RK12M Kanyakumari District

Common First Revision Test - January 2024

Standard - 12

MATHEMATICS

Time Allowed: 3.00 Hours Maximum Marks: 90

PART - I

Note: 1. **Answer all the questions.** **20×1=20**

2. **Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer.**

1) If $|adj\ (adj\ A)| = |A|^{16}$, then the order of the square matrix A is

a) 3 b) 4 c) 2 d) 5

2) The value of $\sum_{n=1}^{11} (i^n + i^{n-1})$ is

a) $1+i$ b) i c) 1 d) 0

3) If f and g are polynomials of degree m and n respectively, and if $h(x) = (fog)(x)$, then the degree of h is

a) mn b) $m+n$ c) m^n d) n^m

4) If $\sin^{-1}x = 2\sin^{-1}\alpha$ has a solution, then

a) $|\alpha| \le \frac{1}{\sqrt{2}}$ b) $|\alpha| \ge \frac{1}{\sqrt{2}}$ c) $|\alpha| < \frac{1}{\sqrt{2}}$ d) $|\alpha| > \frac{1}{\sqrt{2}}$

5) The radius of the circle $3x^2+by^2+4bx-6by+b^2=0$ is

a) 1 b) 3 c) $\sqrt{10}$ d) $\sqrt{11}$

6) The volume of the parallelepiped with its edges represented by the vectors $\hat{i}+\hat{j}, \hat{i}+2\hat{j}, \hat{i}+\hat{j}+\pi\hat{k}$ is

a) $\pi/2$ b) $\pi/3$ c) π d) $\pi/4$

7) The slope of the tangent to the curve $f(x) = 2\cos 4x$ at $x = \pi/12$ is

a) $-4\sqrt{3}$ b) -4 c) $\frac{\sqrt{3}}{12}$ d) $4\sqrt{3}$

8) If $w(x, y) = x^y$, $x > 0$, then $\frac{\partial w}{\partial y}$ is equal to

a) $x^y \log x$ b) $y \log x$ c) $y\,x^{y-1}$ d) $x \log y$

9) The value of $\int_0^1 x(1-x)^{99}\,dx$ is

a) $\frac{1}{11000}$ b) $\frac{1}{10100}$ c) $\frac{1}{10010}$ d) $\frac{1}{10001}$

10) The integrating factor of the differential equation $\frac{dy}{dx} + P(x)y = Q(x)$ is x, then P(x) is

a) x b) $x^2/2$ c) $1/x$ d) $1/x^2$

11) If $P(X = 0) = 1 - P(X = 1)$. If $E(X) = 3Var(X)$, then $P(X = 0)$ is

a) $2/3$ b) $2/5$ c) $1/5$ d) $1/3$

12) Which one of the following is a binary operation on N

a) Addition b) Multiplicaiton

c) Both (1) and (2) d) None of the above

13) Which of the following is a continuous random variable?

I. The number of cars crossing a particular signal in a day

II. The number of customers in a queue to buy train tickets at a moment

III. The time taken to complete a telephone call

a) I and II b) II only c) III only d) II and III

14) The order and degree of the differential equation $\frac{d^2y}{dx^2} + y^{1/3} = 0$ are respectively

a) 2, 3 b) 3, 3 c) 2, 6 d) 2, 1

15) The value of $\int_0^{\pi/6} \cos^3 3x\,dx$ is

a) $\frac{2}{3}$ b) $\frac{2}{9}$ c) $\frac{1}{9}$ d) $\frac{1}{3}$

16) The curve $y = x^4+1$ has point of inflection at

a) (0, 0) b) (0, 1) c) (0, 4) d) nowhere

17) If $\bar{a}$ and $\bar{b}$ are parallel vectors, then $[\bar{a}, \bar{c}, \bar{b}]$ is equal to

a) 2 b) −1 c) 1 d) 0

18) If $x + y = k$ is a normal to the parabola $y^2 = 12x$, then the value of k is

a) 3 b) −1 c) 1 d) 9

19) The value of $\sin^{-1}(\cos x)$, $0 \le x \le \pi$ is

a) $\pi - x$ b) $x - \pi/2$ c) $\pi/2 - x$ d) $x - \pi$

20) The principal argument of $(\sin 40° + i \cos 40°)^5$ is

a) −110° b) −70° c) 70° d) 110°

PART - II

nswer any seven questions. Question No. 30 is compulsory. **7×2=14**

21) If $\omega \neq 1$ is a cube root of unity, show that $(1-\omega+\omega^2)^6+(1+\omega-\omega^2)^6 = 128$

22) Show that, if p, q, r are rational, the roots of the equation $x^2-2px+p^2-q^2+2qr-r^2 = 0$ are rational.

23) Find the value of $\sin^{-1}[\sin 5]$

24) Examine the position of the point (2, 3) with respect to the circle $x^2+y^2-6x-8y+12 = 0$.

25) Verify whether the line $\frac{x-3}{-4} = \frac{y-4}{-7} = \frac{z+3}{12}$ lies in the plane $5x-y+z = 8$

26) Evaluate: $\lim_{x \to 0} \frac{1-\cos x}{x^2}$

27) If $U = (x-y)(y-z)(z-x)$ find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$

28) Evaluate: $\int_{-1}^{1} x \cos\left(\frac{e^x - 1}{e^x + 1}\right) dx$

29) Solve: $\tan y \frac{dy}{dx} = \cos(x + y) + \cos(x - y)$

30) Find A^{-1} if adj $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

PART - III

Answer any seven questions. Question No. 40 is compulsory. **7×3=21**

31) Find the domain of $\sin^{-1}\left(\frac{2 + \cos x}{3}\right)$

32) If $\frac{z + 3}{z - 5i} = \frac{1 + 4i}{2}$, find the complex number z in the rectangular form.

33) A room 34 m long is constructed to be a whispering gallery. The room has an ellipitcal ceiling. If the maximum height of the ceilling in 8m, determinie where the foci are located.

34) Prove that $[\vec{a} - \vec{b}, \vec{b} - \vec{c}, \vec{c} - \vec{a}] = 0$

35) Find two positive numbers whose sum is 12 and their product is maximum.

36) If $u(x, y) = \frac{x^2 + y^2}{\sqrt{x + y}}$, find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$

37) Find the area of the region bounded by y = cos x, y = sin x, the lines $x = \pi/4$ and $x = 5\pi/4$

38) Show that y = a cos (log x) + b sin (log x), x > 0 is a solution of the differential equation $x^2y'' + xy' + y = 0$

39) A random variable X has the following probability mass function:

x	1	2	3	4	5	6
f(x)	k	2k	6k	5k	6k	10k

Find (i) $P(X \leq 4)$ (ii) $P(3 < X)$

40) Show that $p \rightarrow q$ and $q \leftrightarrow p$ are not equivalent.

PART - IV

Answer all the questions: **7×5=35**

41) a] Four men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix inversion method.

(OR)

b] Show that the line x−y+4 = 0 is a tangent to the ellipse $x^2+3y^2 = 12$. Also find the coordinates of the point of contact.

42) a] Find the value of $\cos\left[\sin^{-1}\left(\frac{4}{5}\right)-\tan^{-1}\left(\frac{3}{4}\right)\right]$

(OR)

b] Solve: $2\sqrt{\frac{x}{a}}+3\sqrt{\frac{a}{x}}=\frac{b}{a}+\frac{6a}{b}$

43) a] Prove by vector method that sin (A–B) = sin A cos B – cos A sin B

(OR)

b] Find all the values of $\left(\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)^{3/4}$

44) a] For the curve $y=\frac{3x}{x^2-1}$, discuss the following:

(i) Domain (ii) Symmetry (iii) Asymptotes
(iv) Intercepts (v) Monotonicity

(OR)

b] Show that $\int_0^1\left[\tan^{-1}x+\tan^{-1}(1-x)\right]dx=\frac{\pi}{2}-\log 2$

45) a] Let g(x, y) = 2y + x², x = 2r–s, y = r²+2s, where r, s ∈ R. Find $\frac{\partial g}{\partial r},\frac{\partial g}{\partial s}$

(OR)

b] Solve $\left(1+2e^{x/y}\right)dx+2e^{x/y}\left(1-\frac{x}{y}\right)dy=0$

46) a] Find the mean and variance of a random variable X, whose probability density function is $f(x)=\begin{cases}\lambda e^{-\lambda x} & \text{for } x\geq 0\\ 0 & \text{otherwise}\end{cases}$

(OR)

b] Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for the operation X_{11} on a subset A = {1, 3, 4, 5, 9} of the set of remainders {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

47) a] Find the volume of a right-circular cone of base radius r and height h.

(OR)

b] Find the parametric vector, non-parametric vector and Cartesian form of the equations of the plane passing through the three non-collinear points (3, 6, –2) (–1, –2, 6) and (6, 4, –2)

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