Class : 12

Register Number 1 2 0 1 4 3

FIRST REVISION EXAMINATION, JANUARY - 2024

Time Allowed : 3.00 Hours] **MATHEMATICS** [Max. Marks : 90

PART - I

1. Answer all the questions by choosing the correct answer from the given 4 alternatives $20 \times 1 = 20$
2. Write question number, correct option and corresponding answer
3. Each question carries 1 mark

1. If $A=\begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$, and $AB = I_2$, then $B=$

(1) $\left(\cos^2\frac{\theta}{2}\right)A$ (2) $\left(\cos^2\frac{\theta}{2}\right)A^T$ (3) $(\cos^2\theta)I$ (4) $\left(\sin^2\frac{\theta}{2}\right)A$

2. If $x^a y^b = e^m$, $x^c y^d = e^n$, $\Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}$, $\Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ then the values of x and y are respectively,

(1) $e^{(\Delta 2/\Delta 1)}, e^{(\Delta 3/\Delta 1)}$ (2) $\log(\Delta 1/\Delta 3), \log(\Delta 2/\Delta 3)$ (3) $\log(\Delta 2/\Delta 1), \log(\Delta 3/\Delta 1)$ (4) $e^{(\Delta 1/\Delta 3)}, e^{(\Delta 2/\Delta 3)}$

3. The solution of the equation $|z| - z = 1 + 2i$ is

(1) $\frac{3}{2} - 2i$ (2) $-\frac{3}{2} + 2i$ (3) $2 - \frac{3}{2}i$ (4) $2 + \frac{3}{2}i$

4. If α, β, and γ are the zeros of $x^3 + px^2 + qx + r$, then $\sum\frac{1}{\alpha}$ is

(1) $-\frac{q}{r}$ (2) $-\frac{p}{r}$ (3) $\frac{q}{r}$ (4) $-\frac{q}{p}$

5. $\sin^{-1}(\cos x) = \frac{\pi}{2} - x$ is valid for

(1) $-\pi \le x \le 0$ (2) $0 \le x \le \pi$ (3) $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ (4) $-\frac{\pi}{4} \le x \le \frac{3\pi}{4}$

6. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if

(1) $15 < m < 65$ (2) $35 < m < 85$ (3) $-85 < m < -35$ (4) $-35 < m < 15$

7. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\vec{b} + \vec{c}}{\sqrt{2}}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(1) $\frac{\pi}{2}$ (2) $\frac{3\pi}{4}$ (3) $\frac{\pi}{4}$ (4) π

8. If the line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lies in the plane $x + 3y - \alpha z + \beta = 0$, then (α, β) is

(1) $(-5, 5)$ (2) $(-6, 7)$ (3) $(5, -5)$ (4) $(6, -7)$

9. The point on the curve $6y = x^3 + 2$ at which y-coordinate changes 8 times as fast as x-coordinate is

(1) $(4, 11)$ (2) $(4, -11)$ (3) $(-4, 11)$ (4) $(-4, -11)$

10. The approximate change in the volume V of a cube of side x metres caused by increasing the side by 1% is

(1) $0.3xdxm^3$ (2) $0.03xm^3$ (3) $0.03x^2m^3$ (4) $0.03x^3m^3$

11. If $f(x) = \frac{x}{x+1}$, then its differential is given by

(1) $\frac{-1}{(x+1)^2}dx$ (2) $\frac{1}{(x+1)^2}dx$ (3) $\frac{1}{x+1}dx$ (4) $\frac{-1}{x+1}dx$

12. The value of $\int_0^1 \frac{dx}{1+5^{\cos x}}$ is

(1) $\frac{\pi}{2}$ (2) π (3) $\frac{3\pi}{2}$ (4) 2π

13. The order and degree of the differential equation $\sqrt{\sin x}(dx + dy) = \sqrt{\cos x}(dx - dy)$ is

(1) 1, 2 (2) 2, 2 (3) 1, 1 (4) 2, 1

14. If in 6 trials, X is a binomial variable which follows the relation 9P(X=4) = P(X=2), then the probability of succes

(1)0.125 (2) 0.25 (3) 0.375 (4) 0.75

15. In the set Q define a ⊙ b = a+b+ab . For what value of y, 3 ⊙ (y ⊙ 5) = 7 ?

(1) $y = \frac{2}{3}$ (2) $y = \frac{-2}{3}$ (3) $y = \frac{-3}{2}$ (4) $y = 4$

16. $\arg(z^n)$ =

(1) $(\arg z)^n$ (2) $n(\arg z)$ (3) $\arg(nz)$ (4) $\frac{1}{n}\arg z$

17. $11x^2 - 25y^2 - 44x + 50y - 256 = 0$ represents

(1) circle (2) Ellipse (3) Parabola (4) Hyperbola

18. Slope of the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$ is

(1) 3 (2) -3 (3) $\frac{1}{3}$ (4) $-\frac{1}{3}$

4x + 3cos x
slope of Normal = $\frac{-1}{4x+3\cos x}$ = $\frac{-1}{3}$

19. $\int_0^{\sqrt{3}} \frac{1}{1+x^2}dx$ =

(1) $\frac{\pi}{12}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{3}$

20. $4 +_6 5$ =

(1) 2 (2) 3 (3) 9 (4) 20

PART - II

1. Answer any 7 questions
2. Each question carries 2 marks
3. Question number 30 is compulsory

21. Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

Eg 1.11 AA^T = A^TA = I_2
Pg : 12 orthogonal

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Ex 2.8 1 pg 92

22. If $\omega \neq 1$ is a cube root of unity, show that $\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}+\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}=-1$.

23. Discuss the maximum possible number of positive and negative roots of the polynomial equation $9x^9-4x^8+4x^7-3x^6+2x^5+x^3+7x^2+7x+2=0$. 4 +ve, 3 −ve pg 127 Ex 3.6 (1)

24. Two fair coins are tossed simultaneously (equivalent to a fair coin is tossed twice). Find the probability mass function for number of heads occurred. Eg 11.5 pg 185 Ex 6.2 (9)

25. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}$, $\hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ are coplanar, prove that c is the geometric mean of a and b.

26. A person learnt 100 words for an English test. The number of words the person remembers in t days after learning is given by $W(t)=100\times(1-0.1t)^2, 0\leq t\leq 10$. What is the rate at which the person forgets the words 2 days after learning? Eg 7.3 Pg 4 Ans 16 words

27. Let $g(x)=x^2+\sin x$. Calculate the differential dg. $(2x+\cos x)\,dx$ Eg 8.6 Pg 67

28. Evaluate: $\int_0^{\frac{\pi}{2}}\sin^2x\cos^4x\,dx$ Ex 9.6 (v) $\frac{\pi}{32}$

29. Find the equation of the tangent at $t=2$ to the parabola $y^2=8x$ Ex 5.4 (5) $x-2y+8=0$

30. Let $M=\left\{\begin{pmatrix}x & x\\ x & x\end{pmatrix}: x\in R-\{0\}\right\}$ and let $\cdot$ be the matrix multiplication. M is closed under $\cdot$. If so, examine the existence of identity, existence of inverse properties for the operation $\cdot$ on M. Ex 12.2 (9)

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PART - III

7 x 3 = 21

1. **Answer any 7 questions**
2. **Each question carries 3 marks**
3. **Question number 40 is compulsory**

31. Find the square root of $6-8i$. $\pm(2\sqrt{2}-i\sqrt{2})$ Eg 2.17 Pg 72

32. Solve the equation $9x^3-36x^2+44x-16=0$ if the roots form an arithmetic progression. Ex 3.3 (2) pg 117 $\frac{2}{3}, \frac{4}{3}, 2$

33. Find the value of $\tan^{-1}(-1)+\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$ $-\frac{\pi}{12}$ Eg 4.10 Pg 147

34. Show that the straight line $x+1=2y=-12z$ and $x=y+2=6z-6$ are skew and hence find the shortest distance between them. Ex 6.5 (5) 2 units

Ex 7.6 1(v)

35. Find the absolute extrema of the following function on the given closed interval : $f(x)=2\cos x+\sin 2x$; $\left[0,\frac{\pi}{2}\right]$

36. Let $U(x,y,z)=x^2-xy+3\sin z$, $x,y,z\in R$. Find the linear approximation for U at $(2,-1,0)$. Eg 8.17 pg 81 $5x-2y+3z-6$

37. Evaluate : $\int_0^1\sqrt{\frac{1-x}{1+x}}\,dx$

38. Solve the differential equation: $\frac{dy}{dx}+\frac{3y}{x}=\frac{1}{x^2}$, given that $y=2$ when $x=1$

$2x^3y = x^2+3$

Ex 10.7 (15)

39. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ be any two boolean matrices of the same type. Find $A \vee B$ and $A \wedge B$.

40. Solve by determinant method : $5x + 2y = 17$, $3x + 7y = 31$

PART - IV

7 x 5 = 35

1. Answer all the questions
2. Each question carries 5 marks

41. a) Find the inverse by Gauss-Jordan method: $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$

(OR)

b) If $2 + i$ and $3 - \sqrt{2}$ are the roots of the equation $x^6 - 13x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140 = 0$. Find all root

42. a) Find the equation of the circle through the points $(1,0), (-1,0)$ and $(0,1)$. Ex 5.1 (6)

(OR)

b) Find the intervals of monotonicity and local extrema of the function $f(x) = \frac{x}{1+x^2}$. Eg 7.56 Pg 39

43. a) Find the area of the region in the first quadrant bounded by the parabola $y^2 = 4x$, the line $x + y = 3$ and y-axis.

Eg 9.58 (OR) Pg 131

b) A random variable X has the following probability mass function.

x	1	2	3	4	5
f(x)	k^2	$2k^2$	$3k^2$	$2k$	$3k$

Find (i) the value of k (ii) $P(2 \le X < 5)$ (iii) $P(3 < X)$

Ex 11.2 (6)

44. a) Show that the points $1, \frac{-1}{2} + i\frac{\sqrt{3}}{2}$, and $\frac{-1}{2} - i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle

Eg 2.14 Pg 70

(OR)

b) Find the number of solution of the equation $\tan^{-1}(x-1) + \tan^{-1}x + \tan^{-1}(x+1) = \tan^{-1}(3x)$. Ex 4.5 (10) Pg 166

45. a) Prove by vector method that $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$.

(OR)

Ex 8.6 (7)

b) Let $U(x,y) = e^x \sin y$, where $x = st^2$, $y = s^2t$, $s, t \in \mathbb{R}$. Find $\frac{\partial U}{\partial s}, \frac{\partial U}{\partial t}$, and evaluate them at $s = t = 1$.

46. a) Solve: $\left(1 + 3e^{\frac{y}{x}}\right)dy + 3e^{\frac{y}{x}}\left(1 - \frac{y}{x}\right)dx = 0$, given that $y = 0$ when $x = 1$ Ex 10.6 (7)

(OR)

b) Using the equivalence property, show that $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$. Eg 12.9 Pg 248

47. a) Find the angle between $y = x^2$ and $y = (x-2)^2$.

(OR)

b) Find the vector and cartesian equation of the plane containing the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{3}$ and $\frac{x-1}{-3} = \frac{y-4}{2} = \frac{z-5}{1}$