Tsi12M

Tenkasi District

First Revision Examination, January - 2024

31-01-2024.

Standard 12

MATHEMATICS

Time Allowed: 3.00 Hours

Maximum Marks: 90

PART - I

20×1=20

Note: i) Answer all the questions.

ii) Choose the most suitable answer from the given four alternatives and write the option code and corresponding answer.

1) If $A = \begin{pmatrix} 3/5 & 4/5 \\ x & 3/5 \end{pmatrix}$ and $A^T = A^{-1}$ then the value of x is

a) $-4/5$ b) $-3/5$ c) $3/5$ d) $4/5$

2) If $|Z| = 1$ then the value of $\frac{1+z}{1+\bar{z}}$ is

a) z b) $\bar{z}$ c) $1/z$ d) 1

3) If $\cot^{-1}2$ and $\cot^{-1}3$ are two angles of a triangle then the third angle is

a) $\pi/4$ b) $3\pi/4$ c) $\pi/6$ d) $\pi/3$

4) The radius of the circle $3x^2+by^2+4bx-6by+b^2 = 0$ is

a) 1 b) 3 c) $\sqrt{10}$ d) $\sqrt{11}$

5) If the length of the perpendicular from the origin to the plane $2x+3y+\lambda z = 1$, $\lambda > 0$ is $1/5$, then the value of λ is

a) $2\sqrt{3}$ b) $3\sqrt{2}$ c) 0 d) 1

6) The number of real numbers in $[0, 2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is

a) 2 b) 4 c) 1 d) ∞

7) If $a+ib = (8-6i) - (2i-7)$ then the value of a & b are

a) 8, −15 b) 8, 15 c) 1, 4 d) 15, −8

8) If the projection of $\vec{a}$ on $\vec{b}$ and projection of $\vec{b}$ on $\vec{a}$ are equal then the angle between $\vec{a}+\vec{b}$ & $\vec{a}-\vec{b}$ is

a) $\pi/2$ b) $\pi/3$ c) $\pi/4$ d) $2\pi/3$

9) If $P = \begin{pmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -2 \end{pmatrix}$ is the adjoint of 3×3 matrix A and $|A| = 4$ then x is

a) 15 b) 12 c) 14 d) 11

10) The locus of a point whose distance from (−2, 0) is $2/3$ times its distance from the line $x = -9/2$ is

a) a parabola b) a hyperbola c) an ellipse d) a circle

11) Angle between $y^2 = x$ and $x^2 = y$ at the origin is

a) $\tan^{-1}\frac{3}{4}$ b) $\tan^{-1}\frac{4}{3}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{4}$

12) The minimum value of the function $|3-x|+9$ is

a) 0 b) 3 c) 6 d) 9

13) If $w(x, y, z) = x^2(y-z) + y^2(z-x)+z^2(x-y)$ then $\frac{\partial w}{\partial x}+\frac{\partial w}{\partial y}+\frac{\partial w}{\partial z}$ is

a) $xy+yz+zx$ b) $x(y+z)$ c) $y(z+x)$ d) 0

14) The operation * defined by $a*b = \frac{ab}{7}$ is not a binary operation on

a) Q^+ b) Z c) R d) C

15) If a compound statement involves 3 simple statements then the number of rows in the truth table is

a) 9 b) 8 c) 6 d) 3

16) Let X have a Bernoulli distribution with mean 0.4, then the variance of $2x-3$ is

a) 0.24 b) 0.48 c) 0.6 d) 0.96

17) The solution of the differential equation $\frac{dy}{dx} = 2xy$ is

a) $y = Ce^{x^2}$ b) $y = 2x^2+C$ c) $y = Ce^{-x^2}+C$ d) $y = x^2+C$

18) If f(x) is an odd function then $\int_{-a}^{a} f(x)dx$ is

a) 0 b) $2\int_0^a f(x)dx$ c) 2 d) $\int_0^a f(x)dx$

19) If $y = Ke^{\lambda x}$ then its differential equation is

a) $\frac{dy}{dx} = \lambda y$ b) $\frac{dy}{dx} = Ky$ c) $\frac{dy}{dx}+Ky = 0$ d) $\frac{dy}{dx} = e^{\lambda x}$

20) The area between $y^2 = 4x$ and its latus rectum is

a) $\frac{2}{3}$ b) $\frac{4}{3}$ c) $\frac{8}{3}$ d) $\frac{5}{3}$

PART - II

Answer any seven questions. Question No. 30 is compulsory. 7×2=14

21) If A is a non-singular matrix of odd order. Prove that |adj A| is positive.

22) Solve: $2x^3+11x^2-9x-18 = 0$

23) If the equation $3x^2+(3-p)xy+qy^2-2px = 8pq$ represents a circle. Find p and q. Also determine the radius of the circle.

24) Prove that $\tan^{-1}\frac{2}{11}+\tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$

25) Find the angle between the planes $\vec{r}.(\hat{i}+\hat{j}-2\hat{k}) = 3$ and $2x-2y+z = 2$

26) Solve: $\frac{dy}{dx}+2y = e^{-x}$

27) If $\int_0^{\infty} e^{-\alpha x^2} x^3\,dx = 32$, $\alpha > 0$ find α

28) Find the points on the curve $y^2-4xy = x^2+5$ for which the tangent is horizontal.

29) On R define * by $a*b = a\sqrt{b}$, $\forall a,b \in R$. Is * binary on R?

30) If ω is a cube root of unity find the value of $(1-\omega+\omega^2)^4+(1+\omega-\omega^2)^4$

PART - III

Answer any seven questions. Question No. 40 is compulsory. **7×3=21**

31) Solve the system of linear equations by matrix inversion method:
$2x+5y = -2$; $x+2y = -3$

32) Find the square root of $-5-12i$.

33) Find the value of $\sin^{-1}\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9}+\cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right)$

34) Find the foot of the perpendicular drawn from the point (5, 4, 2) to the line $\frac{x+1}{2}=\frac{y-3}{3}=\frac{z-1}{-1}$

35) Find the equation of the circle through the points (1, 0) (−1, 0) and (0, 1)

36) If $\upsilon(x, y) = x^2-xy+\frac{1}{4}y^2+7$, $x, y \in R$ find the differential $d\upsilon$.

37) Evaluate $\int_0^1 |5x-3|\,dx$ using properties of integration.

38) Find the differential equation of the curve represented by $xy = ae^x+be^{-x}+x^2$

39) The probability density function of x is given by $f(x) = \begin{cases} Kxe^{-2x} & x > 0 \\ 0 & x \le 0 \end{cases}$ find the value of K.

40) Find the value of a so that the curves $y = 3e^x$ and $y = \frac{a}{3}e^{-x}$

PART - IV

Answer all the questions: **7×5=35**

41) a] Investigate the values of λ and μ the system of linear equations $2x+3y+5z = 9$, $7x+3y-5z = 8$, $2x+3y+\lambda z = \mu$ have (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

(OR)

b] A conical water tank with vertex down of 12 meters height has a radius of 5 meters at the top. If the water flows into the tank at a rate 10 cubic m/min, how fast is the depth of the water increases when the water is 8 meters deep?

42) a] Prove $p \to (q \to r) \equiv (p \wedge q) \to r$ using truth table.

(OR)

b] If $z = x+iy$ is a complex number such that $\text{Im}\left(\frac{2z+1}{iz+1}\right) = 0$, show that the locus of z is $2x^2+2y^2+x-2y = 0$

43) a] If the roots of $x^3+px^2+qx+r = 0$ are in H.P. Prove that $9pqr = 27r^2+2q^3$, $p, q, r \neq 0$

(OR)

b] Find the vertex, focus, directrix and length of the latus rectum of the parabola $x^2-4x-5y-1 = 0$

44) a] Find the non-parametric form of vector equation and cartesian equation of the plane passing through the point (1, −2, 4) and perpendicular to the plane $x+2y-3z = 11$ and parallel to the line $\frac{x+7}{3} = \frac{y+3}{-1} = \frac{z}{1}$

(OR)

b] If x is the random variable with probability density function f(x) given by

$$f(x) = \begin{cases} x+1 & -1 \leq x < 0 \\ -x+1 & 0 \leq x < 1 \\ 0 & \text{otherwise} \end{cases}$$ then find (i) the distribution function f(x)

(ii) $p(-0.5 \leq x \leq 0.5)$

45) a] Find the population of a city at any time t, given that the rate of increase of population is proportional to the population at that instant and that in a period of 40 years the population increased from 3,00,000 to 4,00,000.

(OR)

b] Evaluate: $\int_0^{\pi} \frac{x \sin x}{1+\sin x} dx$

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46) a] Prove by vector method that $\sin(\alpha+\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

(OR)

b] Solve $(2x+3y)dx + (y-x)dy = 0$

47) a] Find the area of the region common to the cricle $x^2+y^2 = 16$ and the parabola $y^2 = 6x$.

(OR)

b] At a water fountain, water atttains a maximum height of 4 m at horizontal distance of 0.5 m from its origin. If the path of water is a parabola, find the height of water at a horizontal distance of 0.75 m from the point of origin.

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