| FIRST REVISION TEST - 2024 | Reg. No. | | | | | | |
|---|---|---|---|---|---|---|---|

## XII - MATHEMATICS

Time Allowed : 3.00 Hrs.                                    Maximum Marks: 90

### Part - I

**I. Choose the correct answer:**                                    20 x 1 = 20

1. If $A = \begin{pmatrix} 7 & 3 \\ 4 & 2 \end{pmatrix}$ then $9I_2 - A =$

   a) $A^{-1}$      b) $\dfrac{A^{-1}}{2}$      c) $3A^{-1}$      d) $2A^{-1}$

2. In the non-homogenous system of equations with 3 unknowns if $\rho(A) = \rho([A/B]) = 2$ then the system has
   a) unique solution      b) one parameter family of solutions
   c) two parameter family of solutions      d) no solution

3. If $|z| = 1$, then the value of $\dfrac{1+z}{1+\bar{z}}$ is

   a) $z$      b) $\bar{z}$      c) $\dfrac{1}{z}$      d) 1

4. The solution of the equation $|z| - z = 1 + 2i$ is

   a) $\dfrac{3}{2} - 2i$      b) $-\dfrac{3}{2} + 2i$      c) $2 - \dfrac{3}{2}i$      d) $2 + \dfrac{3}{2}i$

5. The number of real numbers in $[0, 2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is
   a) 2      b) 4      c) 1      d) $\infty$

6. If $\cot^{-1} x = \dfrac{2\pi}{5}$ for some $x \in R$, then value of $\tan^{-1} x$ is

   a) $\dfrac{-\pi}{10}$      b) $\dfrac{\pi}{5}$      c) $\dfrac{\pi}{10}$      d) $\dfrac{-\pi}{5}$

7. The radius of the circle $3x^2 + by^2 + 4bx - 6by + b^2 = 0$ is
   a) 1      b) 3      c) $\sqrt{10}$      d) $\sqrt{11}$

8. If the parabola $y^2 = 4ax$ passes through the point (3,2) then the length of its latus rectum is

   a) $\dfrac{2}{3}$      b) $\dfrac{4}{3}$      c) $\dfrac{1}{3}$      d) 1

9. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $[\vec{a}, \vec{b}, \vec{a} \times \vec{b}] = \dfrac{1}{4}$, then the angle between $\vec{a}$ and $\vec{b}$ is

   a) $\dfrac{\pi}{6}$      b) $\dfrac{\pi}{4}$      c) $\dfrac{\pi}{3}$      d) $\dfrac{\pi}{2}$

10. The coordinates of the point where the line $\vec{r} = (6\hat{i} - \hat{j} - 3\hat{k}) + t(-\hat{i} + 4\hat{k})$ meets the plane $\vec{r} \cdot (\hat{i} + \hat{j} - \hat{k}) = 3$ are
    a) (2,1,0)      b) (7,−1,−7)      c) (1,2,−6)      d) (5,−1,1)

11. What is the value of the limit $\lim\limits_{x \to 0} \left( \cot x - \dfrac{1}{x} \right)$

    a) 0      b) 1      c) 2      d) −1

12. The point of inflection of the curve $y = (x - 1)^3$ is

    a) (0,0)                b) (0,1)            c) (1,0)            d) (1,1)

13. If $f(x, y) = e^{xy}$, then $\dfrac{\partial^2 f}{\partial x\, \partial y}$ is equal to

    a) $xy\, e^{xy}$         b) $(1 + xy)\, e^{xy}$      c) $(1 + y)\, e^{xy}$    d) $(1 + x)\, e^{xy}$

14. The value of $\displaystyle\int_{-1}^{2} |x|\, dx$ is

    a) $\frac{1}{2}$              b) $\frac{3}{2}$            c) $\frac{5}{2}$           d) $\frac{7}{2}$

15. The value of $\displaystyle\int_{0}^{1} x^2 (1 - x)^3\, dx$ is

    a) $\frac{1}{30}$           b) $\frac{1}{20}$         c) $\frac{1}{60}$         d) $\frac{1}{2}$

16. The order and degree of the differential equation $\sqrt{\sin x}\,(dx + dy) = \sqrt{\cos x}\,(dx - dy)$

    a) 1,2            b) 2,2          c) 1,1          d) 2,1

17. Integrating factor of the differential equation $x\dfrac{dy}{dx} - y = 2x^2$ is

    a) $e^{-x}$           b) $x$             c) $\frac{1}{x}$           d) $e^{-y}$

18. Let X have a Bernoulli distribution with mean 0.4, then the variance of (2X–3) is

    a) 0.24         b) 0.48         c) 0.6          d) 0.96

19. The value of Var(3X – 5) is

    a) 3Var(X)      b) –5          c) 25Var(X)      d) 9Var(X)

20. The operation * defined by $a * b = \dfrac{ab}{7}$ is not a binary operation on

    a) Q+            b) Z           c) R          d) C

## Part - II

II. Answer any 7 questions. (Q.No.30 is compulsory)        7 x 2 = 14

21. If matrix $A = \begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$, then verify that $\left(A^{T}\right)^{-1} = \left(A^{-1}\right)^{T}$

22. Find the additive and multiplicative inverse of $z_1 = 1 + i$

23. If p and q are the roots of the equation $lx^2 + nx + n = 0$, show that $\sqrt{\dfrac{p}{q}} + \sqrt{\dfrac{q}{p}} + \sqrt{\dfrac{n}{l}} = 0$

24. Find the acute angle between the following lines.

    $\vec{r} = \left(4\hat{i} - \hat{j}\right) + t\left(\hat{i} + 2\hat{j} - 2\hat{k}\right), \quad \vec{r} = \left(\hat{i} - 2\hat{j} + 4\hat{k}\right) + s\left(-\hat{i} - 2\hat{j} + 2\hat{k}\right)$

25. Prove that the function $f(x) = x^2 - 2x - 3$ is strictly increasing in $(2,\infty)$

26. Use linear approximation to find an approximate value of $\sqrt{9.2}$ without using a calculator.

27. Evaluate: $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^3 \cos x\, dx$

12    Maths - 2

28. Fill in the following table so that the binary operation * on A = {a, b, c} is commutative.

| * | a | b | c |
|---|---|---|---|
| a | b |   |   |
| b | c | b | a |
| c | a |   | c |

29. The probability that a certain kind of component will survive an electrical test is $\frac{3}{4}$. Find the probability that exactly 3 of the 5 components tested survive.

30. Find the value of $\cot^{-1}(-1)$

## Part - III

**III. Answer any 7 questions. (Q.No.40 is compulsory)**                    7 x 3 = 21

31. Find the rank of the matrix by row reduction method: $\begin{bmatrix} 4 & 3 & 1 & -2 \\ -3 & -1 & -2 & 4 \\ 6 & 7 & -1 & 2 \end{bmatrix}$

32. Obtain the Cartesian equation for the locus of $z = x + iy$, if $|z + 3i| = |z - 3i|$

33. Find the value of $\sin\left[\tan^{-1}\left(\frac{1}{2}\right) - \cos^{-1}\left(\frac{4}{5}\right)\right]$

34. Obtain the equation of the circle for which (3, 4) and (2, −7) are the ends of a diameter.

35. Find the magnitude and the direction cosines of the torque about the point (2, 0, −1) of a force $2\hat{i} + \hat{j} - \hat{k}$ whose line of action passes through the origin.

36. Write the Maclaurin series expansion of $\log(1 + x)$; $-1 \le x \le 1$

37. Evaluate $\displaystyle\int_{2}^{3} \frac{\sqrt{x}}{\sqrt{5 - x} - \sqrt{x}}\,dx$

38. If $u(x, y) = x^2y + 3xy^4$, $x = e^t$ and $y = \sin t$, find $\dfrac{du}{dt}$.

39. Two balls are chosen randomly from an urn containing 8 white and 4 black balls. Suppose that we win Rs.20 for each black ball selected and we lose Rs.10 for each white ball selected. Find the expected winning amount and variance.

40. From the differential equation by eliminating the arbitary constants A and B from $y = A\cos 5x + B\sin 5x$

## Part - IV

                                                          7 x 5 = 35

**IV. Answer all the questions.**

41. a)   Investigate the value of $\lambda$ and $\mu$ the system of linear equations $2x + 3y + 5z = 9$; $7x + 3y - 5z = 8$ and $2x + 3y + \lambda z = \mu$ have i) no solution ii) a unique solution iii) an infinite number of solutions.

**(OR)**

   b)   If $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$ then prove that

   i)   $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)$

   ii)   $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha + \beta + \gamma)$

42. a) Solve the equation $(x - 5)(x - 7)(x + 6)(x + 4) = 504$

**(OR)**

b) Find the number of solution of the equation
$\tan^{-1}(x - 1) + \tan^{-1} x + \tan^{-1}(x + 1) = \tan^{-1}(3x)$

43. a) Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent.

**(OR)**

b) Two coast guard stations are located 600 km apart at points A(0, 0) and B(0, 600). A distress signal from a ship at P is received at slightly different times by two stations. It is determined that the ship is 200 km farther from station A than it is from station B. Determine the equation of hyperbola that passes through the location of the ship.

44. a) Find the non–parametric form of vector equation and Cartesian equation of the plane passing through the point (0,1, −5) and parallel to the straight lines
$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + s(2\hat{i} + 3\hat{j} + 6\hat{k})$ and $\vec{r} = (\hat{i} - 3\hat{j} + 5\hat{k}) + t(\hat{i} + \hat{j} - \hat{k})$

**(OR)**

b) If the curves $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$ intersect each other orthogonally then prove that $\dfrac{1}{a} - \dfrac{1}{b} = \dfrac{1}{c} - \dfrac{1}{d}$

45. a) A hollow cone with base radius 'a' cm and height 'b' cm is placed on a table. Show that the volume of the largest cylinder that can be hidden underneath is $\dfrac{4}{9}$ times volume of the cone.

**(OR)**

b) If $U(x, y, z) = \log(x^3 + y^3 + z^3)$, find $\dfrac{\partial U}{\partial x} + \dfrac{\partial U}{\partial y} + \dfrac{\partial U}{\partial z}$

46. a) Find the area of the region bounded by the curve $2 + x - x^2 + y = 0$, x–axis, x = −3 and x = 3.

**(OR)**

b) A tank initially contains 50 liters of pure water. Starting at time t = 0 a brine containing with 2 grams of dissolved salt per liter flows into the tank at the rate of 3 liters per minute. The mixture is kept uniform by stirring and the well–stirred mixture simultaneously flows out of the tank at the same rate. Find the amount of salt present in the tank at any time t > 0.

47. a) Find the mean and variance of the random variable X with the given probability mass function $f(x) = \begin{cases} \dfrac{1}{2} e^{\frac{-x}{2}} & \text{for } x > 0 \\ 0 & \text{elsewhere} \end{cases}$

**(OR)**

b) Let A be Q − {1}. Define * on A by x*y = x + y − xy. Is * binary on A? If so, examine the
(i) Commutative property
(ii) Associative property
(iii) Existence of Identity
(iv) Existence of inverse property for the operation * on A.