PART - I

Marks: 90

Choose the correct Answer **20x1=20**

1) The principal argument of $\frac{-2}{1+i\sqrt{3}}$ is

a) $\pi/3$ b) $2\pi/3$ c) $-2\pi/3$ d) $-\pi/2$

2) If A^TA^{-1} is symmetric then A^2 =

a) A^{-1} b) $(A^T)^2$ c) A^T d) $(A^{-1})^2$

3) If x+y = K is a normal to the parabola $y^2 = 12x$ then the value of K is

a) 3 b) -1 c) 1 d) 9

4) According to the rational root theorem which number is not possible rational zero of $4x^7+2x^4-10x^3-5$?

a) -1 b) $5/4$ c) $4/5$ d) 5

5) The angle between the line $\vec{r} = (\hat{i} + 2\hat{j} - 3\hat{k}) + t(2\hat{i} + \hat{j} - 2\hat{k})$ and the plane $\vec{r}.(\hat{i} + \hat{j}) + 4 = 0$ is

a) 0° b) 30° c) 45° d) 90°

6) If $\sin^{-1} x/5 + \operatorname{cosec}^{-1} 5/4 = \pi/2$ then the value of x is

a) 4 b) 5 c) 2 d) 3

7) The eccentricity of the ellipse $(x-3)^2 + (y-4)^2 = y^2/9$ is

a) $\sqrt{3}/2$ b) $1/3$ c) $1/3\sqrt{2}$ d) $1/\sqrt{3}$

8) The product of all four values of $\left(\cos\pi/3 + i\sin\pi/3\right)^{3/4}$ is

a) -2 b) -1 c) 1 d) 2

9) If $\rho(A) = \rho(A/B)$ then the system Ax = B of linear equations is

a) consistent and has unique solution b) consistent
c) consistent and has infinitely many solutions d) inconsistent

10) $2\hat{i} - \hat{j} + 3\hat{k}$, $3\hat{i} + 2\hat{j} + \hat{k}$, $\hat{i} + m\hat{j} + 4\hat{k}$ are coplanar then m is

a) 3 b) 0 c) -3 d) 1

11) Substraction is not a binary operation in

a) R b) Z c) N d) Q

12) If the function $f(x) = 1/12$ for a < x < b represents a probability density function of a continuous random variable x then which of the following cannot be the value of a & b

a) 0 & b b) 5 & 17 c) 7 & 19 d) 16 & 24

13) The volume when the curve $y = \sqrt{3 + x^2}$ from x = 0 to x = 4 is rotated about x axis is

a) 100π b) $\frac{100\pi}{9}$ c) $\frac{100\pi}{3}$ d) $\frac{100}{3}$

14) P is the amount of certain substance left in after time t. If the rate of evaporation of the substance is proportional to the amount remaining then

a) $P = Ce^{kt}$ b) $P = Ce^{kt}$ c) $P = cKt$ d) $Pt = c$

15) If $f(x, y) = e^{xy}$ then $\frac{\partial^2 f}{\partial x \partial y}$ is

a) xye^{xy} b) $(1+xy)e^{xy}$ c) $(1+y)e^{xy}$ d) $(1+x)e^{xy}$

16) Point of inflection of the curve $y = x^4$ is at

a) x = 0 b) x = 1 c) x = 12 d) nowhere

17) The value of $\int_{-1}^{2} |x| dx$

a) $\frac{1}{2}$ b) $\frac{3}{2}$ c) $\frac{5}{2}$ d) $\frac{7}{2}$

18) The integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{\lambda}$ is

a) $\frac{x}{e^{\lambda}}$ b) $\frac{e^{\lambda}}{x}$ c) λe^{x} d) e^{x}

19) The mean of a binomial distribution is 5 and its standard deviation is 2 then the value of n and p are

a) $\left(\frac{4}{5}, 25\right)$ b) $\left(25, \frac{4}{5}\right)$ c) $\left(\frac{1}{5}, 25\right)$ d) $\left(25, \frac{1}{5}\right)$

20) The maximum value of the function x^2e^{-2x}, x > 0 is

a) $\frac{1}{e}$ b) $\frac{1}{2e}$ c) $\frac{1}{e^2}$ d) $\frac{4}{e^4}$

PART - II

7x2=14

Answer any 7 questions. Qn.no. 30 is compulsory.

21) Find the inverse of $A = \begin{pmatrix} 2 & -1 \\ 5 & -2 \end{pmatrix}$ by Gauss-Jordan method.

22) Show that $|3z-5+i| = 4$ represents a circle and find its centre and radius.

23) Find the domain of $\tan^{-1}\sqrt{9 - x^2}$

24) Examine the position of the point (2, 3) with respect to the circle $x^2+y^2-6x-8y+12 = 0$

25) Find the length of the perpendicular from the point (1, -2, 3) to the plane x-y+z = 5

26) Solve: $x^3-3x^2-33x+35 = 0$.

27) Evaluate: $\int_0^1 x^2 (1-x)^3 dx$

28) The mean and variance of a binomial variate x are respectively 2 and 1.5 find P(X = 0)

29) $A = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ be any two boolean matrices of the same type find $A \vee B$ and $A \wedge B$

30) Find the asymptotes of the curve $f(x) = \frac{x^2}{x+1}$

PART - III **7x3=21**

Answer any 7 questions. Qn. no. 40 is compulsory.

31) Solve $2x+2y+z = 5$, $x-y+z = 1$, $3x+y+2z = 4$ by rank method.

32) If $\omega \neq 1$ is a cube root of unity. Show that $(1-\omega+\omega^2)^6+(1+\omega-\omega^2)^6 = 128$.

33) Prove that an angle in a semi circle is a right angle.

34) The orbit of Halley's comet is an ellipse 36.18 astronomical units long and by 9.12 astronomical units wide. Find its eccentricity.

35) Evaluate: $\lim_{x \to \pi/2} \frac{\sec x}{\tan x}$

36) If $v(x, y, z) = \log(x^3+y^3+z^3)$ find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$

37) Evaluate: $\int_{-\pi/2}^{\pi/2} x \cos x \, dx$

38) A six sided die is marked 2 on one face, 3 on two of its faces and 4 on remaining three faces. The die is thrown twice. If x denotes the total score in two throws, find the values of the random variable and number of points in its inverse images.

39) Construct the truth table $(p \vee q) \wedge \neg q$

40) Form the differential equation by eliminating the arbitrary constants A and B from $y = A \cos x + B \sin x$

PART - IV **7x5=35**

Answer all the questions:

41) Investigate for what values of λ and μ the system of linear equations $x+2y+z = 7$, $x+y+\lambda z = \mu$, $x+3y-5z = 5$ has (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

(OR)

A garden is to be laidout in a rectangular area and protected by wire fence. What is the largest possible area of the fenced garden with 40 meters of wire.

42) If $z = x+iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$. Show that $x^2+y^2+3x-3y+2 = 0$

(OR)

Find the number of solution of the equation

$\tan^{-1}(x^{-1})+\tan^{-1}(x)+\tan^{-1}(x+1) = \tan^{-1}(3x)$

43) Find the sum of squares of roots of the equation $2x^4-8x^3+6x^2-3 = 0$

(OR)

$z(x, y) = x \tan^{-1}(x y)$, $x = t^2$, $y = se^t$, s, t, $\in$ R find $\frac{\partial z}{\partial s}$ at $s = t = 1$

44) Find the area of the region bounded by $y = \cos x$, $y = \sin x$, the lines $x = \pi/4$ and $x = 5\pi/4$

(OR)

Find the equations of tangents to the hyperbola $\frac{x^2}{16} - \frac{y^2}{64} = 1$ which are parallel to $10x-3y+9 = 0$.

45) Find the parametric form of vector equation and Cartesian equations of the plane passing through the points (2, 2, 1) (9, 3, 6) and perpendicular to the plane $2x+6y+6z = 9$

(OR)

The probability density function of the random variable x is given by

$f(x) = \begin{cases} 16xe^{-4x} & \text{for } x > 0 \\ 0 & \text{for } x \le 0 \end{cases}$ find the mean and variance of X.

46) Solve $\frac{dy}{dx} + 2y \cot x = 3x^2 \operatorname{cosec}^2 x$

(OR)

A bridge has a parabolic arch that is 10 m high in the centre and 30 m wide at the bottom. Find the height of the arch 6 m from the centre on either sides.

47) Suppose a person deposits Rs.10,000 in a bank account at the rate of 5% per annum compounded continuously. How much money will be in his bank account 18 months later?

(OR)

$M = \left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in R - \{0\} \right\}$ and let * be the matrix multiplication. Determine whether m is closed under *. If so examine the commutative and associative properties, existence of identity, existence of inverse properties for the operation * on M

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