KK12M

Kanyakumari District

Common Half Yearly Examination - December 2023

Standard 12

MATHEMATICS

Time Allowed: 3.00 Hours Maximum Marks: 90

PART - I

Note: i) Answer all the questions. **20×1=20**

ii) Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer.

1) If α, β and γ are the zeros of x^3+px^2+qx+r then $\sum \frac{1}{\alpha}$ is

a) $-q/r$ b) $-p/r$ c) q/r d) $-q/p$

2) If $\omega \neq 1$ is a cubic root of unity and $(1+\omega)^7 = A+B\omega$, then (A, B) equals

a) (1, 0) b) (−1, 1) c) (0, 1) d) (1, 1)

3) Find centre and radius of the circle $|z+2-i| < 2$

a) −2+i, 2 b) −2−i, −2 c) 2−1, 2 d) 2+i, 2

4) If $A = \begin{bmatrix} 1 & \tan\theta/2 \\ -\tan\theta/2 & 1 \end{bmatrix}$ and $AB = I_2$, then B =

a) $\left(\cos^2\theta/2\right)A$ b) $\left(\cos^2\theta/2\right)A^T$ c) $\left(\cos^2\theta\right)I$ d) $\left(\sin^2\theta/2\right)A$

5) Let $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and $4B = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{bmatrix}$. If B is the inverse of A, then the value of x is

a) 2 b) 4 c) 3 c) 1

6) If $\bar{a}, \bar{b}, \bar{c}$ are non-coplanar, non-zero vectors such that $\left[\bar{a}, \bar{b}, \bar{c}\right] = 3$ then $\left\{\left[\bar{a}\times\bar{b}, \bar{b}\times\bar{c}, \bar{c}\times\bar{a}\right]\right\}^2$ is equal to

a) 81 b) 9 c) 27 18

7) Distance from the origin to the plane $3x-6y+2z+7 = 0$ is

a) 0 b) 1 c) 2 d) 3

8) If the length of the perpendicular from the origin to the plane $2x+3y+\lambda z = 1, \lambda > 0$ is $1/5$, then the value of λ is

a) $2\sqrt{3}$ b) $3\sqrt{2}$ c) 0 d) 1

9) The area of quadrilateral formed with foci of the hyperbolas $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$ is

a) $4(a^2+b^2)$ b) $2(a^2+b^2)$ c) a^2+b^2 d) $1/2\,(a^2+b^2)$

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10) The eccentricity of the ellipse $(x-3)^2+(y-4)^2=\frac{y^2}{9}$ is

a) $\frac{\sqrt{3}}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{3\sqrt{2}}$ d) $\frac{1}{\sqrt{3}}$

11) If $\sin^{-1}x+\sin^{-1}y=\frac{2\pi}{3}$ then $\cos^{-1}x+\cos^{-1}y$ is equal to

a) $\frac{2\pi}{3}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{6}$ d) π

12) If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\frac{3\pi}{2}$,

the value of $x^{2017}+y^{2018}+z^{2019}-\frac{9}{x^{101}+y^{101}+z^{101}}$ is

a) 0 b) 1 c) 2 d) 3

13) If sin x is the integrating factor of the linear differential equation $\frac{dy}{dx}+Py=Q$ then P is

a) log sin x b) cos x c) tan x d) cot x

14) Find order and degree of differential equation $x=e^{xy\frac{dy}{dx}}$

a) 1, 2 b) 2, 1 c) 1, 1 d) 2, 2

15) The value of $\int_0^1 x(1-x)^{99}\,dx$ is

a) $\frac{1}{11000}$ b) $\frac{1}{10100}$ c) $\frac{1}{10010}$ d) $\frac{1}{10001}$

16) Find the area of the region bounded by ellipse $\frac{x^2+y^2}{a^2+b^2}=1$ is

a) πa^2 b) πb^2 c) πab d) πa

17) If $f(x,y,z)=xy+yz+zx$, then f_x-f_z is equal to

a) z−x b) y−z c) x−z d) y−x

18) If $u(x,y)=\frac{x+y}{\sqrt{x}+\sqrt{y}}$ then the degree of 'u' is

a) 1 b) 2 c) $\frac{1}{2}$ d) $\frac{1}{4}$

19) The slope of the line normal to the curve $f(x)=2\cos 4x$ at $x=\frac{\pi}{12}$ is

a) $-4\sqrt{3}$ b) −4 c) $\frac{\sqrt{3}}{12}$ d) $4\sqrt{3}$

20) The maximum value of the function $x^2e^{-2x}, x>0$ is

a) $\frac{1}{e}$ b) $\frac{1}{2e}$ c) $\frac{1}{e^2}$ d) $\frac{4}{e^2}$

PART - II

Answer any seven questions. Question No. 30 is compulsory. **7×2=14**

21) Find the angle between the planes $\vec{r}.(\hat{i}+\hat{j}+\hat{k}) = 5$ and $x-y+z = 4$

22) Show that the percentage error in the n^{th} root of a number is approximately $\frac{1}{n}$ times the percentage error in the number.

23) Evaluate: $\int_0^{\pi/2} \begin{vmatrix} \cos^4 x & 7 \\ \sin^3 x & 3 \end{vmatrix} dx$

24) Show that $y = ax + \frac{b}{x}$, $x \neq 0$ is a solution of the differential equation $x^2y'' + xy' - y = 0$

25) If adj $A = \begin{vmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix}$ find A^{-1}.

26) Show that $(2+i\sqrt{3})^{10} - (2-i\sqrt{3})^{10}$ is purely imaginary.

27) Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5} - \sqrt{3}$ as a root.

28) Find the value of $\sin^{-1}(10)$.

29) Find the equation of the parabola whose vertex is $(1, -2)$ and focus is $(4, -2)$

30) Find the critical point of the function $f(x) = |x-17|$

PART - III

Answer any seven questions. Question No. 40 is compulsory. **7×3=21**

31) Obtain the Cartesian form of the locus of z if $|2z-3-i| = 3$.

32) Find the value of $\sec^{-1}\left(-\frac{2\sqrt{3}}{3}\right)$

33) Find the torque of the resultant of three forces represented by $-3\hat{i}+6\hat{j}-3\hat{k}$, $4\hat{i}-10\hat{j}+12\hat{k}$ and $4\hat{i}+7\hat{j}$ acting the point with position vector $8\hat{i}-6\hat{j}-4\hat{k}$ about the point with position vector $18\hat{i}+3\hat{j}-9\hat{k}$

34) Prove that the point of intersection of the tangents at t_1 and t_2 on the parabola $y^2 = 4ax$ is $[at_1t_2, a(t_1+t_2)]$

35) Evaluate: $\int_{-\log 2}^{\log 2} e^{|x|} dx$

36) Suppose f(x) is a differentiable funciton for all x with $f'(x) \leq 29$ and $f(2) = 17$ What is the maximum value of f(7)?

37) Solve $\frac{dy}{dx} = e^{x+y} + x^3e^y$

38) Solve $\sin^2 x - 5\sin x + 4 = 0$

39) Solve by Cramer's rule: $\frac{3}{x} + 2y = 12$, $\frac{2}{x} + 3y = 13$

40) Find the area of the circle of radius r.

PART - IV

Answer all the questions: **7×5=35**

41) a] Solve $(x-5)(x-7)(x+6)(x+4) = 504$

(OR)

b] Find the number of solutions of the equations.
$\tan^{-1}(x-1) + \tan^{-1}x + \tan^{-1}(x+1) = \tan^{-1}(3x)$

42) a] Prove by vector method that the perpendiculars from the vertices to the opposit sides of a triangle are concurrent.

(OR)

b] A semi elliptical archway over a one-way road has a height of 3 m and a width of 12 m. The truck has a width of 3 m and a height of 2.7 m. Will the truck clear the opening of the archway?

43) a] A pot of boiling water at 100°C is removed from a store at time t = 0 and left to cool in the kitchen. After 5 minutes, the water temperature has decreased to 80°C and another 5 minutes later it has dropped to 65°C. Determine the temperature of the kitchen.

(OR)

b] A watermelon has an ellipsoid shape which can be obtained by revolving an ellipse with major axis 20 cm and minor axis 10 cm about its major-axis. Find its volume using integration.

44) a] Derive the intercept form of the equation of a plane using vector method.

(OR)

b] Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is 10 meter high?

45) a] If $u = \tan^{-1}\left(\frac{x^3+y^3}{x+y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$

(OR)

b] Solve $z^4 = 1 - \sqrt{3}\,i$.

46) a] If ax^2+bx+c is divided by $x+3$, $x-5$ and $x-1$ the remainders are 21, 61 and 9 respectively. Find a, b and c. (Use Gaussian Elimination method)

(OR)

b] Find the equation of the circular passing through the points.
(1, 1) (2, −1) and (3, 2)

47) a] Prove that $\int_0^{\frac{\pi}{4}} \log(1+\tan x)\,dx = \frac{\pi}{8}\log 2$

(OR)

b] A rectangular page is to contain 24 cm^2 of print. the margins at the top and bottom of the page are 1.5 cm and the margins at other sides of the page is 1 cm. What should be the dimensions of the page so that the area of the paper used is minimum.

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