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COMMON FIRST REVISION EXAMINATION - 2024

Std - XII

MATHEMATICS

Time : 3.00 Hours Marks: 90

Part - I

i) All questions are compulsory.

ii) Choose the most suitable answer from the given four alternatives and write the option code and the corresponding answer: 20 x 1 = 20

1 If $A\begin{bmatrix}1 & -2\\ 1 & 4\end{bmatrix}=\begin{bmatrix}6 & 0\\ 0 & 6\end{bmatrix}$, then A =

a) $\begin{bmatrix}1 & -2\\ 1 & 4\end{bmatrix}$ b) $\begin{bmatrix}1 & 2\\ -1 & 4\end{bmatrix}$ c) $\begin{bmatrix}4 & 2\\ -1 & 1\end{bmatrix}$ d) $\begin{bmatrix}4 & -1\\ 2 & 1\end{bmatrix}$

2 If $A=\begin{bmatrix}2 & 3\\ 5 & -2\end{bmatrix}$ be such that $\lambda A^{-1} = A$, then λ is

a) 17 b) 14 c) 19 d) 21

3 In the system of 3 linear equations with three unknowns then the system

a) has unique solution

b) reduces to 2 equations and has infinitely many solutions

c) reduces be a single equation and has infinitely many solution

d) is inconsistent

4 The solution of the equation $|z| = -z = 1 + 2i$ is

a) $\frac{3}{2} - 2i$ b) $-\frac{3}{2} + 2i$ c) $2 - \frac{3}{2}i$ d) $2 + \frac{3}{2}i$

5 The value of $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ is

a) $\text{cis}\,\frac{2\pi}{3}$ b) $\text{cis}\,\frac{4\pi}{3}$ c) $-\text{cis}\,\frac{2\pi}{3}$ d) $-\text{cis}\,\frac{4\pi}{3}$

6 Which one of the following is not true

a) $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$ b) $\overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$ c) $\text{Re}(z) = \frac{\bar{z} + z}{2}$ d) $\text{Im}(z) = \frac{\bar{z} - z}{2i}$

7 If f and g are polynomials of degrees m and n respectively, and if h(x) = (fog) (x), then the degree of h is

a) mn b) m+n c) m^n d) n^m

8 If $x^3 + 12x^2 + 10ax + 1999$ definitely has a positive zero. If and only if

a) $a \geq 0$ b) $a>0$ c) $a<0$ d) $a \leq 0$

9 Which one of the following is can't be a root of a reciprocal equation?

a) $3+2\sqrt{2}$ b) $-\frac{1}{2}$ c) 0 d) 1

10 If $\cot^{-1} x = \frac{2\pi}{5}$ for some $x \in R$, the value of $\tan^{-1} x$ is

a) $-\frac{\pi}{10}$ b) $\frac{\pi}{5}$ c) $\frac{\pi}{10}$ d) $-\frac{\pi}{5}$

11 $\sin(\tan^{-1} x)$, $|x|<1$ is equal to

a) $\frac{x}{\sqrt{1-x^2}}$ b) $\frac{1}{\sqrt{1-x^2}}$ c) $\frac{1}{\sqrt{1+x^2}}$ d) $\frac{x}{\sqrt{1+x^2}}$

12 If $\sin^{-1} x = y$ then

a) $0 \le y \le \pi$ b) $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ c) $0 < y < \pi$ d) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

13 The equation of the circle passing through (1,5) and (4, 1) and touching y-axis is $x^2 + y^2 - 5x - 6y + 9 + \lambda(4x + 3y - 19) = 0$ where λ is equal to

a) 0, $-\frac{40}{9}$ b) 0 c) $\frac{40}{9}$ d) $-\frac{40}{9}$

14 If $x + y = k$ is a normal to the parabola $y^2 = 12x$, then the value of k is

a) 3 b) -1 c) 1 d) 9

15 The values of m for which the line $y = mx + 2\sqrt{5}$ touches the hyperbola $16x^2 - 9y^2 = 144$ are the roots of $x^2 - (a+b)x + 4 = 0$, then the value of $(a+b)$ is

a) 2 b) 4 c) 0 d) -2

16 The normal at the point t_1 on the parabola meets the parabola $y^2 = 4ax$ again at t_2 then t_2

a) t_1^2 b) $\frac{1}{t_1}$ c) t_1 d) $t_1 + \frac{2}{t_1}$

17 If $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$ then the value of $[\vec{a}, \vec{b}, \vec{c}]$ is

a) $|\vec{a}||\vec{b}||\vec{c}|$ b) $\frac{1}{3}|\vec{a}||\vec{b}||\vec{c}|$ c) 1 d) -1

18 If the volume of the parallelepiped with $\vec{a} \times \vec{b}$, $\vec{b} \times \vec{c}$, $\vec{c} \times \vec{a}$ as coterminous edges is 8 cubic units then the volume of the parallelepiped with $(\vec{a} \times \vec{b}) \times (\vec{b} \times \vec{c})$, $(\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a})$ and $(\vec{c} \times \vec{a}) \times (\vec{a} \times \vec{b})$ as coterminous edges is

a) 8 cubic units b) 512 cubic units c) 64 cubic units d) 24 cubic units

19 If the direction cosines of a line are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$ then

a) $c = \pm 3$ b) $c = \pm\sqrt{3}$ c) $c > 0$ d) $0 < c < 1$

20 If $|\vec{a} + \vec{b}| = 60$, $|\vec{a} - \vec{b}| = 40$ and $|\vec{b}| = 46$ then the value of $|\vec{a}|$ is

a) 22 b) 21 c) 18 d) 11

Part - II

7 x 2 = 14

Answer any seven Questions. Question No. 30 is compulsory.

21. If A is symmetric, prove that adj A is also symmetric.

22. Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

23. If $|z| = 2$ show that $3 \leq |z+3+4i| \leq 7$

24. Write in polar form of the complex number $3+i\sqrt{3}$

25. Construct a cubic equation with roots 1, 1 and -2

26. Discuss the nature of the roots of the polynomial $x^{2019} + 1947x^{1950} + 15x^8 + 26x^6 + 2019 = 0$

27. Find the principal value of $\sin^{-1}(2)$, if it exists.

28. Find the principal value of $\text{cosec}^{-1}(-1)$

29. Find the equation of the hyperbola with foci $(\pm 2, 0)$ and $e = 3/2$

30. The volume of the parallelepiped whose coterminus edges are $7\vec{i} + \lambda\vec{j} - 3\vec{k}$, $\vec{i} + 2\vec{j} - \vec{k}$ and $-3\vec{i} + 7\vec{j} + 5\vec{k}$ is 90 cubic units. Find the value of λ.

Part - III

7 x 3 = 21

Answer any seven questions. Question No. 40 is compulsory.

31. Find the rank of the matrix $\begin{bmatrix} 2 & -2 & 4 & 3 \\ 3 & 4 & 2 & 1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$ by reducing it to an echelon form.

32. Test for consistency and if possible, solve the following systems of equations by rank method $2x + 2y + z = 5$, $x - y + z = 1$, $3x + y + 2z = 4$

33. Find the square root of $-7 + 24i$

34. If $z_1 = 2-i$ and $z_2 = -4+3i$, find the inverse of $\frac{z_1}{z_2}$

35. Find solution, if any, of the equation $2\cos^2 x - 9\cos x + 4 = 0$

36. Find the domain of $\cos^{-1}\left(\frac{2+\sin x}{3}\right)$

37. Find the centre and radius of the circle $3x^2 + (a+1)y^2 + 6x - 9y + a + 4 = 0$

38. Find the equations of tangent and normal to the parabola $x^2 + 6x + 4y + 5 = 0$ at $(1, -3)$

39. Forces of magnitudes $5\sqrt{2}$ and $10\sqrt{2}$ units acting in the directions $3\vec{i} + 4\vec{j} + 5\vec{k}$ and $10\vec{i} + 6\vec{j} - 8\vec{k}$, respectively act on a particle which is displaced from the point with position vector $4\vec{i} - 3\vec{j} - 2\vec{k}$ to the point with position vector $6\vec{i} + \vec{j} - 3\vec{k}$. Find the work done by the forces.

$\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k}$, $\vec{b} = -\vec{i} + 2\vec{j} - 4\vec{k}$, $\vec{c} = \vec{i} + \vec{j} + \vec{k}$ then find the value of $(\vec{a} \times \vec{b}) \cdot (\vec{a} \times \vec{c})$

Part - IV

Answer all the questions. 7 x 5 = 35

41 a) A fish tank can be filled in 10 minutes using both pumps A and B simultaneously. However, pump B can pump water in or out of the same rate. If pump B is inadvertently run in reverse, then the tank will be filled in 30 minutes. How long would it take each pump to fill the tank by itself? (Use Cramer's rule to solve the problem) **(OR)**

b) If $A = \frac{1}{7}\begin{bmatrix} 6 & -3 & a \\ b & -2 & 6 \\ 2 & c & 3 \end{bmatrix}$ is orthogonal, find a, b and c, and hence A^{-1}

42 a) Using vector method, prove that $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$. **(OR)**

b) By using Gaussian elimination method, balance the chemical reaction equation $C_2H_2 + O_2 \rightarrow CO_2 + H_2O$

43 a) Let z_1, z_2 and z_3 be complex numbers such that $|z_1| = |z_2| = |z_3| = r > 0$ and $z_1 + z_2 + z_3 \neq 0$. Prove that $\left|\frac{z_1z_2 + z_2z_3 + z_3z_1}{z_1 + z_2 + z_3}\right| = r$ **(OR)**

b) Solve the equation $z^3 + 27 = 0$

44 a) If the equations $x^2 + px + q = 0$ and $x^2 + p'x + q' = 0$ have a common root, show that it must be equal to $\frac{pq' - p'q}{q - q'}$ or $\frac{q - q'}{p' - p}$ **(OR)**

b) Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that 1/3 is a solution

45 a) Solve $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$ **(OR)**

b) Find the value of $\cos\left(\sin^{-1}\left(\frac{4}{5}\right) - \tan^{-1}\left(\frac{3}{4}\right)\right)$

46 a) Find the vector parametric, vector non-parametric and Cartesian form of the equation of the plane passing through the points (-1, 2, 0), (2, 2, -1) and parallel to the straight line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$ **(OR)**

b) Show that the lines $\frac{x-3}{3} = \frac{y-3}{1}, z - 1 = 0$ and $\frac{x-6}{2} = \frac{z-1}{3}, y - 2 = 0$ intersect. Also find the point of intersection.

47 a) A tunnel through a mountain for a four lane highway is to have a elliptical opening. The total width of the highway (not the opening) is to be 16m, and the height at the edge of the road must be sufficient for a truck 4m high to clear if the highest point of the opening is to be 5m approximately. How wide must the opening be? **(OR)**

b) Find the vertex, focus, directrix and length of the latus rectum of the parabola $x^2 - 4x + 4y = 0$