V11M

Virudhunagar District
Common Half Yearly Examination - December 2023

Standard 11

Time Allowed: 3.00 Hours   **MATHEMATICS**   Maximum Marks: 90

PART - A

Choose the best answer: **20×1=20**

1) The number of relations on a set containing 3 elements is
   a) 9   b) 81   c) 512   d) 1024

2) If the function f:[−3, 3] → S defined by f(x) = x^2 is onto, then S is
   a) [−9, 9]   b) R   c) [−3, 3]   d) [0, 9]

3) If |x−9| < 2 then
   a) 7 < x < 11   b) x = 11   c) 7 > x > 11   d) x ≠ 7

4) The number of solution of x^2+|x−1| = 1 is
   a) 1   b) 0   c) 2   d) 3

5) The square root of $7 - 4\sqrt{3}$ is
   a) $2+\sqrt{3}$ b) $3-\sqrt{2}$ c) $2-\sqrt{3}$ d) $\sqrt{3}+2$

6) The maximum value of 4 $\sin^2 x$ + 3 $\cos^2 x$ + $\sin\frac{x}{2}$ + $\cos\frac{x}{2}$ is
   a) $4+\sqrt{2}$ b) $\sqrt{3}+2$ c) 9   d) 4

7) cos 1°+cos 2° + cos 3° + ... + cos 179° =
   a) 0   b) 1   c) −1   d) 89

8) The value of 1+3+5+7+....+17 is
   a) 101   b) 81   c) 71   d) 61

9) Number of sides of a polygon having 44 diagonal is
   a) 4   b) 4!   c) 11   d) 22

10) The remainder when 38^{15} is divided by 13 is
    a) 12   b) 1   c) 11   d) 5

11) The coefficient of x^5 in the series e^{-2x} is
    a) $\frac{2}{3}$ b) $\frac{3}{2}$ c) $-\frac{4}{15}$ d) $\frac{4}{15}$

12) The image of the part (2, 3) in the line y = −x is
    a) (−3, −2)   b) (−3, 2)   c) (−2, −3)   d) (3, 2)

13) If the points (x, −2) (5, 2) (8, 8) are collinear then the value of x is
    a) −3   b) $\frac{1}{3}$ c) 1   d) 3

14) If A is a square matrix and |A| = 2 then the value of |AA^T| is
    a) 0   b) 1   c) 2   d) 4

15) $\vec{a}+2\vec{b}$ and $3\vec{a}+m\vec{b}$ are parallel then the value of m is
    a) 3   b) $\frac{1}{3}$ c) 6   c) $\frac{1}{6}$

16) If $\vec{a} = \vec{i}+\vec{j}+\vec{k}$, $\vec{b} = 2\vec{i}+x\vec{j}+\vec{k}$, $\vec{c} = \vec{i}-\vec{j}+4\vec{k}$ and $\vec{a}\cdot(\vec{b}\times\vec{c}) = 70$ then the value of x is
    a) 5   b) 7   c) 26   d) 10

17) The value of $\lim\limits_{x \to 0} \dfrac{\sin x}{\sqrt{x^2}}$ is
    a) 1   b) −1   c) 0   d) ∞

18) $\lim_{n \to \infty} \left(\dfrac{1}{n^2} + \dfrac{2}{n^2} + \dfrac{3}{n^3} + \ldots + \dfrac{n}{n^2} \right) =$

a) $\frac{1}{2}$ b) 0 c) 1 d) ∞

19) If $y = \dfrac{1}{a-z}$ then $\dfrac{dz}{dy}$ is

a) $(a-z)^2$ b) $-(z-a)^2$ c) $(z+a)^2$ d) $-(z+a)^2$

20) If $Pv = 81$ then the value of $\dfrac{dp}{dv}$ at $v = 9$ is

a) 1 b) –1 c) 2 d) –2

PART - B

i) **Answer any seven questions only.**

ii) **Answer the question number 30 compulsory.** 7×2=14

21) If $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$ find $n((A \cup B) \times (A \cap B) \times (A \Delta B))$

22) Find the general solution of $\sin \theta = -\frac{\sqrt{3}}{2}$

23) If $(n+2)P_4 = 42 \times nP_2$ find n

24) Find $\sqrt[3]{1001}$ approximately (two decimal places)

25) Find the distance between the parallel lines $12x+5y = 7$ and $12x+5y+7 = 0$

26) Prove that $\begin{vmatrix} \sec^2\theta & \tan^2\theta & 1 \\ \tan^2\theta & \sec^2\theta & -1 \\ 38 & 36 & 2 \end{vmatrix} = 0$

27) Find the area of the parallelogram whose adjacent sides are $\bar{a} = 3\bar{i} + \bar{j} + 4\bar{k}$ and $\bar{b} = \bar{i} - \bar{j} + \bar{k}$

28) Evaluate: $\lim_{x \to 0} (1 + \sin x)^{2\cosec x}$

29) Differentiate: (i) $y = \sin(x^2)$ (ii) $y = \sin^2 x$

30) Find the complete set of values of a, for which the quadratic $x^2-ax+a+2 = 0$ has equal roots.

PART - C

i) **Answer any seven questions only.**

ii) **Answer the question number 40 compulsory.** 7×3=21

31) From the curve $y = |x|$ draw
(i) $y = |x-1|+1$ (ii) $y = |x+1|-1$ (iii) $y = |x+2|-3$

32) If $\log_2 x + \log_4 x + \log_{16} x = \frac{7}{2}$, find the value of x.

33) Find $\sin(x-y)$ given that $\sin x = \frac{8}{17}$ with $0 < x < \frac{\pi}{2}$ and $\cos y = \frac{-24}{25}$ with $\pi < y < \frac{3\pi}{2}$

34) Find the rank of the word IITJEE

35) If a, b, c are in the geometric progression and if $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$ then prove that x, y, z are in arithmatic progression.

36) If $\cos 2\theta = 0$, determine $\begin{vmatrix} \theta & \cos\theta & \sin\theta \\ \cos\theta & \sin\theta & 0 \\ \sin\theta & 0 & \cos\theta \end{vmatrix}^2$

37) If D and E are the midpoints of the sides AB and AC of a triangle ABC, prove that $\overrightarrow{BE} + \overrightarrow{DC} = \dfrac{3}{2}\overrightarrow{BC}$

38) Prove that $\displaystyle\lim_{x \to a} \dfrac{x^n - a^n}{x - a} = na^{n-1}$

39) If $y = e^{\tan^{-1}x}$, show that $(1+x^2)y^{11}+(2x-1)y^1 = 0$

40) Rewrite $\sqrt{3}x + y + 4 = 0$ into normal form.

PART - D

Answer all the questions.   7×5=35

41) If $f:R \to R$ is defined by $f(x) = 3x-5$, prove that f is bijective and find its inverse.

(OR)

If $y = \dfrac{\sin^{-1}x}{\sqrt{1-x^2}}$, show that $(1-x^2)y_2 - 3xy_1 - y = 0$

42) Resolve into partial fractions: $\dfrac{x+12}{(x+1)^2(x-2)}$

(OR)

State and prove Napier's formula.

43) Find all values of x that satisfies the inquality $\dfrac{2x-3}{(x-2)(x-4)} < 0$

(OR)

Prove that for any natural number n, $a^n - b^n$ is divisble by $a-b$, when $a > b$.

44) Prove that points whose position vectors $2\vec{i}+4\vec{j}+3\vec{k}$, $4\vec{i}+\vec{j}+9\vec{k}$ and $10\vec{i}-\vec{j}+6\vec{k}$ form a right angled triangle.

(OR)

A 150 m long train is moving with constant velocity of 12.5 m/s. Find (i) the equation of the motion of the train (ii) time taken to cross a pole (iii) the time taken to cross the bridge of length 850 m is?

45) Show that $\displaystyle\lim_{x \to 0^+} x\left[\left\lfloor\dfrac{1}{x}\right\rfloor + \left\lfloor\dfrac{2}{x}\right\rfloor + \cdots + \left\lfloor\dfrac{15}{x}\right\rfloor\right] = 120$

(OR)

Prove that $|A| = \begin{vmatrix} (q+r)^2 & p^2 & p^2 \\ q^2 & (r+p)^2 & q^2 \\ r^2 & r^2 & (p+q)^2 \end{vmatrix} = 2pqr(p+q+r)^3$

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46) 7 relatives of a man comprises 4 ladies and 3 gentlemen, his wife also has 7 relatives, 3 of them are ladies and 4 gentlemen. In how many ways can they invite a dinner party of 3 ladies and 3 gentlemen so that there are 3 of man's relative and 3 of the wife's relatives?

(OR)

Prove that $\sqrt[3]{x^3 + 6} - \sqrt[3]{x^3 + 3}$ is approximately equal to $1/x^2$ when x is sufficiently large.

47) Show that the equation $9x^2 - 24xy + 16y^2 - 12x + 16y - 12 = 0$ represents a pair no parallel lines. Find the distance between them.

(OR)

Prove that the medians of a triangle are concurrent.

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