Math Café

+2-QUESTION BANK

FULL PORTION TEST

PREPARED BY: E.PALAVESAM.M.Sc.,B.Ed.,

Math Café

Mathematics Time: 3.00Hrs.

STD: **XII** **MODEL FULL PORTION-1** Marks: **90**

PART-A

I. Choose the best Answers **20 x 1 =20**

1. The integrating factor of $\frac{dy}{dx} + y\cot x = 1$ *is*

   (1) $\cot x$ (2) $\tan x$ (3) $\sin x$ (4) $\cos x$

2. The value of $\int_0^1 \log\left(\frac{x}{1-x}\right)dx$

   (1) 0 (2) 2 (3) 4 (4) 5

3. The vertex of the parabola $x^2 - 8y - 1$ is

   (1) $\frac{1}{8}, 0$ (2) $-\frac{1}{8}, 0$ (3) $-6, \frac{9}{2}$ (4) $\frac{9}{2}, -6$

4. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

   (1) $2ab$ (2) ab (3) $\sqrt{ab}$ (4) $\frac{a}{b}$

5. $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$ is equal to

   (1) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ (2) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ (3) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ (4) $\tan^{-1}\left(\frac{1}{2}\right)$

6. The polynomial $x^3 - kx^2 + 9x$ has three real zeros if and only if, k satisfies

   (1) $|k| \le 6$ (2) $k = 0$ (3) $|k| > 6$ (4) $|k| \ge 6$

7. The conjugate of a complex number is $\frac{1}{i-2}$. Then, the complex number is

   (1) $\frac{1}{i+2}$ (2) $\frac{-1}{i+2}$ (3) $\frac{-1}{i-2}$ (4) $\frac{1}{i-2}$

8. The product of all four values of $\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^{\frac{3}{4}}$ is

   (1) -2 (2) -1 (3) 1 (4) 2

9. Subtraction is not a binary operation in

   (1) $\mathbb{R}$ (2) $\mathbb{Z}$ (3) $\mathbb{N}$ (4) $\mathbb{Q}$

10. Which one is the inverse of the statement $(p \vee q) \rightarrow (p \wedge q)$?

    (1) $(p \wedge q) \rightarrow (p \vee q)$ (2) $\neg(p \vee q) \rightarrow (p \wedge q)$

    (3) $(\neg p \vee \neg q) \rightarrow (\neg p \wedge \neg q)$ (4) $(\neg p \wedge \neg q) \rightarrow (\neg p \vee \neg q)$

11. The value of $\int_0^\infty e^{-3x}x^2 dx$ is

    (1) $\frac{7}{27}$ (2) $\frac{5}{27}$ (3) $\frac{4}{27}$ (4) $\frac{2}{27}$

12. A random variable X has binomial distribution with $n = 25$ and $p = 0.8$ then standard deviation of X is

(1) 6 (2) 4 (3) 3 (4) 2

13. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $[\vec{a}, \vec{b}, \vec{a} \times \vec{b}] = \frac{1}{4}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

14. If the planes $\vec{r} \cdot (2\hat{\imath} - \lambda\hat{\jmath} + \hat{k}) = 3$ and $\vec{r} \cdot (4\hat{\imath} + \hat{\jmath} - \mu\hat{k}) = 5$ are parallel, then the value of λ and μ are

(1) $\frac{1}{2}, -2$ (2) $-\frac{1}{2}, 2$ (3) $-\frac{1}{2}, -2$ (4) $\frac{1}{2}, 2$

15. If $u(x, y) = e^{x^2+y^2}$, then $\frac{\partial u}{\partial x}$ is equal to

(1) $e^{x^2+y^2}$ (2) $2xu$ (3) x^2u (4) y^2u

16. The position of a particle moving along a horizontal line of any time t is given by $s(t) = 3t^2 - 2t - 8$. The time at which the particle is at rest is

(1) $t = 0$ (2) $t = \frac{1}{3}$ (3) $t = 1$ (4) $t = 3$

17. If $A = \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$ then $|\text{adj}(AB)| =$

(1) -40 (2) -80 (3) -60 (4) -20

18. Let A be a non-singular matrix then which one of the following is false?

(1) $(adj\ A)^{-1} = \frac{A}{|A|}$ (2) I is an orthogonal matrix

(3) $adj(adj A) = |A|^n A$ (4) if A is symmetric then adj A is symmetric

19. Angle between $y^2 = x$ and $x^2 = y$ at the origin is

(1) $\tan^{-1}\frac{3}{4}$ (2) $\tan^{-1}\left(\frac{4}{3}\right)$ (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{4}$

20. The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/3} + x^{1/4} = 0$ are respectively

(1) 2, 3 (2) 3, 3 (3) 2, 6 (4) 2, 4

PART-B

II. Answer any 7questions.(Q.no.30 is compulsory) 7 x 2 =14

21. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors, prove that $[\vec{a} + \vec{c}, \vec{a} + \vec{b}, \vec{a} + \vec{b} + \vec{c}] = [\vec{a}, \vec{b}, \vec{c}]$.

22. Let $f(x) = \sqrt[3]{x}$. Find the linear approximation at $x = 27$. Use the linear approximation to approximate $\sqrt[3]{27.2}$

23. Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

24. Write the Maclaurin series expansion of the function e^x

25. Construct the truth table for the statement $\neg(p \wedge \neg q)$.

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26. Discuss the maximum possible number of positive and negative roots of the polynomial equation $9x^9 - 4x^8 + 4x^7 - 3x^6 + 2x^5 + x^3 + 7x^2 + 7x + 2 = 0$.

27. Find the value of $\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$.

28. Simplify $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$. into rectangular form

29. Find the differential equation of the curve represented by $xy = ae^x + be^{-x} + x^2$.

30. Evaluate $\int_0^{\frac{\pi}{2}} \sin^{10} x\, dx$

PART-C

III. Answer any 7 questions.(Q.no.40 is Compulsory) **7 x 3 =21**

31. Find the equation of the hyperbola foci $(\pm 3,5)$, eccentricity $= 2$.

32. Solve : $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

33. Find the square roots of $6 - 8i$

34. The mean and variance of a binomial variate X are respectively 2 and 1.5. Find $P(X = 0)$.

35. Evaluate. $\lim_{x\to 0}\left(\frac{1}{\sin x} - \frac{1}{x}\right)$

36. If α and β are the roots of the quadratic equation $2x^2 - 7x + 13 = 0$, construct a quadratic equation whose roots are α^2 and β^2.

37. Find the value of $\sin^{-1}\left(\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right)$

38. Find the acute angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$.

39. Evaluate $\int_0^1 \frac{2x}{1+x^2}\, dx$

40. If $A = \begin{bmatrix} -3 & -2 \\ \lambda & -2 \end{bmatrix}$, find the value of λ so that $A^2 = \lambda A - 2I$.

PART-D

IV. Answer all the questions **7 x 5 = 35**

41. a) Investigate the values of λ and μ the system of linear equations $2x + 3y + 5z = 9$, $7x + 3y - 5z = 8$, $2x + 3y + \lambda z = \mu$, have

(i) no solution (ii) a unique solution (iii) an infinite number of solutions. **(OR)**

b) Solve the following equation: $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$.

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42. a) If $z = x + iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, show that $x^2 + y^2 + 3x - 3y + 2 = 0$. **(OR)**

b) On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of $4m$ when it is $6m$ away from the point of projection. Finally it reaches the ground 12 m away from the starting point. Find the angle of projection.

43. a) Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point (2,3,6) and parallel to the straight lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-3}{1}$ and $\frac{x+3}{2} = \frac{y-3}{-5} = \frac{z+1}{-3}$ **(OR)**

b) A random variable X has the following probability mass function.

x	1	2	3	4	5	6
$f(x)$	k	$2k$	$6k$	$5k$	$6k$	$10k$

Find (i) $P(2 < X < 6)$ (ii) $P(2 \le X < 5)$ (iii) $P(X \le 4)$ (iv) $P(3 < X)$

44. a) Find the angle between the curves $y = x^2$ and $x = y^2$ at their points of intersection (0,0) and (1,1) **(OR)**

b) Prove that $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$.

45. a) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$. **(OR)**

b) Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for the operation $+_5$ on $\mathbb{Z}_5$ using table corresponding to addition modulo 5.

46. a) Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent. **(OR)**

b) For the ellipse $4x^2 + y^2 + 24x - 2y + 21 = 0$, find the centre, vertices, and the foci. Also prove that the length of latus rectum is 2 .

47. a)Evaluate the definite integral $\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{1+\sqrt{\tan x}}\,dx$ **(OR)**

b) A hollow cone with base radius a cm and height b cm is placed on a table. Show that the volume of the largest cylinder that can be hidden underneath is $\frac{4}{9}$ times volume of the cone.

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