# FIRST REVISION TEST - 2024

**12 - Std** **MATHEMATICS**

Time : 3.00 Hrs Mraks : 90

## PART - I

**Choose the correct answer.** 20 X 1 = 20

1. The solution of the equation $|z| - z = 1 + 2i$ is

   a) $\frac{3}{2} - 2i$ b) $\frac{-3}{2} + 2i$ c) $2 - \frac{3i}{2}$ d) $2 + \frac{3i}{2}$

2. If $\omega \neq 1$ is a cubic root of unity and $(1+\omega)^7 = A + B\omega$, then (A, B) equals

   a) (1, 0) b) (−1, 1) c) (0, 1) d) (1, 1)

3. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$ is a) 1 b) 2 c) 4 d) 3

4. If $A\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$ then A = a) $\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$ c) $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$

5. If $x^3 + 12x^2 + 10ax + 1999$ definitely has a positive zero, if and only if

   a) $a \geq 0$ b) $a > 0$ c) $a < 0$ d) $a \leq 0$

6. If $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$; then $\cos^{-1}x + \cos^{-1}y$ is equal to

   a) $\frac{2\pi}{3}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{6}$ d) $\pi$

7. Find the value of $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right)$ a) 0 b) 1 c) $\frac{1}{2}$ d) $\frac{\pi}{3}$

8. If the coordinates at one end of a diameter of the circle $x^2 + y^2 - 8x - 4y + c = 0$ are (11, 2) the coordinates of the other end are

   a) (−5, 2) b) (−3, 2) c) (5, −2) d) (-2, 5)

9. If $\vec{a}$ and $\vec{b}$ are parallel vectors, then $\left[\vec{a}, \vec{c}, \vec{b}\right]$ is equal to

   a) 2 b) −1 c) 1 d) 0

10. Find the value of $\left[\hat{i}, \hat{j}, \hat{k}\right]$ a) 0 b) 2 c) 1 d) 3

11. The slope of the line normal to the curve f (x) = 2 cos4x at $x = \frac{\pi}{12}$ is

a) $-4\sqrt{3}$ b) -4 c) $\frac{\sqrt{3}}{12}$ d) $4\sqrt{3}$

12, The maximum value of the product of two positive numbers, when their sum of the square is 200, is a) 100 b) $25\sqrt{7}$ c) 28 d) $24\sqrt{14}$

13. The approximate change in the volume V of a cube of side x metres caused by increasing the side by 1% is
a) 0.3x dx $m^3$ b) 0.03x $m^3$ c) 0.03 $x^2m^3$ d) 0.03$x^3m^3$

14. Find the degree of $F(x, y) = \frac{x^2 + 5xy - 10y^2}{5x - 5y}$ a) 1 b) 2 c) 5 d) -10

15. The value of $\int_0^{\pi} \frac{dx}{1 + 5^{\cos x}}$ is a) $\frac{\pi}{2}$ b) $\pi$ c) $\frac{3\pi}{2}$ d) $2\pi$

16. The order and degree of the differential equation
$\sqrt{\sin x}\,(dx + dy) = \sqrt{\cos x}\,(dx - dy)$ is
a) 1, 2 b) 2, 2 c) 1, 1 d) 2, 1

17. If P ( X = 0) = 1 - P (X = 1). If E (X) = 3 Var (x) then P (X = 0) is

a) $\frac{2}{3}$ b) $\frac{2}{5}$ c) $\frac{1}{5}$ d) $\frac{1}{3}$

18. Standard deviation of binomial distribution is ....... a) np b) n c) npq d) $\sqrt{npq}$

19. Subtraction is not a binary operation in a) R b) Z c) N d) Q

20. Which one of the following statements has the truth value T?
a) sin x is an even function b) Every square matrix is non - singular.
c) The product of complex number and its conjugate is purely imaginary.
d) $\sqrt{5}$ is an irrational number.

**PART - II**

**Answer any 7 questions. Q.No. 30 is compulsory:-** 7 X 2 = 14

21. Find the monic polynomial equation of minimum degree with real coefficients having $2 - \sqrt{3}i$ as a root.

22. Find the square root of 6-8i .

23. If $adj\, A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ find $A^{-1}$.

24. Explain why Rolle's theorem is not applicable to the following functions in the respective intervals f (x) = tan x, $x \in [0, \pi]$.

25. If the radius of a sphere, with radius 10cm, has to decrease by 0.1cm, approximately how much will its volume decrease?

26. If $2\hat{i}-\hat{j}+3\hat{k},\ 3\hat{i}+2\hat{j}+\hat{k},\ \hat{i}+m\hat{j}+4\hat{k}$ are coplanar, find the value of m.

27. Find the equation of the parabola with focus $(-\sqrt{2},\ 0)$ and directrix $x=\sqrt{2}$.

28. Evaluate : $\int_{-\pi/2}^{\pi/2} x\cos x\ dx$.

29. The probability density functionof x is given by $f(x)=\begin{cases} kxe^{-2x} & for\ x>0 \\ 0 & for\ x\le 0 \end{cases}$ find the value of k.

30. Prove that, In an algebraic structure the identity element (if exists) must be unique.

**PART - III**

**Answer any 7 questions. Q.No. 40 is compulsory.** 7 X 3 = 21

31. Find the rank of the matrix $\begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$ by reducing it to an Echelon form.

32. Simplify $\left(\sin\frac{\pi}{6}+i\cos\frac{\pi}{6}\right)^{18}$.

33. Find the centre and radius of the circle $3x^2+(a+1)y^2+6x-9y+a+4=0$.

34. For the random variable x with the given probability mass function as below, find the mean and variance. $f(x)=\left\{\frac{4-x}{6},\ x=1,2,3\right.$.

35. Find two positive numbers whose sum is 12 and their product is maximum.

36. Find the domain of $\sin^{-1}(2-3x^2)$.

37. Solve : $2xydx+(x^2+2y^2)\,dy=0$.

38. Establish the equivalence property connecting the bi - conditional with conditional $p\leftrightarrow q\equiv(p\rightarrow q)\wedge(q\rightarrow p)$.

39. Show that the equation $x^9-5x^5+4x^4+2x^2+1=0$ has atleast 6 imaginary solutions.

40. Find the torque of the resultant of the three forces represented by $-3\hat{i}+6\hat{j}-3\hat{k}$, $4\hat{i}-10\hat{j}+12\hat{k}$ and $4\hat{i}+7\hat{j}$ acting at the point with position vector $8\hat{i}-6\hat{j}-4\hat{k}$ about the point with position vector $18\hat{i}+3\hat{j}-9\hat{k}$.

## PART- IV

**Answer all the questions.** 7 x 5 = 35

41. a) On the average 20% of the products manufactured by ABC company are found to be defective. If we select 6 of these products at random and x denotes the number of defective products find the probability that (i) two products are defective (ii) at most one product is defective (iii) at least two products are defective. ***(OR)*** b) Find the centre, foci and eccentricity of the hyperbola $11x^2 - 25y^2 - 44x + 50y - 256 = 0$.

42. a) Verify (i) closure property (ii) commutative property (iii) associative property (iv) existence of identity and (v) existence of inverse for the operation $X_{11}$ on a subset $A = \{1, 3, 4, 5, 9\}$ of the set of remainders $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ ***(OR)***

b) If $z = x + iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, show that $x^2 + y^2 + 3x - 3y + 2 = 0$.

43. a) Expand $\log(1 + x)$ as a Maclaurin's series upto 4 non - zero terms for $-1 < x \leq 1$. ***(OR)*** b) If $2 + i$ and $3 - \sqrt{2}$ are roots of he equation $x^6 - 13x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140 = 0$, find all roots.

44. a) Find the value of $\tan\left(\cos^{-1}\left(\frac{1}{2}\right) - \sin^{-1}\left(\frac{-1}{2}\right)\right)$. ***(OR)***

b) Find the volume of the solid formed by revolving the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a > b$ about the major axis.

45. a) Test for consistency of the following system of linear equations and if possible solve : $x + 2y - z = 3$, $3x - y + 2z = 1$, $x - 2y + 3z = 3$, $x - y + z + 1 = 0$. **(OR)** b) Find the parametric vector, non - parametric vector and Cartesion form of the equations of the plane passing through the three non - collinear points $(3, 6, -2)$, $(-1, -2, 6)$ and $(6, 4, -2)$.

46. a) Find the population of a city at any time t, given that the rate of increase of population is proportional to the population at that instant and that in a period of 40 years the population increased from 3,00,000 to 4,00,000. ***(OR)***
At a water fountain, water attains a maximum height of 4m at horizontal distance of 0.5m from its origin. If the path of water is a parabola, find the height of water at a horizontal distance of 0.75 m from the point of origin.

47. a) If $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$, $\vec{b} = 3\hat{i} + 5\hat{j} + 2\hat{k}$ $\vec{c} = -\hat{i} - 2\hat{j} + 3\hat{k}$ verify that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$ **(OR)**

b) If $w(x, y) = xy + \sin(xy)$ then prove that $\frac{\partial^2 w}{\partial y \partial x} = \frac{\partial^2 w}{\partial x \partial y}$.