MODEL QUESTION PAPER – I 2024

CLASS: XII MATHEMATICS MARKS:90

I CHOOSE THE CORRECT ANSWER

20X1=20

1. If $A^T A^{-1}$ is symmetric then A^2 = a) A^{-1} b) $(A^T)^2$ c) A^T d) $(A^{-1})^2$
2. If A, B and C are invertible matrices of some order, then which one of the following true? a) adj A =|A|A^{-1} b) adj(AB)= (adjA)(adjB) c) det A^{-1} = $(\det A)^{-1}$ d) $(ABC)^{-1} = C^{-1} B^{-1} A^{-1}$
3. If z is a complex number such that z∈ C/R and $z+\frac{1}{z}$ ∈ R, then |z| is a) 0 b) 1 c) 2 d) 3
4. $\text{cis}\frac{28}{5}\pi$ is equal to a) $\text{cis}(-\frac{2\pi}{5})$ b) $\text{cis}(\frac{2\pi}{5})$ c) $\text{cis}(\frac{3\pi}{5})$ d) $\text{cis}(-\frac{3\pi}{5})$
5. According to the rational root theorem, which number is not possible rational zero of $4x^7+2x^5-10x^3-5$? a) -1 b) 5/4 c) 4/5 d) 5
6. The value of sin-1(cos x), $0\leq x \leq \pi$ is a) π- x b) x-$\frac{\pi}{2}$ c) $\frac{\pi}{2}-$x d) x - π
7. The locus of a point whose distance from (-2,0) is 2/3 times of x at (3,0) passing through the line x=-9/2 is a) a parabola b) a hyperbola c) an ellipse d) a circle
8. The number of tangents to the circle from inside the circle is a) 2 real b) 0 c) 2 imaginary d) can't determined
9. Distance from origin to the planes 3x-6y+2z+7=0 is a) 0 b) 1 c) 2 d) 3
10. If $\vec{a}$ and $\vec{b}$ are parallel vectors, then $[\hat{a}\,,\hat{b}\;\hat{c}]$ is equal to a) 2 b) -1 c) 1 d) 0
11. The maximum value of the function x^2e^{-2x} , x>0 is a) 1/e b) 1/2e c) $1/e^2$ d) $4/e^4$
12. Angle between y^2=x and x^2=y at the origin is a) tan-1 (3/4) b tan-1(4/3) c) $\frac{\pi}{4}$ d) $\frac{\pi}{2}$
13. The percentage error of fifth root of 31 is approximately how many times the percentage error in 31? a)1/31 b) 1/5 c) 5 d) 31
14. The value of $\int_{-1}^{2}|x|$ dx is a) ½ b) 3/2 c) 5/2 d) 7/2
15. The area between y^2=4x and its latus rectum is a) 2/3 b) 4/3 c) 8/3 d) 5/3
16. The solution of differential equation $2x\frac{dy}{dx}-$y=3 represents a) straight line b) circles c) parabola d) ellipse
17. Integrating factor of the differential equation $\frac{dy}{dx}=\frac{x+y+1}{x+1}$ is a) $\frac{1}{x+1}$ b) x+1 c) $\frac{1}{\sqrt{x+1}}$ d) $\sqrt{x+1}$
18. A random variable X has binomial distribution with n=25 and p=0.8 then standard deviation of x is a) 6 b) 4 c) 3 d) 2
19. If a compound statement involves 3 simple statements, then the number of rows in the truth table is a) 9 b) 8 c) 6 d) 3
20. The operation '– ' is binary on a) N b) Q\{0} c) R\{0} d) Q

II ANSWER ANY 7 (Q.NO.3O IS COMPULSORY)

7X2=14

21. If A is symmetric prove that adj A is also symmetric.
22. Find the square root of -6+8i
23. If y=$2\sqrt{2}$ x+c is a tangent to the circle $x^2+y^2=16$, find the value of c.
24. A is acted upon by the forces $3\hat{\imath}$ -$2\hat{\jmath}$ +$2\hat{k}$ and $2\hat{\imath}$+$\hat{\jmath}$ -$\hat{k}$ is displaced from the point (1,3,-1) to the point (4,-1,λ) . If the work done by the forces is 16 units, find the value of λ
25. Evaluate $\lim\limits_{x\to\infty}\frac{x}{\log x}$
26. If the radius of a sphere with radius 1cm, has to decrease by .1 cm, approximately how much will its volume decrease?
27. Evaluate $\int_0^1 x^3\,(1-x)^4$ dx
28. The probability that a certain kind of component will survive a electrical test is ¾. Find the probability that exactly 3 of the 5 component tested survive.
29. Let A=$\begin{bmatrix}1&0&1&0\\0&1&0&1\\1&0&0&1\end{bmatrix}$ B=$\begin{bmatrix}0&1&0&1\\1&0&1&0\\1&0&0&1\end{bmatrix}$ be two Boolean matrices of the same type find A V B
30. If $\cot^{-1}$ 2 and $\cot^{-1}$ 3 are the two angles of a triangle, then find the third angle.

III ANSWER ANY 7 (Q.NO 4O IS COMPULSORY) 7X3=21

31. A= $\begin{bmatrix} 1 & tanx \\ -tanx & 1 \end{bmatrix}$ show that $A^T A^{-1} = \begin{bmatrix} cos2x & -sin2x \\ sin2x & cos2x \end{bmatrix}$
32. If |z|=2 show that 3≤|z+3+4i|≤ 7
33. Find the domain of $\cos^{-1}\left(\frac{2+sinx}{3}\right)$
34. If $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, $\vec{b}=2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\vec{c}=3\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{a}\, X(\vec{b}\, X\, \vec{c}\,)$
35. Prove among all the rectangle of the given area square has least perimeter.
36. Let $u(x,y) = e^{-2y}\cos 2x$ for all $(x,y) \in R^2$ prove that u is a harmonic function in R^2
37. Find the length of latus rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$
38. Show that $y=mx+\frac{7}{m}$, $m\neq o$ is a solution of the differential equation $xy'+ 7\frac{1}{y'} - y= o$
39. Construct the truth table for (p V q) V ¬ q
40. Find the exact number of real zeros and imaginary of the polynomial $x^9 +9x^7+7x^5 +5x^3 +3x$

IV ANSWER ALL THE QUESTIONS 7X5=35

41. a) By using Gaussian elimination method, balance the chemical equation $C_5H_8+O_2 \rightarrow CO_2+H_2O$ (OR)
    (b) $\cos^{-1}x + \cos^{-1} y + \cos^{-1} z = \pi$ and 0<x,y,z<1 show that $x^2+y^2+z^2+2xyz=1$

42. a) find the cube roots of unity. (OR)

    b) if $v(x,y) = \log\frac{x^2+y^2}{x+y}$ prove that $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} =1$

43. a) solve $x^4-10x^3+26x^2-10x+1=0$ (OR)

    b) solve $\frac{dy}{dx}+ 2y\cot x = 3x^2\csc^2 x$

44. a) Evaluate $\lim_{x\to\infty}(1 + 2x)^{\frac{1}{2logx}}$ (OR)

    b) Find the parametric, non parametric and cartesian equation of the place passing through non collinear points (2,6-2), (-1,-2,6) and (6,-4,-2)

45. a) Identity the type of conic and find the centre, foci, vertices and directrices of $9x^2-y^2-36x-6y+18=0$ (OR)

    b) Suppose that f(x) given below represent a probability mass function find (i) the value of c (ii) Mean and variance

X	1	2	3	4	5	6
f(x)	C^2	$2c^2$	$3c^2$	$4c^2$	c	2c

46 a) Show that the straight line x+1=2y=-12z and x=y+2=6z-6 are skew and hence find the distance between them (OR)

b) Find the area of the region bounded by the parabola y2=x and the line y=x-2

47 a) If the curves $ax^2+by^2=1$ and $cx^2+dy^2=1$ intersect each other orthogonally then show that $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$ (OR)

b) Verify (i) closure property (ii) commutative property (iii) associative property (iv) existence of identity (iv) existence of inverse for the operation $+_5$ on Z_5 with table corresponding modulo

Model Question Paper I 2024

Class XII Mathematics Marks: 90

I Choose

1. b) $(A^T)^2$
2. a) adj(AB) = (adj A)(adj B)
3. b) 1
4. a) $cis\left(-\frac{2\pi}{3}\right)$
5. a) 1
6. c) $\frac{\pi}{2}$
7. c) an ellipse
8. c) 2 imaginary
9. b) 1
10. d) 0
11. c) $\frac{1}{e^2}$
12. d) $\frac{\pi}{2}$
13. b) $\frac{1}{5}$
14. c) $\frac{5}{2}$
15. c) $\frac{8}{3}$
16. c) parabola
17. a) $\frac{1}{1+x}$
18. d) 2
19. b) 8
20. d) 0

II Answer any 7 Q.NO. 30 compulsory $7\times2=14$

21. Suppose A is symmetric. Then $A^T = A$ and so, by theorem we get $adj(A^T) = (adj\,A)^T$

$adj\,A = (adj\,A)^T \Rightarrow adj\,A$ is Symmetric

22. $-6+8i$ $a=-6$ $b=8$

$\sqrt{a+ib} = \pm\left[\sqrt{\frac{|z|+a}{2}} + i\frac{b}{|b|}\sqrt{\frac{|z|-a}{2}}\right]$

$\sqrt{-6+8i} = \pm\left[\sqrt{\frac{10-6}{2}} + i\frac{8}{8}\sqrt{\frac{10+6}{2}}\right]$

$|z| = \sqrt{6^2+8^2} = 10$ $= \pm(\sqrt{2} + 2\sqrt{2}\,i)$

$= \pm\sqrt{2}(1+2i)$

23. $y = 2\sqrt{2}x + c$ $m = 2\sqrt{2}$

$y = mx + c$

$x^2+y^2 = 16$ $a^2 = 16$

$x^2+y^2 = a^2$

Tangent to the circle

$c^2 = a^2(1+m^2)$

$c^2 = 16(1+(2\sqrt{2})^2)$

$= 16(1+8) = 16(9)$

$c = \pm 4\times 3$ $\boxed{c = \pm 12}$

24. $\vec{r} = 3\hat{i} - 2\hat{j} + 2\hat{k} + 2\hat{i} + \hat{j} - \hat{k}$

$= 5\hat{i} - \hat{j} + \hat{k}$

$\vec{d} = 4\hat{i} - \hat{j} + \lambda\hat{k} - \hat{i} - 3\hat{j} + \hat{k}$

$= 3\hat{i} - 4\hat{j} + (\lambda+1)\hat{k}$

$\vec{r}\cdot\vec{d} = 16$ $(5\hat{i} - \hat{j} + \hat{k})\cdot(3\hat{i} - 4\hat{j} + (\lambda+1)\hat{k}) = 16$

$15 + 4 + \lambda + 1 = 16$ $\lambda = 16 - 20$

$\boxed{\lambda = -4}$

25. Evaluate $\lim_{x\to\infty} \frac{x}{\log x} = \frac{\infty}{\infty}$

Interminate form, Applying L' Hopital Rule

$\lim_{x\to\infty} \frac{1}{\frac{1}{x}} = \lim_{x\to\infty} x = \infty$

26. $r = 10cm$ $dr = 0.1cm$

Vol of sphere $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = \frac{4}{3}\pi 3r^2 \Rightarrow \cancel{dV}$

$dV = \frac{4}{\cancel{3}}\times\pi\times\cancel{3}(10)^2(0.1)$

$= 40\pi\ cm^3$

27. $\int_0^1 x^3(1-x)^4\,dx$

$\int_0^1 x^m(1-x)^n\,dx = \frac{m!\times n!}{(m+n+1)!}$

$\int_0^1 x^3(1-x)^4\,dx = \frac{3!\times 4!}{(3+4+1)!}$

$= \frac{3\times2\times1\times4\times3\times2\times1}{8\times7\times6\times4\times3\times2\times5\times1} = \frac{1}{280}$

28. $p = \frac{3}{4}$ $q = \frac{1}{4}$ $n = 5$

$P(X=3) = {}^{n}C_x\, p^x\, q^{n-x}$

$= {}^5C_3 \left(\frac{3}{4}\right)^3 \left(\frac{1}{4}\right)^2$

$= \frac{5\times4\times3}{3\times2\times1}\ \frac{3\times3\times3}{4\times4\times4}\times\frac{1}{4\times4}$

$= \frac{270}{1024}$

29. $A \vee B = \begin{pmatrix}1&0&1&0\\0&1&0&1\\1&0&0&1\end{pmatrix} \vee \begin{pmatrix}0&1&0&1\\1&0&1&0\\1&0&0&1\end{pmatrix}$

$= \begin{pmatrix}1&1&1&1\\1&1&1&1\\1&0&0&1\end{pmatrix}$

30. $\cot^{-1}2 + \cot^{-1}3 + \cot^{-1}\theta = \pi$

$\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} + \tan^{-1}x = \pi$

$\tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}}\right) + \tan^{-1}x = \pi$

$\tan^{-1}\frac{\frac{5}{6}}{\frac{5}{6}} + \tan^{-1}x = \pi$

$\tan^{-1}\left(\frac{1+x}{1-x}\right) = \pi$

$\frac{1+x}{1-x} = \tan\pi$ $\quad \frac{1+x}{1-x} = 0$

$1+x = 0$ $\quad \boxed{x = -1}$

$\tan^{-1}(-1) = \theta$ $\quad \boxed{\theta = \frac{3\pi}{4}}$

III Answer any 7 Q.NO.40 Compulsory $7\times3 = 21$

31. $A^{-1} = \frac{1}{|A|}\,\mathrm{adj}\,A$

$= \frac{1}{1+\tan^2 x}\begin{bmatrix}1 & -\tan x\\ \tan x & 1\end{bmatrix}$

LHS

$A^T A^{-1} = \begin{bmatrix}1 & -\tan x\\ \tan x & 1\end{bmatrix}\frac{1}{1+\tan^2 x}\begin{bmatrix}1 & -\tan x\\ \tan x & 1\end{bmatrix}$

$= \begin{bmatrix}\frac{1-\tan^2 x}{1+\tan^2 x} & \frac{-\tan x - \tan x}{1+\tan^2 x}\\ \frac{\tan x + \tan x}{1+\tan^2 x} & \frac{-\tan^2 x + 1}{1+\tan^2 x}\end{bmatrix}$

$= \begin{bmatrix}\cos 2x & -\sin 2x\\ \sin 2x & \cos 2x\end{bmatrix} = \text{RHS}$

Hence proved.

32. $|z| = 2$.

$|z+3+4i| \le |z| + |3+4i|$

$\le 2 + \sqrt{3^2+4^2}$

$\le 2 + 5$

≤ 7

$|z+3+4i| \ge \big||z| - |3+4i|\big|$

$\ge |2-5| \ge 3$

$\therefore 3 \le |z+3+4i| \le 7$.

33. let $y = \cos^{-1}x$.

$-1 \le x \le 1$

$-1 \le \frac{2+\sin x}{3} \le 1$

$-3 \le 2+\sin x \le 3$

$-5 \le \sin x \le 1$ reduces to

$-1 \le \sin x \le 1$ or $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

$\therefore$ domain is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

34. $\vec{a}\times(\vec{b}\times\vec{c}) = l\vec{a}+m\vec{b}+n\vec{c}$

$(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c} = l\vec{a}+m\vec{b}+n\vec{c}$

$\therefore l=0 \quad m=\vec{a}\cdot\vec{c}$

$n=-(\vec{a}\cdot\vec{b})$

$m=-(\vec{i}+2\vec{j}+3\vec{k})\cdot(2\vec{i}-\vec{j}+\vec{k})$

$= 2-2+3=-3$

$m=(\vec{i}+2\vec{j}+3\vec{k})\cdot(3\vec{i}+2\vec{j}+\vec{k})$

$=3+4+3=10$

35. $A=lb \qquad P=2l+2b$

$\frac{A}{b}=l \qquad P=\frac{2A}{b}+2b$

$l=\frac{A}{\sqrt{A}} \qquad =\frac{2A+2b^2}{b}$

$l=\sqrt{A} \qquad P'=2\left(\frac{-A}{b^2}+1\right)$

$-\frac{A}{b^2}+2=0$

$\boxed{l=b=\sqrt{A}} \qquad \frac{-A}{b^2}=-1$

$A=b^2$

$b=\sqrt{A}$

$\therefore$ minimum perimeter rectangle of a given area is square.

36. $u(x,y)=e^{-2y}\cos 2x$

$u_x(x,y)=e^{-2y}(-2)\sin 2x$

$u_{xx}=e^{-2y}(-2)(2)(\cos 2x)$

$u_y=e^{-2y}(-2)\cos 2x$

$u_{yy}=(-2)(-2)e^{-2y}\cos 2x$

$u_{xx}+u_{yy}=-4e^{-2y}\cos 2x + 4e^{-2y}\cos 2x=0$

$\therefore u$ is a harmonic function in R^2.

37. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ passing through $S(ae,0)$

L is (ae,y_1)

$\frac{a^2e^2}{a^2}+\frac{y_1^2}{b^2}=1$

$\frac{y_1^2}{b^2}=1-e^2$

$y_1^2=b^2(1-e^2)=b^2\left(\frac{b^2}{a^2}\right)$

$y_1=\pm\frac{b^2}{a}$

$LL'=\frac{2b^2}{a}$

38. $y=mx+\frac{7}{m}$

$y'=m+0 \qquad y'=m$

$xy'+\frac{7}{y'}-y=xm+\frac{7}{m}-xm-\frac{7}{m}$

$=0$

$\therefore xy'+7\frac{1}{y'}-y=0$ is a solution.

39.

p	q	$p\vee q$	$\neg q$	$p\vee q \vee \neg q$
T	T	T	F	T
T	F	T	T	T
F	T	T	F	T
F	F	F	T	T

40. $P(x)=x^9+9x^7+7x^5+5x^3+3x$

No sign changes in $p(x)$

No positive zeros.

$P(-x)=-x^9-9x^7-7x^5-5x^3-3x$

No sign changes in $p(-x)$

No negative zeros.

clearly 0 is a [illegible] zero

$\therefore$ No of real [illegible] $=1$

No. of Imaginary zeros $=9-1=8$

VI Answer All Questions

41 a)

$x_1 C_5H_8 + x_2 O_2 \rightarrow x_3 CO_2 + x_4 H_2O$

Carbon atoms

$5x_1 = x_3 \quad 5x_1 - x_3 = 0$ — (1)

Hydrogen atoms

$8x_1 = 2x_4 \quad 4x_1 - x_4 = 0$ — (2)

Oxygen atoms

$2x_2 = 2x_3 + x_4$

$2x_2 - 2x_3 - x_4 = 0$ — (3)

$AX = B \Rightarrow \begin{bmatrix} 5 & 0 & -1 & 0 \\ 4 & 0 & 0 & -1 \\ 0 & +2 & -2 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$

$[A|B] = \left[\begin{array}{cccc|c} 5 & 0 & -1 & 0 & 0 \\ 4 & 0 & 0 & -1 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{array}\right]$

$[A|B] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cccc|c} 4 & 0 & 0 & -1 & 0 \\ 5 & 0 & -1 & 0 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{array}\right]$

$\xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{cccc|c} 4 & 0 & 0 & -1 & 0 \\ 0 & 2 & -2 & -1 & 0 \\ 5 & 0 & -1 & 0 & 0 \end{array}\right]$

$\xrightarrow{R_3 \to 4R_3 - 5R_1} \left[\begin{array}{cccc|c} 4 & 0 & 0 & -1 & 0 \\ 0 & 2 & -2 & -1 & 0 \\ 0 & 0 & -4 & 5 & 0 \end{array}\right]$

$\rho(A) = \rho(A|B) = 3 < 4$

The system is consistent and has infinite no. of solutions.

$4x_1 - x_4 = 0, \quad 2x_2 - 2x_3 - x_4 = 0$

$-4x_3 + 5x_4 = 0. \quad x_4 = t$

$x_3 = \frac{5t}{4} \quad x_2 = \frac{7t}{4} \quad x_1 = \frac{t}{4}$

$t = 4 \quad x_1 = 1, x_2 = 7, x_3 = 5$

$x_4 = 4$

$C_5H_8 + 7O_2 \rightarrow 5CO_2 + 4H_2O$

b) $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$

$\cos^{-1}x = \alpha \quad \cos^{-1}y = \beta \quad \cos^{-1}z = \gamma$

$\cos\alpha = x \quad \cos\beta = y \quad \cos\gamma = z$

$\sin\alpha = \sqrt{1-x^2} \quad \sin\beta = \sqrt{1-y^2} \quad \sin\gamma = \sqrt{1-z^2}$

$\cos^{-1}(\cos\alpha\cos\beta - \sin\alpha\sin\beta) = \pi - \cos^{-1}z$

$\cos^{-1}(xy - \sqrt{1-x^2}\sqrt{1-y^2}) = \pi - \gamma$

$\cos^{-1}(xy - \sqrt{1-x^2}\sqrt{1-y^2}) = \pi - \gamma$

$xy - \sqrt{1-x^2}\sqrt{1-y^2} = \cos\pi - \gamma$

$xy - \sqrt{1-x^2}\sqrt{1-y^2} = -\cos\gamma$

$xy - \sqrt{1-x^2}\sqrt{1-y^2} = -z$

$(xy + z)^2 = (\sqrt{1-x^2}\sqrt{1-y^2})^2$

$x^2y^2 + 2xyz + z^2 = (1-x^2)(1-y^2)$

$x^2y^2 + 2xyz + z^2 = 1 - y^2 - x^2 + x^2y^2$

$\boxed{x^2 + y^2 + z^2 + 2xyz = 1}$

Hence proved.

42 a) $z = 1^{1/3} \quad z^3 = 1.$

$z^3 = \cos(0 + 2k\pi) + i\sin(0 + 2k\pi)$

$= e^{i2k}, \quad k = 0, 1, 2 \ldots$

$z = \cos\frac{2k\pi}{3} + i\sin\frac{2k\pi}{3} = e^{i\frac{2k\pi}{3}}$

$k = 0, 1, 2.$

$k = 0$

$z = \cos 0 + i\sin 0 = 1$

$k = 1 \quad z = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$

$z = \cos(\pi - \frac{\pi}{3}) + i\sin(\pi - \frac{\pi}{3})$

$= -\frac{1}{2} + i\frac{\sqrt{3}}{2}$

$k = 2$

$z = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$

$= \cos(\pi + \frac{\pi}{3}) + i\sin(\pi + \frac{\pi}{3})$

$= \frac{1}{2} - i\frac{\sqrt{3}}{2}$

Cube roots of unity are

$1, \frac{1+i\sqrt{3}}{2}, \frac{1-i\sqrt{3}}{2} \Rightarrow 1, \omega, \omega^2.$

b) $V(x,y) = \log \frac{x^2+y^2}{x+y}$

V is not homogenous.

$f(x,y) = \frac{x^2+y^2}{x+y} = e^V$

is homogenous.

$f(tx,ty) = \frac{t^2(x^2+y^2)}{t(x+y)} = t\, f(x,y)$

f is homogenous of degree with 1

By Euler's Theorem

$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 1\, f(x,y)$

$x\frac{\partial(e^V)}{\partial x} + y\frac{\partial(e^V)}{\partial y} = e^V$

$x\frac{\partial V}{\partial x}e^V + y\frac{\partial V}{\partial y}e^V = e^V$

$(\div e^V)$

$x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y} = 1$

Hence proved

43 a) $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$

This equation is Type I even degree reciprocal equation.

$x^2\left[\left(x^2+\frac{1}{x^2}\right) - 10\left(x+\frac{1}{x}\right) + 26\right] = 0.$

$y = x + \frac{1}{x}$ $\quad y^2 - 2 = x^2 + \frac{1}{x^2}$

$(y^2-2) - 10y + 26 = 0.$

$y^2 - 10y + 24 = 0$

$(y-6)(y-4) = 0$ $\quad y = 6$ or $y = 4$

$x + \frac{1}{x} = 6$ $\quad x + \frac{1}{x} = 4$

$x^2 + 1 - 6x = 0$ $\quad x^2 + 1 - 4x = 0$

$x^2 - 6x + 1 = 0$ $\quad x^2 - 4x + 1 = 0.$

$x = 3+2\sqrt{2},\ 3-2\sqrt{2}$ $\quad x = 2+\sqrt{3},\ 2-\sqrt{3}.$

Roots are $3 \pm 2\sqrt{2},\ 2 \pm \sqrt{3}.$

b) $\frac{dy}{dx} + 2y\cot x = 3x^2 \operatorname{cosec}^2 x$

$\frac{dy}{dx} + Py = Q$. It is Linear Differential Equation

$P = 2\cot x$ $\quad Q = 3x^2 \operatorname{cosec}^2 x$

$I.F = e^{\int P dx} = e^{2\int \cot x} = e^{2(\log \sin x)} = \sin^2 x$

$y(I.F) = \int Q(I.F)\,dx + c$

$y \sin^2 x = \int 3x^2 \operatorname{cosec}^2 x \cdot \sin^2 x + c$

$= \frac{3x^3}{3} + c$

$\boxed{y\sin^2 x = x^3 + c}$

44 a) $A = \lim_{x\to\infty} (1+2x)^{\frac{1}{2\log x}} = \infty^0$

Indeterminate form.

Applying L Hopital rule.

Taking log on Both side.

$\log A = \lim_{x\to\infty} \frac{1}{2\log x} \log(1+2x).$

$= \lim_{x\to\infty} \frac{\log(1+2x)}{2\log x} \quad \left(\frac{\infty}{\infty}\right)$

$= \lim \frac{\frac{2}{1+2x}}{\frac{2}{x}}$ (by L Hopital rule)

$= \lim \frac{x}{1+2x} \quad \left(\frac{\infty}{\infty}\right)$

$= \lim_{x\to\infty}\left(\frac{1}{2}\right) = \frac{1}{2}.$

$\log A = \frac{1}{2}$ $\quad A = e^{\frac{1}{2}} = \sqrt{e}.$

b) $\vec{a} = 3\hat{i} + 6\hat{j} - 2\hat{k}$ $\quad \vec{b} = -\hat{i} - 2\hat{j} + 6\hat{k}$

$\vec{c} = 6\hat{i} - 4\hat{j} - 2\hat{k}.$

P.V $\vec{r} = \vec{a} + t(\vec{b} - \vec{a}) + s(\vec{c} - \vec{a})$

$\vec{r} = 3\hat{i} + 6\hat{j} - 2\hat{k} + t(-4\hat{i} - 8\hat{j} + 8\hat{k}) + s(3\hat{i} - 10\hat{j} + 0\hat{k})$

Non parametric form:

$(\vec{r}-\vec{a})\cdot[(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})]=0$

$\vec{r}-(3\vec{i}+6\vec{j}-2\vec{k})\cdot$ $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -4 & -8 & 8 \\ 3 & -10 & 0 \end{vmatrix} = 0.$

$\vec{r}-(3\vec{i}+6\vec{j}-2\vec{k})\cdot[\vec{i}(0+80)$
$\vec{j}(0-24)+\vec{k}(40+24)=0$

$\vec{r}-(3\vec{i}+6\vec{j}-2\vec{k})\cdot[+80\vec{i}+24\vec{j}$
$+64\vec{k})=0$
$(\div 8)$

$\vec{r}-(3\vec{i}+6\vec{j}-2\vec{k})\cdot(10\vec{i}+3\vec{j}+8\vec{k})=0$

$\vec{r}\cdot(10\vec{i}-3\vec{j}-8\vec{k})-(30+18+16)=0$

$\vec{r}\cdot(10\vec{i}+3\vec{j}+8\vec{k})=32$

cartesian vector

$10x+3y+8z=32.$

45 a) $9x^2-36x-y^2-6x+18=0$

$9(x^2-4x)-(y^2+6x)+18=0$

$9(x^2-4x+4-4)-(y^2+6x+9-9)+18$
$=0$

$9(x-2)^2-36-(y+3)^2+9+18=0$

$\frac{9(x-2)^2}{9}-\frac{(y+3)^2}{9}=\frac{9}{9}$

$\frac{(x-2)^2}{1^2}-\frac{(y+3)^2}{3^2}=1$

This is hyperbola

Centre $(h,k)=(2,-3)$

$c^2=a^2+b^2=1+9=10$ $c=\sqrt{10}$

vertices $(\pm a+h,k)=(\pm1+2,-3)$
$=(3,-3)\ (1,-3)$

foci $=(\pm c+h,k)$
$=(\pm\sqrt{10}+2,-3)$

vertices$_x = \pm\frac{a}{e}+h = \pm\frac{a^2}{c}+h$

$\boxed{x=\pm\frac{1}{\sqrt{10}}\,2}$

b)

$\Sigma f(x)=1$

$c^2+2c^2+3c^2+4c^2+c+2c=1$

$10c^2+3c-1=0.$

$c=-\frac{1}{2},\ \frac{1}{5}$

-10
$\wedge$
3

$\frac{5}{10}\ \frac{-2}{10}$

i) $\boxed{c=\frac{1}{5}}$.

x	1	2	3	4	5	6
$f(x)$	$\frac{1}{25}$	$\frac{2}{25}$	$\frac{3}{25}$	$\frac{4}{25}$	$\frac{1}{5}$	$\frac{2}{5}$

ii) Mean $=\Sigma x f(x)=$

$\frac{1}{25}+\frac{4}{25}+\frac{9}{25}+\frac{16}{25}+\frac{5}{5}+\frac{12}{5}$

$=\frac{30}{25}+\frac{25+60}{25}=\frac{115}{25}=\frac{23}{5}$

$=4.6$

iii) Variance $\Sigma x^2f(x)-(\Sigma x f x)^2$

$=\frac{1}{25}+\frac{8}{25}+\frac{27}{25}+\frac{64}{25}+\frac{25}{5}$
$+\frac{72}{5}$

$=\frac{100}{25}+\frac{125+360}{25}=\frac{585}{25}=\frac{117}{5}$

$\sigma^2=\frac{585}{25}-\frac{23^2}{5^2}=\frac{585}{25}-\frac{529}{25}$

$=\frac{56}{25}=2.24.$

46 a) $x+1 = 2y = -12z$,

$$\frac{x+1}{1} = \frac{y-0}{1/2} = \frac{z-0}{-1/12}$$

$x = y+2 = 6z-6$

$$\frac{x-0}{1} = \frac{y+2}{1} = \frac{z-1}{1/6}$$

$\vec{a} = \hat{i}$ $\quad \vec{b} = \hat{i} + \frac{\hat{j}}{2} - \frac{1}{12}\hat{k}$

$\vec{c} = 2\hat{j} - \hat{k}$ $\quad \vec{d} = \hat{i} + \hat{j} + \frac{1}{6}\hat{k}$

$\vec{b} \neq \lambda\vec{d}$ ∴ not parallel.

$\vec{c} - \vec{a} = -\hat{i} + 2\hat{j} - \hat{k}$

$$\vec{b} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1/2 & -1/12 \\ 1 & 1 & 1/6 \end{vmatrix}$$

$= \hat{i}\left(\frac{1}{12} + \frac{1}{12}\right) - \hat{j}\left(\frac{1}{6} + \frac{1}{12}\right) + \hat{k}\left(1 - \frac{1}{2}\right)$

$= \hat{i}\left(\frac{2}{12}\right) - \hat{j}\left(\frac{3}{12}\right) + \frac{1}{2}\hat{k}$

$= \frac{1}{6}\hat{i} - \frac{1}{4}\hat{j} + \frac{1}{2}\hat{k}$

$(\vec{c} - \vec{a}) \cdot (\vec{b} \times \vec{d}) = -\frac{1}{6} - \frac{2}{4} - \frac{1}{2}$

$= \frac{-2-6-6}{12} = \frac{-14}{12} \neq 0$

∴ They are not perpendicular

∴ They are skew lines.

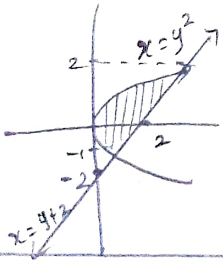
$$\delta = \frac{|(\vec{c} - \vec{a}) \cdot \vec{b} \times \vec{d}|}{|\vec{b} \times \vec{d}|}$$

$$= \frac{\left|-\frac{14}{12}\right|}{\sqrt{\left(\frac{1}{6}\right)^2 + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{2}\right)^2}} = \frac{\frac{14}{12}}{\sqrt{\frac{49}{144}}}$$

$= \frac{14}{12} \times \frac{12}{7} = 2$ units.

b) $y = x - 2$

x	0	2
y	-2	0



$y^2 = y+2 \Rightarrow y^2 - y - 2 = 0$

$y = 2$ or -1

$$A = \int_{-1}^{2} (y+2) - (y^2)\, dy$$

$$= \left[\frac{y^2}{2} + 2y - \frac{y^3}{3}\right]_{-1}^{2}$$

$= \frac{4}{2} + 4 - \frac{8}{3} - \frac{1}{2} + 2 - \frac{1}{3}$

$= 2 + 4 - \frac{9}{3} + \frac{3}{2} = 6 - 3 + \frac{3}{2}$

$= 3 + \frac{3}{2} = \frac{9}{2}$ sq.units.

47 a) $ax^2 + by^2 = 1$ —(1)

$cx^2 + dy^2 = 1$ —(2)

(1) − (2) $(a-c)x^2 + (b-d)y^2 = 0$ —(3)

$ax^2 + by^2 = 1 \Rightarrow a(2x) + b(2y)\frac{dy}{dx} = 0$ $(\div 2)$

$m_1 = \frac{dy}{dx} = \frac{-ax}{by}$

$cx^2 + dy^2 = 1 \Rightarrow c(2x) + d(2y)\frac{dy}{dx} = 0$ $(\div 2)$

$m_2 = \frac{dy}{dx} = -\frac{cx}{dy}$

$m_1 \times m_2 = -1$ (orthogonally)

$-\frac{ax}{by} \times -\frac{cx}{dy} = -1 \Rightarrow acx^2 = -bdy^2$

$\therefore\ y^2 + bdy^2 = 0$ —(4)

Compare (3) and (4)

$\frac{a-c}{ac} + \frac{b-d}{bd} = 0.$

$\frac{a}{ac} - \frac{c}{ac} + \frac{b}{bd} - \frac{d}{bd} = 0.$

$\frac{1}{c} - \frac{1}{a} + \frac{1}{d} - \frac{1}{b} = 0.$

$\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$

Hence proved.

b) $+_5$

$+_5$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

i) $a +_5 b$ is unique.

hence $+_5$ is binary

ii) $a +_5 b = b +_5 a$

$0 +_5 1 = 1 +_5 0$

$1 = 1$

Symmetrically entries are placed ∴ It has

∴ commutative property.

iii) $(a +_5 b) +_5 c = a +_5 (b +_5 c)$

LHS $(2 +_5 3) +_5 4 = 0 +_5 = 4$

RHS

$2 +_5 (3 +_5 4) = 2 +_5 2$

$= 4$

∴ It has statisfy associative property

iv) $a +_5 e = a$

$1 +_5 e = 1$

$e = 0$

∴ Identity element exist

v) Inverse of 0 is 0,

1 is 4, 2 is 3, 3 is 2,

4 is 1

∴ Inverse also exist.