Class : 12

Register Number

FIRST REVISION EXAMINATION, JANUARY - 2024

Time Allowed : 3.00 Hours] **MATHEMATICS** [Max. Marks : 90

PART – I

I. Answer the following: 20x1=20

1. If A and B are orthogonal then $(AB)^T(AB)$ is

a) A b) B c) I d) A^T

2. A is non singular matrix. If $A^{-1} = \begin{pmatrix} 5 & 3 \\ -2 & -1 \end{pmatrix}$ then $(A^T)^{-1}$ = ----------

a) $\begin{pmatrix} -5 & 3 \\ 2 & 1 \end{pmatrix}$ b) $\begin{pmatrix} 5 & 3 \\ -2 & -1 \end{pmatrix}$ c) $\begin{pmatrix} -1 & -3 \\ 2 & 5 \end{pmatrix}$ d) $\begin{pmatrix} 5 & -2 \\ 3 & -1 \end{pmatrix}$

3. Value of $i^n + i^{n+1} + i^{n+2}$ + --------

a) 0 b) 1 c) -1 d) i

4. The range of $\sec^{-1} x$ is

a) $[0, \pi] - [\pi/2]$ b) $[0, \pi]$ c) $[-\pi/2, \pi/2]$ d) $[-\pi/2, \pi/2] - \{0\}$

5. The angle between the curves $y^2=x$ and x^2y at the origin is

a) $\tan^{-1}(3/4)$ b) $\tan^{-1}(4/3)$ c) $\pi/2$ d) $\pi/4$

6. The locus of a point whose distance from (-2,0) is $2/3$ times its distance from the line $x=-9/2$ is

a) a parabola b) a hyperbola c) an ellipse d) a circle

7. $\sin^{-1}(\cos x) = \pi/2 - x$ is valid for

a) $-\pi \le x \le 0$ b) $0 \le x \le \pi$ c) $-\pi/2 \le x \le \pi/2$ d) $-\pi/4 \le x \le 3\pi/4$

8. The least value of the function $|3-x| + 9$

a) 0 b) 3 c) 6 d) 9

9. The differential equation of family of curves $y = Ae^x + Be^{-x}$ where A, B are constants is

a) $\frac{d^2y}{dx^2} + y = 0$ b) $\frac{d^2y}{dx^2} - y = 0$ c) $\frac{dy}{dx} + y = 0$ d) $\frac{dy}{dx} - y = 0$

10. The vertex of the parabola $x^2 = 8y-1$ is

a) $(-1/8, 0)$ b) $(1/8, 0)$ c) $(-6, 9/2)$ d) $(9/2, -6)$

11. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + \hat{j}$, $\vec{c} = \hat{i}$ and $(\vec{a} \times \vec{b}) \times \vec{c} = \lambda\vec{a} + \mu\vec{b}$ then the value of $\lambda+\mu$ is

a) 0 b) 1 c) 6 d) 3

12. The number given by the mean value theorem for the function $1/x$, $x \in [1,9]$ is

a) 2 b) 2.5 c) 3 d) 3.5

13. If $f(x,y) = e^{xy}$ then $\partial^2 f / \partial x \partial y$ is equal to

a) xye^{xy} b) $(1+xy)e^{xy}$ c) $(1+y)e^{xy}$ d) $(1+x)e^{xy}$

14. If the function $f(x) = 1/12$ for $a<x<b$ represents a probability density function of a continuous random variable x then which of the following cannot be the value of a and b?

a) 0 and 12 b) 5 and 17 c) 7 and 19 d) 16 and 24

15. In the last column of the truth table for $\neg (P \vee \neg q)$ the number of final outcomes of the truth value 'F' are

a) 1 b) 2 c) 3 d) 4

16. If $f(x) = \int_0^x t \cos t \, dt$ then df/dx ------

a) Cos x - x Sin x b) Sin x + x Cos x c) x Cos x d) x Sin x

17. Value of $\int_{-\pi/2}^{\pi/2} \frac{\text{Sin } x}{2 + \text{Cos } x} dx$

a) 0 b) 2 c) log 2 d) log 4

18. Which of the following is not true?

a) $|z_1 + z_2| \leq |z_1| + |z_2|$ b) $|z_1 - z_2| \leq |z_1| - |z_2|$

c) $\text{Re}(z) \leq |z|$ d) $\text{Im}(z) \leq |z|$

19. $x^2 + px + 3p/4 = 0$ roots of this equation are α, β. If $|\alpha - \beta| = \sqrt{10}$ then the value of p --------

a) {2, -5} b) {-3,2} c) {-2,5} d) {3,-5}

20. The directrices of the hyperbola $x^2 - 4(y-3)^2 = 16$ are

a) $y = \pm 8/\sqrt{5}$ b) $x = \pm 8/\sqrt{5}$ c) $y = \pm \sqrt{5}/8$ d) $x = \pm \sqrt{5}/8$

PART - B

II. Answer any Seven questions. Question No. 30 is compulsory. **7x2=14**

21. Find the value of $\tan^{-1}[\tan[-\pi/4]]$.

22. Find the centre and radius of the circle $3x^2+3y^2-12x+6y-9=0$.

23. If $f(x,y) = x^3-3x^2+y^2+5x+6$ then find fx at (1,-2).

24. Evaluate $\int_0^{\pi/2} \frac{\text{Sin } x}{1 + \text{Cos}^2 x} dx$

25. Solve the following system of linear equations by Inverse Method. $2x-y= 8$; $3x+2y = -2$.

26. The probability density function of the random variable X is $f(x) = \begin{cases} 3e^{-3x} & x>0 \\ 0 & \text{elsewhere} \end{cases}$

find the mean of X.

27. Find the angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$.

28. If 1 is cube root of unity and $(1+\omega)^7 = A+b\omega$ where $\omega \neq 1$ then find the value of A and B.

29. $z_1 = 2-i$ and $z_2 = -4+3i$ then find the inverse of z_1/z_2.

30. Let $A = \{a+\sqrt{5}b, a, b \in z\}$ check whether the usual multiplication is a binary operation on A.

PART - C

III Answer any Seven questions. Question No. 40 is compulsory. **7x3=21**

31. For $A = \begin{bmatrix} 1 & 3 \\ 2 & -5 \end{bmatrix}$ find adj A. And also prove that A(adj A) = (adj A) A = |A| I.

32. Prove that 1, $-1/2 + i\sqrt{3}/2$ and $-1/2 - i\sqrt{3}/2$ are the vertices of equilateral triangle.

33. Find two positive numbers whose product is 20 and their sum is minimum.

34. Find the area of the region bounded by $2x-y+1=0$, $y=-1$, $y=3$ and y axis.

35. Solve: $dy/dx = e^{x+y} + x^2 e^{x^3+y}$.

36. Evaluate: $\sum_{k=1}^{8} \left(\text{Cos}\frac{2k\pi}{9} + i\,\text{Sin}\frac{2k\pi}{9}\right)$

37. Prove that length of Latus rectum of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{2b^2}{a}$.

38. Solve: Cos x Cos y dy - Sin x Sin y dx = 0.

39. If μ and σ^2 are the mean and variance of the discrete random variable X. Also $E(X+3) = 10$ and $E(X+3)^2 = 116$. Find μ and σ^2.

40. Prove that (i) $P \vee (\neg P)$ is tautology and (ii) $P \wedge (\neg P)$ is contradiction.

PART - D

IV Answer all the questions. **7x5=35**

41. a) If $ax^2 + bx + c$ is divided by x+3, x-5 and x-1, the remainders are 21, 61 and 9 respectively. Find a, b, c (Use Gaussian Elimination Method).

(OR)

b) Simplify: $(-\sqrt{3}+3i)^{31}$.

42. a) Find the centre vertices and Foci of the ellipse $4x^2+y^2+24x-2y+21=0$. Also find the latus rectum.

(OR)

b) Prove by vector method $\cos(\alpha+\beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$

43. a) Find a polynomial equation of minimum degree with rational coefficients having $\sqrt{5} - \sqrt{3}$ as a root.

(OR)

b) Solve $\cos(\tan^{-1} x) = \sin(\cot^{-1} 3/4)$

44. a) Prove that $\int_0^{\pi/4} \log(1+\tan x) = \pi/8 \log 2$.

(OR)

b) Solve: $\frac{dy}{dx} = \frac{x-y+5}{2(x-y)+7}$

45. a) Show that the straight lines $x+1 = 2y = -12z$ and $x = y+2 = 6z-6$ are skew and hence find the shortest distance between them.

(OR)

b) If we blow air into a balloon of a spherical shape at a rate of 1000 cm^3 per second, at what rate the radius of the balloon changes when the radius is 7 cm? Also compute the rate at which the surface area changes.

46. a) The sum of the mean and variance of a binomial distribution for five trials is 1.8. Find the distribution.

(OR)

b) Prove that $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

47. a) Find the absolute extrema and minima for the function $f(x) = 3x^4 - 4x^3$ at the interval $[-1,2]$.

(OR)

b) If the curves $ax^2+by^2=1$ and $cx^2+dy^2=1$ are cuts orthogonally then prove that $1/a - 1/b = 1/c - 1/d$.

ST. JOSEPH'S HR. SEC. SCHOOL - CHENGALPATTU

முதல் திருப்புதல் தேர்வு, ஜனவரி - 2024

C. SELVAM, M.Sc., M.Ed., P.G. ASST. (MATHS)

வகுப்பு : 12 கணிதம் - KEY மதிப்பெண்கள் : 90

10.01.24

I

1	இ	1
2	ஈ	$\begin{pmatrix} 5 & -2 \\ 3 & -1 \end{pmatrix}$
3	அ	0
4	அ	$[0,\pi]-\{\pi/2\}$
5	இ	$\pi/2$
6	இ	நீள்வட்டம்
7	ஆ	$0 \le x \le \pi$
8	ஈ	9
9	ஆ	$\frac{d^2y}{dx^2}-y=0$
10		$(0, 1/8)$ M.A
11	அ	0
12	இ	3
13	ஆ	$(1+xy)e^{xy}$
14	ஈ	16 மற்றும் 24
15	இ	3
16	இ	$x\cos x$
17	அ	0
18	ஆ	$\lvert z_1-z_2\rvert \le \lvert z_1\rvert-\lvert z_2\rvert$
19	இ	$\{-2,5\}$
20	ஆ	$x=\pm 8/\sqrt{5}$

II

21. $-\pi/4$

22. மையம் : $C(-g,-f)=(2,-1)$

ஆரம் $r=\sqrt{g^2+f^2-c}=\sqrt{4+1+3}=\sqrt{8}=2\sqrt{2}$

23. $f_x=3x^2-6x+5$

$f_{x(1,-2)}=3-6+5=2$

24. $t=\cos x$

$dt=-\sin x\,dx$

x	0	$\pi/2$
t	1	0

$=-\int \frac{dt}{1+t^2}=-[\tan^{-1}t]_1^0=-(0-\pi/4)=\pi/4$

25. $A=\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$, $|A|=4+3=7$, $adjA=\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$

$A^{-1}=\frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$

26. $E(X)=\int_0^\infty x\cdot 3e^{-3x}dx = 3\times\frac{1!}{3^2}=1/3$ ($n=1$, $a=3$)

27. $\div 6$ $\frac{x}{3}=\frac{y}{2}=\frac{z}{-6}$, $\frac{x}{2}=\frac{y}{-12}=\frac{z}{-3}$

$\cos\theta=\frac{6+24+18}{|\vec{n_1}||\vec{n_2}|}=0 \Rightarrow \theta=\pi/2$

28. $1+\omega+\omega^2=0 \Rightarrow 1+\omega=-\omega^2$

$(-\omega^2)^7=A+b\omega$

$1=A+b\omega$

$A=1, b=0$

29. $\left(\frac{z_1}{z_2}\right)^{-1}=\frac{z_2}{z_1}=\frac{-4+3i}{2-i}\times\frac{2+i}{2+i}=\frac{-8-3+2i}{5}=\frac{-11+2i}{5}$

30. $(a+\sqrt{5}b)(c+\sqrt{5}d)=(ac+5bd)+(ad+bc)\sqrt{5}$ $\in Z$

ஆகும், A-ன் மீது அடைவு பெற்றுள்ளது

IV

31. $A=\begin{bmatrix} 1 & 3 \\ 2 & -5 \end{bmatrix}$, $adjA=\begin{bmatrix} -5 & -3 \\ -2 & 1 \end{bmatrix}$, $|A|=-11$

$A(adjA)=\begin{bmatrix} -11 & 0 \\ 0 & -11 \end{bmatrix}=(adjA)A=|A|I$

32. $z_1=1$, $z_2=-\frac{1}{2}+i\frac{\sqrt{3}}{2}$, $z_3=-\frac{1}{2}-i\frac{\sqrt{3}}{2}$

$d_1=|z_1-z_2|=\left|1+\frac{1}{2}-i\frac{\sqrt{3}}{2}\right|=\left|\frac{3}{2}-i\frac{\sqrt{3}}{2}\right|$

$=\frac{1}{2}\sqrt{9+3}=\frac{2\sqrt{3}}{2}=\sqrt{3}=d_1$

$d_2=\sqrt{3}$, $d_3=\sqrt{3}$

33. $xy=20$, $y=\frac{20}{x}$, $S=x+y \Rightarrow x+\frac{20}{x}$

$\frac{dS}{dx}=1-20/x^2 \Rightarrow 1-\frac{20}{x^2}=0 \Rightarrow x^2=20$

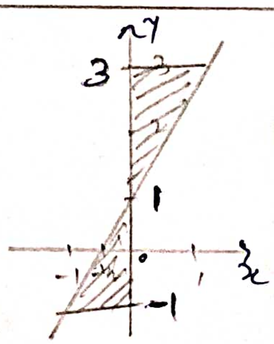
$x=2\sqrt{5}$

$\frac{d^2S}{dx^2}=\frac{40}{x^3}$ (+ve) சிறுமம்

$x=2\sqrt{5}$, $y=2\sqrt{5}$

34. $y=2x+1$

x	0	$-1/2$
y	1	0



$A=\int_{-1}^{1}-x\,dy+\int_1^3 x\,dy$

$=-\int_{-1}^{1}\frac{1}{2}(y-1)dy+\int_1^3\frac{1}{2}(y-1)dy$

$=+\frac{1}{2}\left\{\left[-\frac{y^2}{2}+y\right]_{-1}^{1}+\left[\frac{y^2}{2}-y\right]_1^3\right\}$

$=\frac{1}{2}\left[\frac{1}{2}+\frac{3}{2}+\frac{9}{2}-3-\frac{1}{2}+1\right]=2$

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35. $\frac{dy}{dx} = e^x e^y + x^2 e^{x^3} e^y$

$\frac{dy}{e^y} = (e^x + x^2 e^{x^3}) dx$

$\int e^{-y} dy = \int e^x dx + \int x^2 e^{x^3} dx.$

$-e^{-y} = e^x + \frac{e^{x^3}}{3} + c$

36. -1

37. ar. Sma = $\frac{2b^2}{a}$

38. $\frac{dy}{dx} = \frac{\sin x \sin y}{\cos x \cos y} \Rightarrow \frac{dy}{dx} = \tan x \cdot \tan y$

$\int \frac{dy}{\tan y} = \int \tan x \, dx$

$\int \cot y \, dy = \int \tan x \, dx$

$\log \sin y = \log \sec x + \log c$

$\sin y = c \cdot \sec x$

39. $E(x) = 7$, $E(x^2) = 65$

$Var(x) = 16$

40. (i) $p \vee \neg p$ (ii) $p \wedge \neg p$

p	¬p	p ∨ ¬p
T	F	T
F	T	T

p	¬p	p ∧ ¬p
T	F	F
F	T	F

41 a.

$9a - 3b + c = 21$

$25a + 5b + c = 61$

$a + b + c = 9$

$\begin{bmatrix} 1 & 1 & 1 & 9 \\ 0 & -5 & -6 & -41 \\ 0 & 0 & 8 & 48 \end{bmatrix}$ $a = 2$, $b = 1$, $c = 6$

b. $-\sqrt{3} + 3i = 2\sqrt{3}\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)$

$(-\sqrt{3} + 3i)^{31} = (2\sqrt{3})^{31}\left[cis \frac{62\pi}{3}\right]$

$= (2\sqrt{3})^{31}\left[cis \frac{2\pi}{3}\right]$

$= (2\sqrt{3})^{31}\left[-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right]$

42. a. $\frac{(x+3)^2}{4} + \frac{(y-1)^2}{16} = 1$, $c^2 = a^2 - b^2 = 12$

$c = \pm 2\sqrt{3}$

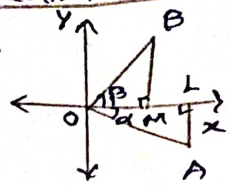
C $(-3, 1)$

S $(-3, 2\sqrt{3}+1)$

V $(3, 5)$ $(3, -3)$

LR $= \frac{2b^2}{a} = 2$

b. $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$



$\hat{a} = \cos\alpha i - \sin\alpha j$

$\hat{b} = \cos\beta i + \sin\beta j$

$\hat{a} \cdot \hat{b} = \cos(\alpha+\beta)$ — ①

$\hat{a} \cdot \hat{b} = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ — ② eqn 1 = 2

43 a. $\sqrt{5} - \sqrt{3}$, $\sqrt{5} + \sqrt{3}$

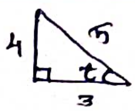
$x^2 - 2\sqrt{5}x + 2 = 0$, $x^2 + 2\sqrt{5}x + 2 = 0$

$[(x^2+2) + 2\sqrt{5}x][(x^2+2) - 2\sqrt{5}x] = 0$

$(x^2+2)^2 - (2\sqrt{5}x)^2 = 0$

$x^4 - 16x^2 + 4 = 0$

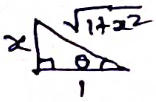
b. $\cos(\tan^{-1} x) = \sin t$



$t = \cot^{-1} 3/4 \Rightarrow \cot t = 3/4$

$\cos(\tan^{-1} x) = 4/5$

put $\tan^{-1} x = \theta \Rightarrow x = \tan\theta$



$\cos\theta = 4/5$

$\frac{1}{\sqrt{1+x^2}} = \frac{4}{5} \Rightarrow 1 + x^2 = \frac{25}{16} \Rightarrow x^2 = \frac{9}{16}$, $\boxed{x = 3/4}$

44 a. $I = \int_0^{\pi/4} \log(1+\tan x)\, dx$ — ① $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$I = \int_0^{\pi/4} \log[1 + \tan(\pi/4 - x)]\, dx$ — ②

① + ② ⇒

$2I = \int_0^{\pi/4} \log\left[1 + \frac{1-\tan x}{1+\tan x}\right] dx = \int [\log 2 - \log(1+\tan x)] dx$

$2I = \pi/4 \log 2 - I$ ⇒ $2I = \pi/4 \log 2 \Rightarrow I = \frac{\pi}{8}\log 2$

b. $\frac{dy}{dx} = \frac{x-y+5}{2(x-y)+7}$ — ①

put $x - y = t \Rightarrow 1 - \frac{dy}{dx} = \frac{dt}{dx} \Rightarrow 1 - \frac{dt}{dx} = \frac{dy}{dx}$

sub in ① ⇒ $1 - \frac{dt}{dx} = \frac{t+5}{2t+7} \Rightarrow 1 - \frac{t+5}{2t+7} = \frac{dt}{dx}$

$\frac{2t+7-t-5}{2t+7} = \frac{dt}{dx} \Rightarrow \frac{t+2}{2t+7} = \frac{dt}{dx}$

$\int dx = \int \frac{2t+7}{t+2} dt \Rightarrow x = \int \frac{2(t+2)}{t+2} + \frac{3}{t+2}\, dt$

$x = 2t + 3\log(t+2) + c$

$x = 2(x-y) + 3\log(x-y+2) + c$

45 a. $x + 1 = 2y = -12z$

$\div 12 \Rightarrow \frac{x+1}{12} = \frac{y}{6} = \frac{z}{-1}$ — ①

$\vec{a} = -\vec{i}$, $\vec{b} = 12\vec{i} + 6\vec{j} - \vec{k}$

$x = y + 2 = 6z - 6$

$\frac{x}{6} = \frac{y+2}{6} = \frac{z-1}{1} \Rightarrow \vec{c} = -2\vec{j} + \vec{k}$

$\vec{d} = 6\vec{i} + 6\vec{j} + \vec{k}$

$$\delta = \frac{|(\vec{c}-\vec{a}).(\vec{b}\times\vec{d})|}{|\vec{b}\times\vec{d}|}$$

$\vec{c}-\vec{a} = \hat{i}-2\hat{j}+\hat{k}$

$\vec{b}\times\vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 12 & 6 & -1 \\ 6 & 6 & 1 \end{vmatrix} = 6(2\hat{i}-3\hat{j}+6\hat{k})$

$(\vec{c}-\vec{a}).(\vec{b}\times\vec{d}) = 84$

தே. தூரம் $= \frac{84}{6\sqrt{49}} = \frac{14}{7} = 2$

(ii) $V = \frac{4}{3}\pi r^3$

$\frac{dv}{dt} = \frac{4}{3}\pi \times 3r^2 \frac{dr}{dt}$

$1000 = 4\pi \times 7 \times 7 \times \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{250}{49\pi}$

$S.A = 4\pi r^2$

$\frac{dA}{dt} = 4\pi \times 2r \frac{dr}{dt}$

$= 8\pi \times 7 \times \frac{250}{49\pi}$

$\frac{dA}{dt} = \frac{2000}{7}\ cm^2$

46. (i) $n=5$

எதிர்பார்ப்பு $= np$, மாறுபாடு $= npq$

$np+npq = 1.8 \Rightarrow np(1+q) = 1.8$

$5p(1+1-p) = 1.8 \Rightarrow 2p-p^2 = \frac{18}{5} = \frac{18}{50} = \frac{9}{25}$

$50p-25p^2 = 9 \Rightarrow 25p^2-50p+9 = 0$

$(5p-9)(5p-1) = 0 \Rightarrow p = 9/5$ (or) $\boxed{p = 1/5}$

$p \geq 1 \Rightarrow \boxed{q = 4/5}$

$P[x=x] = nCx\, p^x q^{n-x}$

$P[x=x] = 5Cx\,(1/5)^x (4/5)^{5-x}$

$x = 0,1,2,3,4,5$

(ii)

p	q	$p \leftrightarrow q$	$p \to q$	$q \to p$	$p \to q \wedge q \to p$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

① $\equiv$ ② (LHS = RHS)

47 (i) $f(x) = 3x^4 - 4x^2$

$f'(x) = 12x^3 - 12x^2$

$f'(x) = 0 \Rightarrow x = 0, 1$

$f(-1) = 7$, $f(2) = 16$, $f(0) = 0$, $f(1) = -1$

மீப்பெரு மதிப்பு $= 16$

மீச்சிறு மதிப்பு $= -1$

(ii) $m_1 = \frac{-ax}{by}$, $m_2 = -\frac{cx}{dy}$

$m_1 m_2 = -1$

$\frac{acx^2}{bdy^2} = -1$, at (x_1, y_1)

$acx_1^2 + bdy_1^2 = 0$

$(a-c)x_1^2 + (b-d)y_1^2 = 0$

$\frac{a-c}{ac} = \frac{b-d}{bd}$

$\Rightarrow \frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$

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