{x}} - e^{\frac{1}{x}}}{\frac{1}{x}} + \dfrac{1-\cos\left(\frac{1}{x}\right)}{\frac{1}{x}}\right]$

	$= \lim_{x\to\infty}\left[\dfrac{3^{\frac{1}{x}} - 1 + 1 - e^{\frac{1}{x}}}{\frac{1}{x}} + \dfrac{1-\cos\left(\frac{1}{x}\right)}{\frac{1}{x}}\right]$ $= \lim_{x\to\infty}\left[\dfrac{\left(3^{\frac{1}{x}} - 1\right) - \left(e^{\frac{1}{x}} - 1\right)}{\frac{1}{x}} + \dfrac{1-\cos\left(\frac{1}{x}\right)}{\frac{1}{x}}\right]$ $= \lim_{x\to\infty}\left[\dfrac{3^{\frac{1}{x}} - 1}{\frac{1}{x}} - \dfrac{e^{\frac{1}{x}} - 1}{\frac{1}{x}} + \dfrac{1-\cos\left(\frac{1}{x}\right)}{\frac{1}{x}}\right]$ Put $y = \frac{1}{x}$, When $x = \infty \Rightarrow y = \frac{1}{\infty} = 0$ $\lim_{x\to\infty} x\left[3^{\frac{1}{x}} + 1 - \cos\left(\frac{1}{x}\right) - e^{\frac{1}{x}}\right] = \lim_{y\to 0}\left[\dfrac{3^y - 1}{y} - \dfrac{e^y - 1}{y} + \dfrac{1-\cos y}{y}\right]$ $= \left(\lim_{y\to 0}\dfrac{3^y - 1}{y}\right) - \left(\lim_{y\to 0}\dfrac{e^y - 1}{y}\right) + \left(\lim_{y\to 0}\dfrac{1-\cos y}{y}\right)$ $= \log 3 - 1 + 0$ $\lim_{x\to\infty} x\left[3^{\frac{1}{x}} + 1 - \cos\left(\frac{1}{x}\right) - e^{\frac{1}{x}}\right] = (\log 3) - 1$
42. a)	**Let logx/y−z = k** **log x = k(y – z)** **log x = ky – kz ——— (1)** **Similarly log y = k(z – x) = kz – kx ——(2)** **log z = k(x – y) = kx – ky ——- (3)** **Adding (1), (2) and (3)** **log x + log y + log z = ky – kz + kz – kx + kx – ky** **log (xyz) = 0** $\quad$ **xyz =** e^0 **xyz = 1**

b)	Let $I=\int\frac{3x+5}{x^2+4x+7}dx$ $3x+5=A\frac{d}{dx}(x^2+4x+7)+B$ $3x+5=A(2x+4)+B$ Comparing the coefficients of like terms, we get $2A=3\Rightarrow A=\frac{3}{2};4A+B=5\Rightarrow B=-1$ $I=\int\frac{\frac{3}{2}(2x+4)-1}{x^2+4x+7}dx$ $I=\frac{3}{2}\int\frac{2x+4}{x^2+4x+7}dx-\int\frac{1}{x^2+4x+7}dx$ $=\frac{3}{2}\log\lvert x^2+4x+7\rvert-\int\frac{1}{(x+2)^2+(\sqrt{3})^2}dx$ $=\frac{3}{2}\log\lvert x^2+4x+7\rvert-\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x+2}{\sqrt{3}}\right)+c$
43. a)	(i) $\tan\frac{A-B}{2}=\frac{a-b}{a+b}\cot\frac{C}{2}$ (ii) $\tan\frac{B-C}{2}=\frac{b-c}{b+c}\cot\frac{A}{2}$ (iii) $\tan\frac{C-A}{2}=\frac{c-a}{c+a}\cot\frac{B}{2}$ We know the sine formula: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=2R$ Now, $\frac{a-b}{a+b}\cot\frac{C}{2}=\frac{2R\sin A-2R\sin B}{2R\sin A+2R\sin B}\cot\frac{C}{2}$ $=\frac{\sin A-\sin B}{\sin A+\sin B}\cot\frac{C}{2}$ $=\frac{2\cos\frac{A+B}{2}\sin\frac{A-B}{2}}{2\sin\frac{A+B}{2}\cos\frac{A-B}{2}}\cot\frac{C}{2}$ $=\cot\frac{A+B}{2}\tan\frac{A-B}{2}\cot\frac{C}{2}$ $=\cot\left(90^\circ-\frac{C}{2}\right)\tan\frac{A-B}{2}\cot\frac{C}{2}$ $=\tan\frac{C}{2}\tan\frac{A-B}{2}\cot\frac{C}{2}=\tan\frac{A-B}{2}$

<table>
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<td>b)
Qtn is Wrong</td>
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$$y = \frac{\sin^{-1} x}{\sqrt{1 - x^2}}$$

$$y\sqrt{1 - x^2} = \sin^{-1} x$$

Differentiating with respect to x

$$y \times \frac{1}{2}\left(1 - x^2\right)^{\frac{1}{2}-1}(-2x) + \sqrt{1 - x^2}\, y_1 = \frac{1}{\sqrt{1 - x^2}}$$

$$-xy\left(1 - x^2\right)^{-\frac{1}{2}} + \sqrt{1 - x^2}\, y_1 = \frac{1}{\sqrt{1 - x^2}}$$

$$\left(1 - x^2\right)^{\frac{1}{2}}\left[-xy\left(1 - x^2\right)^{-\frac{1}{2}} + \left(1 - x^2\right)^{\frac{1}{2}} y_1\right] = 1$$

$$-xy\left(1 - x^2\right)^{-\frac{1}{2}} \times \left(1 - x^2\right)^{\frac{1}{2}} + \left(1 - x^2\right)^{\frac{1}{2}} \cdot \left(1 - x^2\right)^{\frac{1}{2}} y_1 = 1$$

$$-xy + (1 - x^2)\, y_1 = 1$$

Differentiating with respect to x, we get

$$-x \cdot y_1 + y(-1) + (1 - x^2)\, y_2 + y_1(0 - 2x) = 0$$

$$-xy_1 - y + (1 - x^2)\, y_2 - 2xy_1 = 0$$

$$(1 - x^2)\, y_2 - 3xy_1 - y = 0$$

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<td>44. a)</td>
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Let,

$$P(n) := 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

Substituting $n = 1$ in the statement we get, $P(1) = \frac{1(1+1)(2(1)+1)}{6} = 1$. Hence, $P(1)$ is true. Let us assume that the statement is true for $n = k$. Then

$$P(k) = 1^2 + 2^2 + 3^2 + \cdots + k^2 = \frac{k(k+1)(2k+1)}{6}.$$

We need to show that $P(k+1)$ is true. Consider

$$\begin{aligned} P(k+1) &= \underbrace{1^2 + 2^2 + 3^2 + \cdots + k^2} + (k+1)^2 \\ &= P(k) + (k+1)^2 \\ &= \frac{k(k+1)(2k+1)}{6} + (k+1)^2 \\ &= \frac{k(k+1)(2k+1) + 6(k+1)^2}{6} \\ &= \frac{(k+1)\,(k(2k+1) + 6(k+1))}{6} \\ &= \frac{(k+1)\,(2k^2 + 7k + 6)}{6} \\ &= \frac{(k+1)\,[(k+2)(2k+3)]}{6} \\ &= \frac{(k+1)\,[((k+1)+1)(2(k+1)+1)]}{6}. \end{aligned}$$

That is,

$$P(k+1) = \frac{(k+1)((k+1)+1)\,(2(k+1)+1)}{6}.$$

This implies, $P(k+1)$ is true. The validity of $P(k+1)$ follows from that of $P(k)$. Therefore by the principle of mathematical induction,

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b) (Qtn is wrong)	$ax^2 + 2hxy + by^2 = 0$ .... (1) Given that the slopes of the lines are m and 3m. $\therefore \quad m + 3m = -\frac{2h}{b}$ $(m)(3m) = \frac{a}{b}$ $4m = -\frac{2h}{b}$ and $3m^2 = \frac{a}{b}$ $m = -\frac{h}{2b} \Rightarrow 3\left(-\frac{h}{2b}\right)^2 = \frac{a}{b}$ $\Rightarrow 3\frac{h^2}{4b^2} = \frac{a}{b}$ $\Rightarrow 3h^2 = 4ab$
45. a)	Let $S_n = 8 + 88 + 888 + 8888 + \ldots\ldots$ up to n terms $= 8(1 + 11 + 111 + \ldots\ldots$ up to n terms$)$ $= \frac{8}{9} \times 9\,(1 + 11 + 111 + \ldots\ldots$ up to n terms$)$ $= \frac{8}{9}(9 + 99 + 999 + \ldots\ldots$ up to n terms $= \frac{8}{9}[(10-1) + (100-1) + (1000-1) + \ldots\ldots$ up to n terms$]$ $= \frac{8}{9}[10 + 100 + 1000 + \ldots\ldots$ n terms$) - (1 + 1 + 1 + 1 + \ldots\ldots + 1$ n times$)]$ $= \frac{8}{9}[10^1 + 10^2 + 10^3 + \ldots\ldots + 10^n - n]$ $= \frac{8}{9}\left[\frac{10(10^n - 1)}{10 - 1} - n\right]$ $= \frac{8}{9}\left[\frac{10(10^n - 1)}{9} - n\right] \qquad \because S_n = \frac{a(r^n - 1)}{r - 1}$ $= \frac{8}{9}\left[\frac{10(10^n - 1) - 9n}{9}\right]$ $= \frac{8}{81}\left[10(10^n - 1) - 9n\right]$

b)	Let O be the origin. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be the position vectors of the points A, B and C respectively Then $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, $\overrightarrow{OC} = \vec{c}$ Given D is the mid point of AB $\therefore \overrightarrow{OD} = \frac{\overrightarrow{OA} + \overrightarrow{OB}}{2}$ $\overrightarrow{OD} = \frac{\vec{a} + \vec{b}}{2}$ Also given E is the mid point of AC $\therefore \overrightarrow{OE} = \frac{\overrightarrow{OA} + \overrightarrow{OC}}{2}$ $\overrightarrow{OE} = \frac{\vec{a} + \vec{c}}{2}$ $\overrightarrow{BE} = \overrightarrow{OE} - \overrightarrow{OB}$ $= \frac{\vec{a} + \vec{c}}{2} - \vec{b}$ $\overrightarrow{BE} = \frac{\vec{a} + \vec{c} - 2\vec{b}}{2}$ $\overrightarrow{DC} = \overrightarrow{OC} - \overrightarrow{OD}$ $= \vec{c} - \frac{\vec{a} + \vec{b}}{2}$ $\overrightarrow{DC} = \frac{2\vec{c} - \vec{a} - \vec{b}}{2}$ $\overrightarrow{BE} + \overrightarrow{DC} = \frac{\vec{a} + \vec{c} - 2\vec{b}}{2} + \frac{2\vec{c} - \vec{a} - \vec{b}}{2}$ $= \frac{\vec{a} + \vec{c} - 2\vec{b} + 2\vec{c} - \vec{a} - \vec{b}}{2}$ $= \frac{3\vec{c} - 3\vec{b}}{2}$ $= \frac{3}{2}(\vec{c} - \vec{b})$ $= \frac{3}{2}(\overrightarrow{OC} - \overrightarrow{OB})$ $\overrightarrow{BE} + \overrightarrow{DC} = \frac{3}{2}\overrightarrow{BC}$

46. a)	$$\begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix}^2 = \begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix} \times \begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix}.$$ $$= \begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix} \times (-1)(-1) \begin{vmatrix} 1 & x & x \\ -x & -1 & -x \\ -x & -x & -1 \end{vmatrix}$$ $$= \begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix} \times \begin{vmatrix} 1 & x & x \\ -x & -1 & -x \\ -x & -x & -1 \end{vmatrix}$$ $$= \begin{vmatrix} 1-x^2-x^2 & x-x-x^2 & x-x^2-x \\ x-x-x^2 & x^2-1-x^2 & x^2-x-x \\ x-x^2-x & x^2-x-x & x^2-x^2-1 \end{vmatrix}$$ $$= \begin{vmatrix} 1-2x^2 & -x^2 & -x^2 \\ -x^2 & -1 & x^2-2x \\ -x^2 & x^2-2x & -1 \end{vmatrix}.$$
b)	We have $x^4+y^4=16$. Differentiating implicitly, $4x^3+4y^3y'=0$ Solving for y' gives $$y' = -\frac{x^3}{y^3}.$$ To find y'' we differentiate this expression for y' using the quotient rule and remembering that y is a function of x. $$y'' = \frac{d}{dx}\left(\frac{-x^3}{y^3}\right) = \frac{-\left[y^3\frac{d}{dx}(x^3)-x^3\frac{d}{dx}(y^3)\right]}{(y^3)^2}$$ $$= -\frac{\left[y^3.3x^2-x^3(3y^2y')\right]}{y^6}$$ $$= -\frac{3x^2y^3-3x^3y^2\left(-\frac{x^3}{y^3}\right)}{y^6}$$ $$= -\frac{3(x^2y^4+x^6)}{y^7} = \frac{-3x^2[x^4+y^4]}{y^7}$$ $$= \frac{-3x^2(16)}{y^7} = \frac{-48x^2}{y^7}.$$

47. a)

Let $\vec{AB} = -\hat{i} - 2\hat{j} - 6\hat{k}$, $\vec{BC} = 2\hat{i} - \hat{j} + \hat{k}$

and $\vec{CA} = -\hat{i} + 3\hat{j} + 5\hat{k}$

$$|\vec{AB}| = |-\hat{i} - 2\hat{j} - 6\hat{k}|$$

$$= \sqrt{(-1)^2 + (-2)^2 + (-6)^2}$$

$$AB = \sqrt{1 + 4 + 36} = \sqrt{41}$$

$$|\vec{BC}| = |2\hat{i} - \hat{j} + \hat{k}|$$

$$= \sqrt{2^2 + (-1)^2 + 1^2}$$

$$BC = \sqrt{4 + 1 + 1} = \sqrt{6}$$

$$|\vec{CA}| = |-\hat{i} + 3\hat{j} + 5\hat{k}|$$

$$= \sqrt{(-1)^2 + 3^2 + 5^2}$$

$CA = \sqrt{35}$

$AB \neq BC + CA$ ∴ The given vectors form a triangle, Also

$AB^2 = 41, BC^2 = 6, CA^2 = 35$

$AB^2 = BC^2 + CA^2$

∴ Δ ABC is a right angled triangle.

b)

$$P(A_2/B) = \frac{P(A_2) \cdot P(B/A_2)}{P(A_1) \cdot P(B/A_1) + P(A_2) \cdot P(B/A_2) + P(A_3) \cdot P(B/A_3)}$$

Given $P(A_1) = \frac{5}{10}$, $P(B/A_1) = 0.4$

$P(A_2) = \frac{3}{10}$ $P(B/A_2) = 0.5$

$P(A_3) = \frac{2}{10}$, $P(B/A_3) = 0.3$

$$P(A_2/B) = \frac{\frac{3}{10} \times 0.5}{\frac{5}{10} \times .0.4 + \frac{3}{10} \times 0.5 + \frac{2}{10} \times 0.3}$$

$$= \frac{\frac{0.15}{10}}{\frac{2.0 + 1.5 + 0.6}{10}}$$

$$P(A_2/B) = \frac{0.15}{4.1} = \frac{15}{41}$$