(b) On the average, 20% of the products manufactured by ABC Company are found to be defective. If we select 6 of these products at random and X denotes the number of defective products find the probability that (i) two products are defective (ii) at most one product is defective (iii) at least two products are defective.

45. (a) Prove by vector method that $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$. **(OR)**

(b) If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -\frac{1}{2}\cot u$

46. (a) Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identify, and (v) existence of inverse for the operation $\times_{11}$ on a subset A = {1,3,4,5,9} of the set of remainders {0,1,2,3,4,5,6,7,8,9,10} **(OR)**

(b) Find the vector parametric, vector non-parametric and Cartesian form of the equation of the plane passing through the points (−1,2,0), (2,2,−1) and parallel to the straight line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$.

47. (a) A tunnel through a mountain for a four lane highway is to have a elliptical opening. The total width of the highway (not the opening) is to be 16m, and the height at the edge of the road must be sufficient for a truck 4m high to clear if the highest point of the opening is to be 5m approximately. How wide must the opening be? **(OR)**

(b) *A conical water tank with vertex down of 12 metres height has a radius of 5 metres at the top. If water flows into the tank at a rate 10 cubic m/min, how fast is the depth of the water increases when the water is 8 metres deep?*

Class : 12 Register Number

FIRST REVISION EXAMINATION, JANUARY - 2024

Time Allowed : 3.00 Hours] **MATHEMATICS** [Max. Marks : 90

SECTION - A

Note: (i) All questions are compulsory.
(ii) Each question carries one mark.
(iii) Choose the most suitable answer from the given four alternatives. *20 x 1 = 20*

1. If the two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles then the locus of P is (1) $2x+1=0$ (2) $x=-1$ (3) $2x-1=0$ (4) $x=1$

2. If $\vec{a}, \vec{b}, \vec{c}$ are non–coplanar, non zero vectors such that $[\vec{a}, \vec{b}, \vec{c}] = 2$, then $\{[\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}]\}^2$ is equal to
(1) 61 (2) 4 (3) 16 (4) 8

3. Angle between $y^2 = x$ and $x^2 = y$ at (1, 1) is
(1) $\tan^{-1}\frac{3}{4}$ (2) $\tan^{-1}\left(\frac{4}{3}\right)$ (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{4}$

4. The maximum product of two positive numbers, when their sum of the squares is 200, is (1) 100 (2) $25\sqrt{7}$ (3) 28 (4) $24\sqrt{14}$

5. If $v(x, y) = \log(e^x + e^y)$, then $\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y}$ is equal to
(1) $e^x + e^y$ (2) $\frac{1}{e^x+e^y}$ (3) 2 (4) 1

6. The value of $\int_{-4}^{4}\left[\sin^{-1}\left(\frac{x^2}{x^2+1}\right) + \sec^{-1}\left(\frac{x^2+1}{x^2}\right)\right]dx$ is
(1) π (2) 4π (3) 3π (4) 2π

7. The value of $\int_0^\infty e^{-3x}x^2\,dx$ is
(1) $\frac{7}{27}$ (2) $\frac{5}{27}$ (3) $\frac{4}{27}$ (4) $\frac{2}{27}$

8. The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{3}} + x^{\frac{1}{4}} = 0$ are respectively.
(1) 2, 3 (2) 3, 3 (3) 2, 6 (4) 2, 4

9. Let X have a Bernoulli distribution with mean 0.4, then the variance of $(2X-3)$ is
(1) 0.24 (2) 0.48 (3) 0.6 (4) 0.96

10. Which one is the contrapositive of the statement $(p \vee q) \to r$?
(1) $\neg r \to (\neg p \wedge \neg q)$ (2) $\neg r \to (p \vee q)$ (3) $r \to (p \wedge q)$ (4) $p \to (q \vee r)$

11. If $A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$, then $9I_2 - A =$
(1) A^{-1} (2) $\frac{A^{-1}}{2}$ (3) $3A^{-1}$ (4) $2A^{-1}$

12. If $\rho(A) = \rho([A|B])$, then the system $AX = B$ of linear equations is
(1) consistent and has a unique solution (2) consistent
(3) consistent and has infinitely many solution (4) inconsistent

13. The value of $\sum_{n=1}^{13}(i^n + i^{n-1})$ is
(1) $1+i$ (2) i (3) 1 (4) 0

14. If $\omega = cis\frac{2\pi}{3}$, then the number of distinct roots of $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$

(1) 1 (2) 2 (3) 3 (4) 4

15. If The number of positive zeros of the polynomial $\sum_{r=0}^{n} {}^{n}_{r}C\,(-1)^r\,x^r$ is

(1) 0 (2) n (3) < n (4) r

16. The general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$

(1) $e^x + e^y = C$ (2) $e^{-x} + e^{-y} = C$ (3) $e^{-x} + e^y = C$ (4) $e^x + e^{-y} = C$

17. A random variable X has binomial distribution with n=25 and p = 0.8 then standard deviation of X is

(1) 6 (2) 4 (3) 3 (4) 2

18. The equation $tan^{-1}x - cot^{-1}x = tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ has

(1) no solution (2) unique solution

(3) two solutions (4) infinite number of solutions

19. If $sin^{-1}x + sin^{-1}y + sin^{-1}z = \frac{3\pi}{2}$, the value of $x^{2017} + y^{2018} + z^{2019} - \frac{9}{x^{101}+y^{101}+z^{101}}$ is

(1) 0 (2) 1 (3) 2 (4) 3

20. If the normals of the parabola $y^2 = 4x$drawn at the end points of its latus rectum are tangents to the circle $(x-3)^2 + (y+2)^2 = r^2$, then the value of r^2 is

(1) 2 (2) 1 (3) 3 (4) 4

SECTION – B

Note: (i) Answer any SEVEN questions. (ii) Question No. 30 is compulsory. ***7 X 2=14***

21. Find the rank of the matrix.

$$\begin{bmatrix} 6 & 0 & -9 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

22. Find the value of $\sum_{k=1}^{8}\left(cos\frac{2k\pi}{9} + i\,sin\frac{2k\pi}{9}\right)$.

23. Find the value of $sec^{-1}\left(-\frac{2\sqrt{3}}{3}\right)$.

24. Find centre and radius of the circle $2x^2 + 2y^2 - 6x + 4y + 2 = 0$

25. If $\vec{a} = \hat{\imath}+2\hat{\jmath}+3\hat{k}$, $\vec{b} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$, $\vec{c} = 3\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{a}\times(\vec{b}\times\vec{c}) = l\vec{a} + m\vec{b} + n\vec{c}$, find the values of l, m, n.

26. Evaluate: $\lim_{x\to 0^+}\left(\frac{1}{x} - \frac{1}{e^x-1}\right)$.

27. Evaluate: $\int_{-\log 2}^{\log 2} e^{-|x|}\,dx$.

28. The probability density function of $X$is given by$f(x) = \begin{cases} kxe^{-2x} & for\ x > 0 \\ 0 & for\ x \le 0 \end{cases}$ Find the value of k.

29. On Z, define $*$by $(m * n) = m^n + n^m : \forall m, n \in Z$. Is $*$binary on Z?.

30. Show that $y = Ae^{3x} + Be^{-3x}$ is a solution of the differential equation $\frac{d^2y}{dx^2} - 9y = 0$.

SECTION - C

Note: (i) Answer any SEVEN Questions. (ii) Question No.40 is compulsory. ***7 X 3 =21***

31. If $z_1 = 2 - i$ and $z_2 = -4 + 3i$, find the inverse of $z_1 z_2$ and $\frac{z_1}{z_2}$.

32. Obtain the condition that the roots of $x^3 + px^2 + qx + r = 0$are in A.P.

33. Find the value, if it exists. If not, give the reason for non-existence $sin^{-1}[sin\,5]$.

34. The parabolic communication antenna has a focus at 2m distance from the vertex of the antenna. Find the width of the antenna 3m from the vertex.

35. Find intervals of concavity and points of inflexion for the function $f(x) = \frac{1}{2}(e^x - e^{-x})$

36. If $v(x,y,z) = x^3 + y^3 + z^3 + 3xyz$, show that $\frac{\partial^2 v}{\partial y \partial z} = \frac{\partial^2 v}{\partial z \partial y}$.

37. Solve : $\frac{dy}{dx} = (3x + y + 4)^2$.

38. Four fair coins are tossed once. Find the probability mass function, mean and variance for number of heads occurred.

39. Show that $p \to q$ and $q \to p$are not equivalent.

40. If $A = \frac{1}{9}\begin{bmatrix} -8 & 1 & a \\ b & 4 & 7 \\ 1 & c & 4 \end{bmatrix}$is orthogonal, find a, b & c and hence A^{-1}.

PART – D

Answer ALL questions: ***7 x 5 = 35***

41. (a) In a murder investigation, a corpse was found by a detective at exactly 8 p.m. Being alert, the detective also measured the body temperature and found it to be 70^{0}F. Two hours later, the detective measured the body temperature again and found it to be 60^{0}F. If the room temperature is 50^{0}F, and assuming that the body temperature of the person before death was 98.6^{0}F, at what time did the murder occur? [log (2.43) = 0.88789; log (0.5) = -0.69315] **(OR)**

(b) Investigate the values of $\lambda$and $\mu$the system of linear equations

$2x + 3y + 5z = 9, 7x + 3y - 5z = 8, 2x + 3y + \lambda z = \mu$, have

i) no solution ii) a unique solution iii) an infinite number of solution

42. (a) If $z = x + iy$ and $arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, show that $x^2 + y^2 + 3x - 3y + 2 = 0$. **(OR)**

(b) The volume of a cylinder is given by the formula $V = \pi r^2 h$. Find the greatest and least values of V if $r + h = 6$.

43. (a) Find the area of the region common to the circle $x^2 + y^2 = 16$and the parabola $y^2 = 6x$. **(OR)**

(b) Solve the equation $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$.

44. (a) Solve: $tan^{-1}\left(\frac{x-1}{x-2}\right) + tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$. **(OR)**