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TVL12M

Tirunelveli District

Common Half Yearly Examination - 2023

Standard 12

MATHEMATICS

Time Allowed: 3.00 Hours Maximum Marks: 90

PART - A

Choose the correct answer: **20×1=20**

1) If $A\begin{bmatrix}1 & -2\\ 1 & 4\end{bmatrix}=\begin{bmatrix}6 & 0\\ 0 & 6\end{bmatrix}$, then A =

a) $\begin{bmatrix}1 & -2\\ 1 & 4\end{bmatrix}$ b) $\begin{bmatrix}4 & 2\\ -1 & 1\end{bmatrix}$ c) $\begin{bmatrix}4 & -1\\ 2 & 1\end{bmatrix}$ d) $\begin{bmatrix}1 & 2\\ -1 & 4\end{bmatrix}$

2) The solution of the equation $|z|-z = 1+2i$ is

a) $\frac{3}{2}+2i$ b) $\frac{3}{2}-2i$ c) $2-\frac{3}{2}i$ d) $2+\frac{3}{2}i$

3) The principal argument of $\frac{3}{-1+i}$ is

a) $\frac{-5\pi}{6}$ b) $\frac{-3\pi}{4}$ c) $\frac{-2\pi}{3}$ d) $\frac{-\pi}{2}$

4) The polynomial x^3-Kx^2+9x has three real zeros if and only if, K satisfies

a) $|K| \le 6$ b) $K = 0$ c) $|K| \ge 6$ d) $|K| > 6$

5) If $\sin^{-1}x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$, then x is equal to

a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{5}}$ c) $\frac{2}{\sqrt{5}}$ d) $\frac{\sqrt{3}}{2}$

6) The eccentricity of the ellipse $(x-3)^2+(y-4)^2 = \frac{y^2}{9}$ is

a) $\frac{\sqrt{3}}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{3\sqrt{2}}$ d) $\frac{1}{\sqrt{3}}$

7) The angle between the lines $\frac{x-2}{3}=\frac{y+1}{-2}, z=2$ and $\frac{x-1}{1}=\frac{2y+3}{3}=\frac{z+5}{2}$ is

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

8) If the length of the perpendicular from the origin to the plane $2x+3y+\lambda z = 1$, $\lambda > 0$ is $\frac{1}{5}$, then the value of λ is

a) $2\sqrt{3}$ b) $3\sqrt{2}$ c) 0 d) 1

9) The number given by the mean value theorem for the function $\frac{1}{x}$, $x \in [1, 9]$ is

a) 2 b) 2.5 c) 3.5 d) 3

10) A stone is thrownup vertically. The height it reaches at time t seconds is given by $x = 80t-16t^2$. The stone reaches the maximum height in time t seconds is given by

a) 2 b) 2.5 c) 3 d) 3.5

11) Linear approximation for $g(x) = \cos x$ at $x = \frac{\pi}{2}$ is

a) $x+\frac{\pi}{2}$ b) $-x+\frac{\pi}{2}$ c) $x-\frac{\pi}{2}$ d) $-x-\frac{\pi}{2}$

12) The area between $y^2 = 4x$ and its latus rectum is

a) $\frac{2}{3}$ b) $\frac{4}{3}$ c) $\frac{8}{3}$ d) $\frac{5}{3}$

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13) The solution of the differential equation $\frac{dy}{dx} + \frac{1}{\sqrt{1-x^2}} = 0$ is

a) $y+\sin^{-1}x = c$ b) $x+\sin^{-1}y = 0$ c) $y^2+2\sin^{-1}x = c$ d) $x^2+2\sin^{-1}y=0$

14) If cos x is the integrating factor of the linear differential equation $\frac{dy}{dx} + py = Q$, then P is

a) log sin x b) $-\tan x$ c) tan x d) $-\cot x$

15) Let X have a binomial distribution with mean 0.4, then the variance of (2X–3) is

a) 0.24 b) 0.48 c) 0.6 d) 0.96

16) If $f(x) = \begin{cases} 2x & 0 \le x \le a \\ 0 & \text{otherwise} \end{cases}$ is a probability density function of a random variable, then the value of 'a' is

a) 1 b) 2 c) 3 d) 4

17) Subtraction is not a binary operation is

a) R b) Z c) N d) Q

18) Which one is the inverse of the statement $(p \vee q) \rightarrow (p \wedge q)$?

a) $(p \wedge q) \rightarrow (p \vee q)$ b) $\neg(p \vee q) \rightarrow (p \wedge q)$

c) $(\neg p \vee \neg q) \rightarrow (\neg p \wedge \neg q)$ d) $(\neg p \wedge \neg q) \rightarrow (\neg p \vee \neg q)$

19) If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ then |adj A| =

a) 36 b) 6 c) 216 d) 16

20) The value of $\int_0^1 x^2 (1-x)^9 \, dx$ is

a) $1/610$ b) $1/630$ c) $1/640$ d) $1/660$

PART - B

Answer seven questions. Question No. 30 is compulsory. **7×2=14**

21) If $A = \begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$, find $(A^{-1})^T$.

22) If $z_1 = 3-2i$ and $z_2 = 6+4i$, find z_1/z_2 in the rectangular form.

23) Find the value of $\tan^{-1}\left[\tan\left(3\pi/4\right)\right]$

24) Find the equation of the hyperbola with vertices (0, ±4) and foci (0, ±6)

25) If the straightlines $\frac{x-5}{5m+2} = \frac{2-y}{5} = \frac{1-z}{-1}$ and $x = \frac{2y+1}{4m} = \frac{1-z}{-3}$ are perpendicular to each other, find the value of m.

26) Use linear approximation to find an approximate value of $\sqrt{9.2}$

27) Evaluate: $\int_0^{\pi/4} \sin^6 2x \, dx$

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28) The probability density function of x is given by $f(x) = \begin{cases} Kxe^{-2x} & \text{for } x > 0 \\ 0 & \text{for } x \le 0 \end{cases}$, find the value of K.

29) On z, define * by $m*n = m^n+n^m$, $\forall$ m, n$\in$z. Is * binary on z?

30) Solve: $\frac{dy}{dx} = \frac{x+y}{x}$

PART - C

Answer any seven questions. Question No. 40 is compulsory. **7×3=21**

31) $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, show that $A^T.A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$

32) Find the sum of squares of roots of the equation $2x^4-8x^3+6x^2-3 = 0$

33) If the normal at the point 't_1' on the parabola $y^2 = 4ax$ meets the parabola again at the point 't_2', then prove that $t_2 = -\left[t_1 + \frac{2}{t_1}\right]$

34) Verify whether the line $\frac{x-3}{-4} = \frac{y-4}{-7} = \frac{z+3}{12}$ lies in the plane $5x-y+z = 8$

35) Find the intervals of concavity of the curve $f(x) = (x-1)^3 (x-5)$, x$\in$R

36) If $w(x, y, z) = x^2+y^2+z^2$, $x = e^t$, $y = e^t \sin t$, $z = e^t \cos t$, Find $\frac{dw}{dt}$.

37) Solve: $\frac{dy}{dx} + \frac{y}{x \log x} = \frac{\sin 2x}{\log x}$

38) The mean and standard deviation of a binomial variate X are 6 and 2. Find (i) the probability mass function (ii) $P(x \ge 2)$

39) Prove that $p \rightarrow q = \neg p \vee q$

40) Find the value of $\left[\frac{1+i}{\sqrt{2}}\right]^8 + \left[\frac{1-i}{\sqrt{2}}\right]^8$

PART - D

Answer all the questions: **7×5=35**

41) Test for consistency and slove: $4x-2y+6z = 8$; $x+y-3z = -1$; $15x-3y+9z = 21$

(OR)

Find the centre, foci, vertices of the ellipse $18x^2+12y^2-144x+48y+120 = 0$

42) If $z = x+iy$ and $\arg\left[\frac{z-i}{z+2}\right] = \frac{\pi}{4}$, show that $x^2+y^2+3x-3y+2 = 0$

(OR)

A pot of boiling water at 100°C is removed from a stove at time t = 0 and left to cool in the kitchen. After 5 minutes, the water temperature has decreased to 80°C, and another 5 minutes later it has dropped to 65°C. Determine the temperature of the kitchen.

43) Determine K and solve the equation $2x^3-6x^2+3x+k = 0$ if one of its roots is twice the sum of the other two roots.

(OR)

Show that the lines $\frac{x-3}{3} = \frac{y-3}{-1}, z-1 = 0$ and $\frac{x-6}{2} = \frac{z-1}{3}, y-2 = 0$ intersect. Also find the point of intersection.

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44) Solve: $\tan^{-1}(2x)+\tan^{-1}(3x) = \pi/4$, if $6x^2 < 1$.

(OR)

Two balls are chosen randomly from an urn containing 8 white and 4 black balls. Suppose that we win Rs. 20 for each black ball is selected and we lose Rs. 10 for each white ball selected. Find the expected winning amount and variance.

45) A hollow cone with base radius a cm and height b cm is placed on a table. Show that the volume of the largest cylinder that can be hidden underneath is $4/9$ times volume of the cone.

(OR)

If $w(x, y) = xy+\sin(xy)$, prove that $\frac{\partial^2 w}{\partial y \partial x} = \frac{\partial^2 w}{\partial x \partial y}$

46) Find the parametric vector, non-parametric vector and Cartesian form of equations of the plane passing through the three non-collinear points (3, 6, −2) (−1, −2, 6) and (6, −4, −2)

(OR)

Evaluate: $\int_0^{\pi} \frac{x \sin x}{1+\sin x} dx$

47) Find the area of the region enclosed by the curves $y = x^2$ and $y = \sqrt{x}$

(OR)

Find the intervals of monotonicity and local extrema of the function $f(x) = 4x^3-6x^2-72x+30$

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