FIRST REVISION TEST - 2024

B

Standard XII

Reg.No. | | | | | | |

MATHEMATICS

Time : 3.00 hrs **Part - I** *Marks : 90*

I. Choose the correct answer: **20 x 1 = 20**

1. If A^TA^{-1} is symmetric, then A^2 =

   a) A^{-1} b) A^T c) $(A^{-1})^2$ d) $(A^T)^2$

2. In the case of Cramer's rule which of the following are correct?

   i) $\Delta = 0$ ii) $\Delta \neq 0$ iii) the system has unique solution

   iv) the system has Infinity infinitely many solutions

   a) (i) and (iv) b) (ii) and (iii) c) all d) none

3. If $\frac{z-1}{z+1}$ is purely imaginary, then $|Z|$ is

   a) $\frac{1}{2}$ b) 2 c) 1 d) 3

4. If the direction cosines of a line are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$ then

   a) $c = \pm\sqrt{3}$ b) $c = \pm 3$ c) $c > 0$ d) $0 < c < 1$

5. Integrating factor of the differential equation $\frac{dy}{dx} = \frac{x+y+1}{x+1}$ is

   a) $\frac{1}{x+1}$ b) $x + 1$ c) $\frac{1}{\sqrt{x+1}}$ d) $\sqrt{x+1}$

6. The tangent to the curve $y^2 - xy + 9 = 0$ is vertical when

   a) $y = 0$ b) $y = \pm\sqrt{3}$ c) $y = \frac{1}{2}$ d) $y = \pm 3$

7. The number of positive zeros of the polynomial $\sum_{r=0}^{n} {}^nC_r (-1)^r x^r$ is

   a) n b) 0 c) < n d) r

8. The volume of solid of revolution of the region bounded by $y^2 = x(a - x)$ about x axis

   a) πa^3 b) $\frac{\pi a^3}{4}$ c) $\frac{\pi a^3}{5}$ d) $\frac{\pi a^3}{6}$

9. Rolle's theorem is applicable only when

   a) $f'(x) \neq 0$ b) $f'(x) < 0$ c) $f'(x) > 0$ d) $f'(x) = 0$

10. Linear approximation for $g(x) = \cos x$ at $x = \frac{\pi}{2}$ is

    a) $x - \frac{\pi}{2}$ b) $x + \frac{\pi}{2}$ c) $-x + \frac{\pi}{2}$ d) $-x - \frac{\pi}{2}$

11. A binary operation on a set S is a function from

    a) $S \to S$ b) $(S \times S) \to S$ c) $S \to (S \times S)$ d) $(S \times S) \to (S \times S)$

12. If a compound statement involves 3 simple statements then the number of rows in the truth table is
a) 9 b) 8 c) 6 d) 3

13. A random variable X has binomial distribution with n = 25 and p = 0.8 then standard deviation of X is
a) 4 b) 3 c) 6 d) 2

14. The product of the roots of n^{th} roots of unity is
a) $(-1)^n$ b) 1 c) $(-1)^{n-1}$ d) 0

15. $\int_0^a \frac{f(x)}{f(x)+f(a-x)}dx$ =
a) 0 b) a c) $a/2$ d) 2a

16. For the parabola $(x-h)^2 = -4a(y-k)$, the equation of the directrix is
a) $y = k$ b) $x = k + a$ c) $y = a$ d) $y = k + a$

17. If $X \sim B(n, p)$ then
a) $\mu = np, \sigma^2 = np(1-p)$ b) $\mu = nq, \sigma = np(1-p)$
c) $\mu = np, \sigma = np(1-p)$ d) $\mu = npq, \sigma = npq$

18. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + \hat{j}$, $\vec{c} = \hat{i}$ and $(\vec{a} \times \vec{b}) \times \vec{c} = \lambda\vec{a} + \mu\vec{b}$, then the value $\lambda + \mu$ is
a) 0 b) 6 c) 1 d) 3

19. $\cos(\cos^{-1}x) = x$ if
a) $|x| < 1$ b) $|x| \le 1$ c) $|x| \ge 1$ d) $|x| = 0$

20. Identify the incorrect statement.
a) $Re(Z) \le |Z|$ b) $|Z|^2 = 1 \Rightarrow \frac{1}{Z} = \bar{Z}$
c) $||Z_1| - |Z_2|| \ge |Z_1 + Z_2|$ d) $|Z^n| = |Z|^n$

Part - II

II. Answer any 7 questions. (Q.No.30 is compulsory) **7 x 2 = 14**

21. If A is a symmetric matrix, prove that adj A is also symmetric.

22. If $\left(\frac{1-i}{1+i}\right)^{100} = a + ib$, find the values of a and b.

23. If $x^2 + 2(k+2)x + 9k = 0$ has equal roots, Find k.

24. Find the principal solution of $\sqrt{3}\sec x = -2$

25. Evaluate : $\int_0^1 x^3(1-x^4)dx$

26. Show that $y = mx + \frac{7}{m}$, $m \neq 0$ is a solution of the differential equation $xy' + 7\frac{1}{y'} - y = 0$

27. The probability function of X is given by $f(x) = \begin{cases} kxe^{-2x} & \text{for } x > 0 \\ 0 & \text{for } x \le 0 \end{cases}$
Find the value of k.

28. Let * be defined on R by (a * b) = a + b + ab – 7. Is * binary on R ? If so, find $3 * \left(\frac{-7}{15}\right)$

29. Show that the percentage error in the n^{th} root of a number is approximately $\frac{1}{n}$ times the percentage error in the number.

30. Evaluate : $[2\hat{i}\ \hat{j}\ \hat{k}]+[\hat{i}\ \hat{k}\ \hat{j}]+[\hat{k}\ \hat{j}\ 2\hat{i}]$

Part - III

III. Answer any 7 questions. (Q.No.40 is compulsory) **7 x 3 = 21**

31. Solve the following system of linear equations by matrix inversion method :
$5x + 2y = 3,\ 3x + 2y = 5$

32. Find the quotient $\frac{\left(\cos\frac{9\pi}{4}+i\sin\frac{9\pi}{4}\right)}{\cos\left(-\frac{3\pi}{2}\right)+i\sin\left(-\frac{3\pi}{2}\right)}$ in rectangular form.

33. Find all real numbers satisfying $4^x - 3(2^{x+2}) + 2^5 = 0$

34. Prove that $\frac{\pi}{2} \le \sin^{-1} x + 2\cos^{-1} x \le \frac{3\pi}{4}$

35. Find the equation of the hyperbola whose foci are (±2, 0) and eccentricity = $\frac{3}{2}$

36. Find the angle between the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane $10x + 2y - 11z = 3$

37. Evaluate : $\lim_{x\to 1^-}\left(\frac{\log(1-x)}{\cot(\pi x)}\right)$

38. If $X \sim B(n, p)$ such that $4P(X = 4) = P(X = 2)$ and $n = 6$. Find the distribution.

39. Using truth table prove that $q \to p \equiv \neg p \to \neg q$

40. Find the area of the region bounded by the line $5x + 3y = 15$, x-axis and the lines $x = -1$ and $x = 2$.

Part - IV

IV. Answer all the questions. **7 x 5 = 35**

41. a) Investigate for what values of λ and μ the system of linear equations
$x + 2y + z = 7,\ x + y + \lambda z = \mu,\ x + 3y - 5z = 5$ has
i) no solution ii) a unique solution
iii) an infinite number of solutions

(OR)

b) Prove that : $\tan\left[\frac{1}{2}\sin^{-1}\left(\frac{2a}{1+a^2}\right)+\frac{1}{2}\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)\right] = \frac{2a}{1-a^2}$

42. a) Solve the equation $(2x - 3)(6x - 1)(3x - 2)(x - 2) - 5 = 0$

(OR)

b) Find the area of the region in the first quadrant enclosed by x-axis. Line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$

43. a) Find the equation of the circle passes through the points (0, 0), (–2, 1) and (–3, 2)

(OR)

b) Find the probability mass function f(x) of the discrete random variable X whose cumulative distribution function F(X) is given by

$$F(x) = \begin{cases} 0 & , -\infty < x < -2 \\ 0.25 & , -2 \le x < -1 \\ 0.60 & , -1 \le x < 0 \\ 0.90 & , 0 \le x < 1 \\ 1 & , 1 \le x < \infty \end{cases}$$

Also find (i) $P(X < 0)$ and (ii) $P(X \ge -1)$

44. a) Find the foot of the perpendicular drawn from the point (5,4,2) to the line $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1}$. Also find the equation of the perpendicular.

(OR)

b) If $z = x + iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, show that $x^2 + y^2 + 3x - 3y + 2 = 0$

45. a) Using vector method, prove that $\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$

(OR)

b) If $f(x, y) = \cos(x^2 - 3xy)$, find f_x, f_y and show that $f_{xy} = f_{yx}$

46. a) A tank contains 1000 litres of water in which 100 grams of salt is dissolved. Brine (Brine is a high-concentration solution of salt [usually sodium chloride] in water) runs in a rate of 10 litres per minute and each litre contains 5 grams of dissolved salt. The mixture of the tank is kept uniform by stirring. Brine runs out at 10 litres per minute. Find the amount of salt at any time t.

(OR)

b) Verify

(i) Closure property (ii) Commutative property

(iii) Associative property (iv) Existence of identity and

(v) . Existence of inverse for the operation X_{11} on a subset A = {1,3,4,5,9} of the set of remainders {0,1,2,3,4,5,6,7,8,9,10}

47. a) A metal box with a square base and vertical sides is to contain 1024 cm^3 of water. The material for the top and bottom costs ₹5 per cm^2 and the material for the sides costs ₹2.50 per cm^2. Find the least cost of the box.

(OR)

b) Solve : $\left(1 + y^2\right) + \left(x - e^{\tan^{-1} y}\right)\frac{dy}{dx} = 0$

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