V12M

Virudhunagar District
Common First Revision Test - January 2024

Standard 12
MATHEMATICS
PART - I

Time Allowed: 3.00 Hours | Maximum Marks: 90

Choose the correct answer: **20×1=20**

1) If A is a non singular matrix such that $A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$, then $(A^T)^{-1}$ =

a) $\begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ c) $\begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$ d) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$

2) If $z = x+iy$ is a complex number such that $|z+2| = |z-2|$, then the locus of z is

a) real axis b) imaginary axis c) ellipse d) circle

3) The number of real numbers in $[0, 2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is

a) 2 b) 4 c) 1 d) ∞

4) The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is

a) [1, 2] b) [−1, 1] c) [0, 1] d) [−1, 0]

5) The equation of the normal to the circle $x^2+y^2-2x-2y+1 = 0$ which is parallel to the line $2x+4y = 3$ is

a) $x+2y = 3$ b) $x+2y+3 = 0$ c) $2x+4y+3 = 0$ d) $x-2y+3 = 0$

6) The angle between the line $\vec{r} = \left(\hat{i} + 2\hat{j} - 3\hat{k}\right) + t\left(2\hat{i} + \hat{j} - 2\hat{k}\right)$ and the plane $\vec{r}.(i + j) + 4 = 0$ is

a) 0° b) 30° c) 45° d) 90°

7) The curve $y = ax^4+bx^2$, with $ab > 0$

a) has no horizontal tangent b) is concave up
c) is concave down d) has no points of inflection

8) If $v(x, y) = \log(e^x+e^y)$, then $\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y}$ is equal to

a) e^x+e^y b) $\frac{1}{e^x + e^y}$ c) 1 d) 2

9) The volume of solid of revolution of the region bounded by $y^2 = x(a-x)$ about x-axis is

a) πa^3 b) $\frac{\pi a^3}{4}$ c) $\frac{\pi a^3}{5}$ d) $\frac{\pi a^3}{6}$

10) Integrating factor of the differential equation $\frac{dy}{dx} = \frac{x + y + 1}{x + 1}$ is

a) $\frac{1}{x+1}$ b) $x+1$ c) $\frac{1}{\sqrt{x+1}}$ d) $\sqrt{x+1}$

11) If $P(X = 0) = 1-P(X = 1)$. If $E(x) = 3\text{var}(X)$ then $P(X = 0)$

a) $\frac{2}{3}$ b) $\frac{2}{5}$ c) $\frac{1}{5}$ d) $\frac{1}{3}$

12) The operation * defined by $a * b = ab/7$ is not a binary operation on

a) Q^+ b) Z c) R d) C

13) If $x = \cos\theta + i\sin\theta$, then the value of $x^n + \frac{1}{x^n}$

a) $2\cos\theta$ b) $2i\sin n\theta$ c) $2\cos n\theta$ d) $2i\cos n\theta$

14) The number of solutions of the equation $\tan^{-1}2x + \tan^{-1}3x = \pi/4$ is

a) 2 b) 3 c) 1 d) none

15) The point of contact of $y^2 = 4ax$ and the tangent $y = mx+c$ is

a) $\left(\frac{2a}{m^2}, \frac{a}{m}\right)$ b) $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$ c) $\left(\frac{a}{m}, \frac{2a}{m^2}\right)$ d) $\left(\frac{-a}{m^2}, \frac{-2a}{m}\right)$

16) If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar, non-zero vectors such that $\left[\vec{a}, \vec{b}, \vec{c}\right] = 3$, then $\left\{\left[\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}\right]\right\}^2$

a) 81 b) 9 c) 27 d) 18

17) The value of the limit $\lim_{x\to 0} \cot x - \frac{1}{x}$ is

a) 0 b) 1 c) 2 d) ∞

18) The value of $\int_0^a \left(\sqrt{a^2 - x^2}\right)^3 dx$ is

a) $\frac{\pi a^3}{16}$ b) $\frac{3\pi a^4}{16}$ d) $\frac{3\pi a^2}{8}$ d) $\frac{3\pi a^4}{8}$

19) If $f(x) = Cx^2$, $0 < x < 2$ is the probability density function of x, then C is

a) $1/3$ b) $4/3$ c) $8/3$ d) $3/8$

20) The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is

a) $xy = K$ b) $y = K\log x$ c) $y = Kx$ d) $\log y = Kx$

PART - II

Answer any seven questions. Question No. 30 is compulsory. **7×2=14**

21) If $A = \begin{bmatrix} -2 & 4 \\ 1 & -3 \end{bmatrix}$ then find A^{-1}.

22) If $Z_1 = 3-2i$ and $Z_2 = 6+4i$, find $\frac{Z_1}{Z_2}$ in the rectangular form.

23) Find the principal value of $\cot^{-1}\left(\sqrt{3}\right)$

24) Find the length of latus rectum of the parabola $y^2 = 4ax$.

25) Find the volume of the parallelopiped whose coterminous edges are represented by the vectors $-6\hat{i} + 14\hat{j} + 10\hat{k}$, $14\hat{i} - 10\hat{j} - 6\hat{k}$ and $2\hat{i} + 4\hat{j} - 2\hat{k}$.

26) Evaluate: $\lim_{x \to 0} \frac{\sin mx}{x}$

27) Evaluate: $\int_0^\infty x^3 e^{-2x}\, dx$

28) Find the order and degree of the differential equation $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + \int y\, dx - x^3$

29) Suppose two coins are tossed once. if x denote number of tails (i) write down the sample space (ii) find the inverse image of 1.

30) Verify associative property of the following operation * defined by $a*b = a^b$, $\forall a, b \in N$

PART - III

Answer any seven questions. Question No. 40 is compulsory. 7×3=21

31) If $|z| = 3$, show that $7 \le |z+6-8i| \le 13$

32) Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.

33) The equation $y = \frac{1}{32}x^2$ models cross sections of parabolic mirrors that are used for solar energy. There is a heating tube located at the focus of each parabola, how high is this tube located above the vertex of the parabola?

34) Find the coordinates of the point where the straight line $\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + t(3\hat{i} + 4\hat{j} + 2\hat{k})$ intersects the plane $x-y+z-5 = 0$

35) Let $u(x, y) = e^{-2y}\cos(2x)\ \forall (x, y) \in R^2$, prove that u is a harmonic function in R^2.

36) Find the area of the region bounded by the line $6x+5y = 30$, x-axis and the lines $x = -1$ and $x = 3$.

37) Solve: $\frac{dy}{dx} + 2y = e^{-x}$

38) A random variable x has the probability mass function

x	1	2	3	4	5
f(x)	K^2	$2K^2$	$3K^2$	2K	3K

(i) Find the value of K

(ii) $P(2 \le X \le 5)$

39) Prove that $p\wedge(q, \wedge r) \equiv (p\wedge q)\wedge r$ using truth table.

40) Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$

PART - IV

Answer all the questions: 7×5=35

41) a] Solve $3x+3y-z = 11$, $2x-y+2z = 9$, $4x+3y+2z = 25$

(OR)

b] Evaluate: $\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{1+\sqrt{\tan x}}\, dx$

42) a] If $2\cos\alpha = x+\frac{1}{x}$ and $2\cos\beta = y+\frac{1}{y}$ show that

(i) $\frac{x}{y}+\frac{y}{x} = 2\cos(\alpha-\beta)$ (ii) $\frac{x^m}{y^n}-\frac{y^n}{x^m} = 2i\sin(m\alpha-n\beta)$

(OR)

b] Solve: $(x^2-3y^2)dx + 2xy\,dy = 0$

43) a] Solve: $6x^4-35x^3+62x^2-35x+6 = 0$

(OR)

b] Find the mean and variance of the random variable x whose probability density function is $f(x) = \begin{cases}\lambda e^{-\lambda x}, & x \geq 0\\ 0, & \text{otherwise}\end{cases}$

44) a] i) Find the value of $\tan^{-1}\left(\sqrt{3}\right)-\sec^{-1}(-2)$

ii) Prove that $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3} = \frac{\pi}{4}$

(OR)

b] Find the centre, foci and vertices of the hyperbola $9x^2-y^2-36x-6y+18 = 0$

45) a] Prove by vector method that the perpendiculars from the vertices to the opposite sides of a triangle are concurrent.

(OR)

b] Sketch the curve $y = f(x) = x^3-6x-9$

46) a] $f(x,y) = \tan^{-1}\left(\frac{x}{y}\right)$, find f_x, f_y and show that $f_{xy} = f_{yx}$

(OR)

b] Verify (i) closure property (ii) commutative property (iii) associative property (iv) existence of identity and (v) existence of inverse for the operation XII on a subset A = {1, 3, 4, 5, 9} of the set of remainders {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

47) a] Find the parametric form of vector equation and Cartesian equation of the plane passing through the points (2, 2, 1) (1, −2, 3) and parallel to the straight line passing through the pants (2, 1, −3) and (−1, 5, −8)

(OR)

b] Using integration find the area of the region bounded by triangle ABC whose vertices A, B and C are (−1, 1) (3, 2) and (0, 5) respectively

- - - - -