FIRST REVISION TEST - 2024 | Exam No.

Time : 3-00 Hours **XII - MATHS** *Marks : 90*

PART - I

Note: i) Answer all the questions.
ii) Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer. ***(20x1=20)***

1. If A and B are orthogonal then $(AB)^T (AB)$ is
   1) A   2) B
   3) I   4) A^T
2. If $0 \le \theta \le \pi$ and the system of equations $x+(\sin\theta)y-(\cos\theta)z=0$, $(\cos\theta)x-y+z=0$, $(\sin\theta)x+y-z=0$ has a non trivial solution then θ is
   1) $2\pi/3$ 2) $3\pi/4$
   3) $5\pi/6$ 4) $\pi/4$
3. The solution of the equation $|z|-z=1+2i$ is
   1) $\frac{3}{2}-2i$ 2) $-\frac{3}{2}+2i$
   3) $2-\frac{3}{2}i$ 4) $2+\frac{3}{2}i$
4. If $\omega=\cos\frac{2\pi}{3}$ then the number of distinct roots of $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix}=0$
   1) 1   2) 2   3) 3   4) 4
5. If $x^3+12x^2+10ax+1999$ definitely has a positive zero, if and only if
   1) $a<0$ 2) $a>0$ 3) $a\le0$ 4) $a\ge0$
6. If $\cot^{-1}2$ and $\cot^{-1}3$ are two angles of a triangle then the third angle is
   1) $\pi/4$ 2) $3\pi/4$
   3) $\pi/3$ 4) $\pi/6$
7. The area of quadrilateral formed with foci of the hyperbolas $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$
   1) $4(a^2+b^2)$ 2) $2(a^2+b^2)$ 3) a^2+b^2 4) $1/2(a^2+b^2)$
8. $y^2-2x-2y+5=0$ is a
   a) circle with centre (1, 1)   2) parabola with vectar (1, 2)
   c) parabola with directrix $x=3/2$ 4) parabola with directrix $x=1/2$
9. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+\hat{j}$, $\vec{c}=\hat{i}$ and $(\vec{a}\times\vec{b})\times\vec{c}=\lambda\vec{a}+\mu\vec{b}$ then the value of $\lambda+\mu$ is
   1) 3   2) 1
   3) 0   4) 6
10. If the length of the perpendicular from the origin to the plane $2x+3y+\lambda z=1$, $\lambda>0$ is 1/5 then the value of λ is
    1) $2\sqrt{3}$ 2) $3\sqrt{2}$
    3) 0   4) 1
11. The maximum slope of the tangent to the curve $y=e^x \sin x$, $x\in[0, 2\pi]$ is at
    1) $x=\pi/4$ 2) $x=\pi/2$
    3) $x=\pi$ 4) $x=3\pi/2$

12. Suppose f(x) is a differentiable function for all x with $f'(x) \leq 29$ and f(2)=17. What is maximum value of f(7)=?
 1) 162 2) 160 3) 152 4) 150
13. The percentage error of fifth root of 31 is approximately how many times the percentage error in 31?
 1) 1/31 2) 1/5 3) 31 4) 5
14. The value of $\int_0^\infty e^{-x}x^n dx$ (n is a non negative integer)
 1) (n-1)! 2) (n+1)! 3) n! 4) $\frac{n!}{a^{n+1}}$
15. The volume of solid of revolution of the region bounded by $y^2=x(a-x)$ about x-axis is
 1) πa^3 2) $\frac{\pi a^3}{4}$ 3) $\frac{\pi a^3}{5}$ 4) $\frac{\pi a^3}{6}$
16. The degree and order of the differential equation $\left(\frac{dx}{dy}\right)^2 + 5y^{\frac{1}{3}} = x$
 1) 1, 2 2) 2, 1
 3) 6, 1 4) 3, 1
17. A rod of lenth 2L is broken into two pieces at random. The probability density function of the shorter of the two pieces is
 $f(x)=\begin{cases} 1/l & 0<x<l \\ 0 & l\leq x<2l \end{cases}$ The mean and variance of the shorter of the two pieces are respectively
 1) $\frac{l}{2}, \frac{l^2}{3}$ 2) $\frac{l}{2}, \frac{l^2}{6}$
 3) $l, \frac{l^2}{12}$ 4) $\frac{l}{2}, \frac{l^2}{12}$
18. Let x have a Bernoulli distribution with mean 0.4, then the varience of (2x–3) is
 1) 0.24 2) 0.48 3) 0.96 4) 0.6
19. If cosx is the integrating factor of the linear differential equation $\frac{dy}{dx}+py=Q$ then p is
 1) log sinx 2) cosx
 3) –tanx 4) cotx
20. Subtraction is not a binary operation in
 1) R 2) Z 3) N 4) Q

PART - II

Note: Answer any seven questions. Question No.30 is compulsory. ***(7x2=14)***

21. If $z_1=2-i$ and $z_2=-4+3i$ find the inverse of $\frac{z_1}{z_2}$.
22. Show that the polynomial $9x^9+2x^5-x^4-7x^2+2=0$ has at least six imaginary roots.
23. State the reason for $\cos^{-1}\left[\cos\left(-\frac{\pi}{6}\right)\right] \neq -\frac{\pi}{6}$.

24. Find the internals of monotoncity for the function $f(x)=x^2-4x+4$

25. Use the linear approximation to find approximate values of $\sqrt[3]{26}$.

26. Show that $y=e^{-x}+mx+n$ is a solution of the differential equation $e^x\left(\frac{d^2y}{dx^2}\right)-1=0$

27. Solve: $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$

28. A random variable x has the following probability mass function.

x	1	2	3	4	5	6
f(x)	K	2K	6K	5K	6K	10K

Find the value of K.

29. Compute $p(x=k)$ for the binomial distribution $B(n, p)$ where $n=10$, $p=1/5$, $k=4$.

30. Expreés $\left[\vec{a}+\vec{b}+\vec{c},\ \vec{a}-\vec{b},\ \vec{c}\right]$ in terms of $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$.

PART - III

Note: Answer any seven questions. Question No.40 is compulsory. ***(7x3=21)***

31. Verify $(AB)^{-1}=B^{-1}A^{-1}$ with $A=\begin{bmatrix}0 & -3\\1 & 4\end{bmatrix}, B=\begin{bmatrix}-2 & -3\\0 & -1\end{bmatrix}$.

32. Prove that $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}=\frac{\pi}{4}$.

33. Find the value of $\sum_{K=1}^{8}\left(\cos\frac{2K\pi}{9}+i\sin\frac{2K\pi}{9}\right)$.

34. Obtain the condition that the roots of $x^3+px^2+qx+r=0$ are in A.P.

35. If $y=2\sqrt{2}x+c$ is a tangent to the circle $x^2+y^2=16$, find the value of c.

36. Find the distance of a point (2, 5, −3) from the plane $\vec{r}.(6\hat{i}-3\hat{j}+2\hat{k})=5$.

37. Assume that the cross section of the artery of human is circular. A drug is given to a patient to dilate his arteries. If the radius of an artery is increased from 2mm to 2.1mm how much is cross sectional area increased approximately?

38. Find by integration the volume of the solid generated by revolving about y-axis the region bounded by the curves $y=\log x$, $y=0$, $x=0$ and $y=2$.

39. The probability density function of x is given by

$$f(x)=\begin{cases}Kxe^{-2x}, & \text{for } x>0\\ 0, & \text{for } x\le 0\end{cases}$$

40. Evaluate: $\lim_{x\to\infty}\frac{\frac{1}{x^2}-2\tan^{-1}\left(\frac{1}{x}\right)}{\frac{1}{x}}$

PART - IV

Note. Answer all the questions. ***(7x5=35)***

41. a) By using Gaussian elimination method balance the chemical reaction equation $C_3H_4+O \rightarrow H_2O_2+CO_3$. ***(OR)***

b) Prove by vector method that $\sin(\alpha+\beta)=\sin\alpha\cos\beta + \cos\alpha \sin\beta$.

42. a) If $z=x+iy$ is a complex number such that $\text{Im}\left(\frac{2z+1}{iz+1}\right)=0$ show that the locus of z is $2x^2+2y^2+x-2y=0$. ***(OR)***

b) Find the points on the unit circle $x^2+y^2=1$ nearest and farthest from (1, 1)

43. a) Find the centre, foci and eccentricity of the hyperbola $11x^2-25y^2-44x+50y-256=0$.

(OR)

b) A camera is accidentally knocked off an edge of a cliff 400ft high. The camera falls a distance of $S=16t^2$ in t seconds.

i) How long does the camera fall before it hits the ground?

ii) What is the average velocity with which the camera falls during the last 2 seconds?

iii) What is the instantaneous velocity of the camera when it hits the ground?

44. a) If $a_1, a_2, a_3, \ldots . a_n$ is an arithmatic progresion with common difference d- prove that $\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+\ldots+\tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right]=\frac{a_n-a_1}{1+a_1a_n}$.

(OR)

b) If $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}, \vec{b}=3\hat{i}+5\hat{j}+2\hat{k}, \vec{c}=-\hat{i}-2\hat{j}+3\hat{k}$ verify that

$\vec{a}x(\vec{b}x\vec{c})=(\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}$

45. a) Solve the equation $6x^4-5x^3-38x^2-5x+6=0$ if it is known that 1/3 is a solution.

(OR)

b) Suppose the amount of milk sold daily at a milk booth is distributed with a minimum of 200 litres and a maximum of 600 litres with probability density function

$$f(x)=\begin{cases} K & 200 \le x \le 600 \\ 0 & \text{otherwise} \end{cases}$$

find i) the value of k ii) the distribution function

iii) The probability that daily sales will fall between 300 litres and 500 litres?

46. a) Solve: $[y(1-x\tan x)+x^2\cos x]dx-xdy=0$.

(OR)

b) Prove that $p\rightarrow(\neg q \vee r) \equiv \neg p \vee(\neg q \vee r)$ using truth table.

47. a) Show that the locus of a point which moves so that the difference of its distance from the points (5, 0) and (–5, 0) is 8 is $9x^2-16y^2=144$.

(OR)

b) Find the area of smaller port of the circle $x^2+y^2=a^2$ cut off by the line $x=\frac{a}{\sqrt{2}}$.