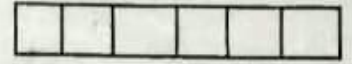


FRM

**FIRST REVISION TEST - 2024****12** - Std**MATHEMATICS**

Time : 3.00 Hrs

Mraks : 90

**PART - I****Choose the correct answer.**

20 X 1 = 20

- The solution of the equation  $|z| - z = 1 + 2i$  is  
 a)  $\frac{3}{2} - 2i$       b)  $\frac{-3}{2} + 2i$       c)  $2 - \frac{3i}{2}$       d)  $2 + \frac{3i}{2}$
- If  $\omega \neq 1$  is a cubic root of unity and  $(1 + \omega)^7 = A + B\omega$ , then  $(A, B)$  equals  
 a)  $(1, 0)$       b)  $(-1, 1)$       c)  $(0, 1)$       d)  $(1, 1)$
- The rank of the matrix  $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$  is a) 1      b) 2      c) 4      d) 3
- If  $A \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$  then  $A =$  a)  $\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$       b)  $\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$       c)  $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$       d)  $\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$
- If  $x^3 + 12x^2 + 10ax + 1999$  definitely has a positive zero, if and only if  
 a)  $a \geq 0$       b)  $a > 0$       c)  $a < 0$       d)  $a \leq 0$
- If  $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$ ; then  $\cos^{-1}x + \cos^{-1}y$  is equal to  
 a)  $\frac{2\pi}{3}$       b)  $\frac{\pi}{3}$       c)  $\frac{\pi}{6}$       d)  $\pi$
- Find the value of  $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right)$       a) 0      b) 1      c)  $\frac{1}{2}$       d)  $\frac{\pi}{3}$
- If the coordinates at one end of a diameter of the circle  $x^2 + y^2 - 8x - 4y + c = 0$  are  $(11, 2)$  the coordinates of the other end are  
 a)  $(-5, 2)$       b)  $(-3, 2)$       c)  $(5, -2)$       d)  $(-2, 5)$
- If  $\vec{a}$  and  $\vec{b}$  are parallel vectors, then  $[\vec{a}, \vec{c}, \vec{b}]$  is equal to  
 a) 2      b) -1      c) 1      d) 0
- Find the value of  $[\hat{i}, \hat{j}, \hat{k}]$       a) 0      b) 2      c) 1      d) 3

11. The slope of the line normal to the curve  $f(x) = 2 \cos 4x$  at  $x = \frac{\pi}{12}$  is
- a)  $-4\sqrt{3}$       b)  $-4$       c)  $\frac{\sqrt{3}}{12}$       d)  $4\sqrt{3}$
12. The maximum value of the product of two positive numbers, when their sum of the square is 200, is
- a) 100      b)  $25\sqrt{7}$       c) 28      d)  $24\sqrt{14}$
13. The approximate change in the volume  $V$  of a cube of side  $x$  metres caused by increasing the side by 1% is
- a)  $0.3x \, dx \, m^3$       b)  $0.03x \, m^3$       c)  $0.03 \, x^2 \, m^3$       d)  $0.03x^3 \, m^3$
14. Find the degree of  $F(x, y) = \frac{x^2 + 5xy - 10y^2}{5x - 5y}$
- a) 1      b) 2      c) 5      d) -10
15. The value of  $\int_0^{\pi} \frac{dx}{1 + 5^{\cos x}}$  is
- a)  $\frac{\pi}{2}$       b)  $\pi$       c)  $\frac{3\pi}{2}$       d)  $2\pi$
16. The order and degree of the differential equation  $\sqrt{\sin x} (dx + dy) = \sqrt{\cos x} (dx - dy)$  is
- a) 1, 2      b) 2, 2      c) 1, 1      d) 2, 1
17. If  $P(X = 0) = 1 - P(X = 1)$ . If  $E(X) = 3$   $\text{Var}(x)$  then  $P(X = 0)$  is
- a)  $\frac{2}{3}$       b)  $\frac{2}{5}$       c)  $\frac{1}{5}$       d)  $\frac{1}{3}$
18. Standard deviation of binomial distribution is .....
- a)  $np$       b)  $n$       c)  $npq$       d)  $\sqrt{npq}$
19. Subtraction is not a binary operation in
- a)  $R$       b)  $Z$       c)  $N$       d)  $Q$
20. Which one of the following statements has the truth value T?
- a)  $\sin x$  is an even function      b) Every square matrix is non-singular.  
c) The product of complex number and its conjugate is purely imaginary.  
d)  $\sqrt{5}$  is an irrational number.

### PART - II

Answer any 7 questions. Q.No. 30 is compulsory:-

7 X 2 = 14

21. Find the monic polynomial equation of minimum degree with real coefficients having  $2 - \sqrt{3}i$  as a root.
22. Find the square root of  $6-8i$ .

23. If  $\text{adj } A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$  find  $A^{-1}$ .

24. Explain why Rolle's theorem is not applicable to the following functions in the respective intervals  $f(x) = \tan x$ ,  $x \in [0, \pi]$ .
25. If the radius of a sphere, with radius 10cm, has to decrease by 0.1cm, approximately how much will its volume decrease?
26. If  $2\hat{i} - \hat{j} + 3\hat{k}$ ,  $3\hat{i} + 2\hat{j} + \hat{k}$ ,  $\hat{i} + m\hat{j} + 4\hat{k}$  are coplanar, find the value of  $m$ .
27. Find the equation of the parabola with focus  $(-\sqrt{2}, 0)$  and directrix  $x = \sqrt{2}$ .
28. Evaluate:  $\int_{-\pi/2}^{\pi/2} x \cos x \, dx$ .
29. The probability density function of  $x$  is given by  $f(x) = \begin{cases} kxe^{-2x} & \text{for } x > 0 \\ 0 & \text{for } x \leq 0 \end{cases}$  find the value of  $k$ .
30. Prove that, In an algebraic structure the identity element (if exists) must be unique.

### PART - III

Answer any 7 questions. Q.No. 40 is compulsory.

7 X 3 = 21

31. Find the rank of the matrix  $\begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$  by reducing it to an Echelon form.
32. Simplify  $\left( \sin \frac{\pi}{6} + i \cos \frac{\pi}{6} \right)^{18}$ .
33. Find the centre and radius of the circle  $3x^2 + (a + 1)y^2 + 6x - 9y + a + 4 = 0$ .
34. For the random variable  $x$  with the given probability mass function as below, find the mean and variance.  $f(x) = \begin{cases} \frac{4-x}{6}, & x = 1, 2, 3. \end{cases}$
35. Find two positive numbers whose sum is 12 and their product is maximum.
36. Find the domain of  $\sin^{-1}(2 - 3x^2)$ .
37. Solve  $\therefore 2xydx + (x^2 + 2y^2) dy = 0$ .
38. Establish the equivalence property connecting the bi - conditional with conditional  $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$ .
39. Show that the equation  $x^9 - 5x^5 + 4x^4 + 2x^2 + 1 = 0$  has atleast 6 imaginary solutions.
40. Find the torque of the resultant of the three forces represented by  $-3\hat{i} + 6\hat{j} - 3\hat{k}$ ,  $4\hat{i} - 10\hat{j} + 12\hat{k}$  and  $4\hat{i} + 7\hat{j}$  acting at the point with position vector  $8\hat{i} - 6\hat{j} - 4\hat{k}$  about the point with position vector  $18\hat{i} + 3\hat{j} - 9\hat{k}$ .

## PART- IV

Answer all the questions.

7 x 5 = 35

41. a) On the average 20% of the products manufactured by ABC company are found to be defective. If we select 6 of these products at random and  $x$  denotes the number of defective products find the probability that (i) two products are defective (ii) at most one product is defective (iii) at least two products are defective. **(OR)** b) Find the centre, foci and eccentricity of the hyperbola  $11x^2 - 25y^2 - 44x + 50y - 256 = 0$ .
42. a) Verify (i) closure property (ii) commutative property (iii) associative property (iv) existence of identity and (v) existence of inverse for the operation  $X_{11}$  on a subset  $A = \{1, 3, 4, 5, 9\}$  of the set of remainders  $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$  **(OR)** b) If  $z = x + iy$  and  $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$ , show that  $x^2 + y^2 + 3x - 3y + 2 = 0$ .
43. a) Expand  $\log(1+x)$  as a Maclaurin's series upto 4 non-zero terms for  $-1 < x \leq 1$ . **(OR)** b) If  $2+i$  and  $3-\sqrt{2}$  are roots of the equation  $x^6 - 13x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140 = 0$ , find all roots.
44. a) Find the value of  $\tan\left(\cos^{-1}\left(\frac{1}{2}\right)\right) \sin^{-1}\left(\frac{-1}{2}\right)$ . **(OR)** b) Find the volume of the solid formed by revolving the region bounded by the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ,  $a > b$  about the major axis.
45. a) Test for consistency of the following system of linear equations and if possible solve:  $x + 2y - z = 3$ ,  $3x - y + 2z = 1$ ,  $x - 2y + 3z = 3$ ,  $x - y + z + 1 = 0$ . **(OR)** b) Find the parametric vector, non-parametric vector and Cartesian form of the equations of the plane passing through the three non-collinear points  $(3, 6, -2)$ ,  $(-1, -2, 6)$  and  $(6, 4, -2)$ .
46. a) Find the population of a city at any time  $t$ , given that the rate of increase of population is proportional to the population at that instant and that in a period of 40 years the population increased from 3,00,000 to 4,00,000. **(OR)** At a water fountain, water attains a maximum height of 4m at horizontal distance of 0.5m from its origin. If the path of water is a parabola, find the height of water at a horizontal distance of 0.75 m from the point of origin.
47. a) If  $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ ,  $\vec{b} = 3\hat{i} + 5\hat{j} + 2\hat{k}$ ,  $\vec{c} = -\hat{i} - 2\hat{j} + 3\hat{k}$  verify that  $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$  **(OR)** b) If  $w(x, y) = xy + \sin(xy)$  then prove that  $\frac{\partial^2 w}{\partial y \partial x} = \frac{\partial^2 w}{\partial x \partial y}$ .