ST. JOSEPH HR. SEC. SCHOOL

ERAIYUR

CLASS 12 - MATHEMATICS

(2, 3 & 5 MARKS)

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1.APPLICATIONS OF MATRICES AND DETERMINANTS

2 Marks

1. If $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is non-singular, find A^{-1}. **(EG 1.2)**
2. If A is a non-singular matrix of odd order, prove that $|adj\ A|$ is positive. **(EG 1.4)**
3. If A is symmetric, prove that then $adj\ A$ is also symmetric. **(EG 1.7)**
4. Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal. **(EG 1.11)**
5. Find the inverse (if it exists) of $\begin{bmatrix} -2 & 4 \\ 1 & -3 \end{bmatrix}$ **(EX 1.1 - 1)**
6. If $adjA=\begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1}. **(EG 1.6)**
7. Verify the property $(A^T)^{-1}=(A^{-1})^T$ with $A=\begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$. **(EG 1.8)**
8. Find the rank of the $(i)\ \begin{bmatrix} 2 & -4 \\ -1 & 2 \end{bmatrix}$ $(ii)\ \begin{bmatrix} -1 & 3 \\ 4 & -7 \\ 3 & -4 \end{bmatrix}$ **(EX 1.2 - 1)**
9. Find the rank of the matrices by minor method: $iii)\ \begin{bmatrix} 1 & -2 & -10 \\ 3 & -6 & -31 \end{bmatrix}$ **(EX 1.2 - 1)**
10. Solve using matrix inversion method: $5x+2y=3, 3x+2y=5$. **(EG 1.22)**
11. Solve the systems by Cramer's rule: $(ii)\ \frac{3}{x}+2y=12, \frac{2}{x}+3y=13$ **(EX 1.4 - 1)**
12. Solve the systems by Cramer's rule: $5x-2y+16=0,\quad x+3y-7=0$ **(EX 1.4 - 1)**

3 Marks

1. Find a matrix A if $adjA=\begin{bmatrix} 7 & 7 & -7 \\ -1 & 11 & 7 \\ 11 & 5 & 7 \end{bmatrix}$. **(EG 1.5)**
2. Verify $(AB)^{-1}=B^{-1}A^{-1}$ with $A=\begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}, B=\begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$. **(EG 1.9)**
3. If $=\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, find x and y such that $A^2+xA+yI_2=0_2$ .Hence, find A^{-1}. **(EG 1.10)**
4. If $A=\frac{1}{7}\begin{bmatrix} 6 & -3 & a \\ b & -2 & 6 \\ 2 & c & 3 \end{bmatrix}$ is orthogonal, find a, b and c, and hence A^{-1}. **(EG 1.12)**
5. If $F(\alpha)=\begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}$, show that $[F(\alpha)]^{-1}=F(-\alpha)$. **(EX 1.1 -3)**
6. If $A=\frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$, prove that $A^{-1}=A^T$. **(EX 1.1 -5)**
7. If $A=\begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, verify that $A(adj\ A)=(adj\ A)A=|A|I_2$. **(EX 1.1 -6)**
8. If $A=\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and $B=\begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$, verify that $(AB)^{-1}=B^{-1}A^{-1}$. **(EX 1.1 -7)**

9. Find $adj(adj\ A)$ if $(adj\ A) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$. **(EX 1.1 -10)**
10. $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, show that $A^T A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$. **(EX 1.1 -11)**
11. Find the matrix A for which $A\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$ **(EX 1.1 -12)**
12. Given $A = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$ find a matrix X such that $AXB = C$. **(EX 1.1 - 13)**
13. If $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, show that $A^{-1} = \frac{1}{2}(A^2 - 3I)$. **(EX 1.1 -14)**
14. Decrypt the received encoded message $[2 \quad -3][20 \quad 4]$ with the encryption matrix $\begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}$ and the decryption matrix as its inverse, where the system of codes are described by the numbers $1 - 26$ to the letters $A - Z$ respectively, and the number 0 to a blank space. **(EX 1.1 -15)**
15. Reduce the matrix $\begin{bmatrix} 0 & 3 & 1 & 6 \\ -1 & 0 & 2 & 5 \\ 4 & 2 & 0 & 0 \end{bmatrix}$ to row-echelon form. **(EG 1.14)**
16. Find the rank of the matrix $\begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$ by reducing it to an echelon form. **(EG 1.18)**
17. Find the inverse of the non-singular matrix $A = \begin{bmatrix} 0 & 5 \\ -1 & 6 \end{bmatrix}$ by Gauss-Jordan method. **(EG 1.20)**
18. In a competetive examination, one mark is awarded for every correct answer while 14 mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly? (Use Cramer's rule to solve the problem). **(EX 1.4 - 2)**
19. Solve the following system: $x + 2y + 3z = 0, 3x + 4y + 4z = 0,\ 7x + 10y + 12z = 0$. **(EG 1.35)**
20. Solve the system: $x + y - 2z = 0, 2x - 3y + z = 0, 3x - 7y + 10z = 0,\ 6x - 9y + 10z = 0$. **(EG 1.37)**
21. Determine the values of λ for which the following system of equations $(3\lambda - 8)x + 3y + 3z = 0, 3x + (3\lambda - 8)y + 3z = 0,\ 3x + 3y + (3\lambda - 8)z = 0$. has a non-trivial solution. **(EG 1.38)**
22. In a competitive examination, one mark is awarded for every correct answer while 14mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly ? (Use Cramer's rule to solve the problem). **(EX 1.4 - 2)**
23. A chemist has one solution which is 50% acid and another solution which is 25% acid. How much each should be mixed to make 10 litres of a 40% acid solution? (Use Cramer's rule to solve the problem). **(EX 1.4 - 3)**
24. A fish tank can be filled in 10 minutes using both pumps A and B simultaneously. However, pump B can pump water in or out at the same rate. If pump B is inadvertently run in reverse, then the tank will be filled in 30 minutes. How long would it take each pump to fill the tank by itself? (Use Cramer's rule to solve the problem). **(EX 1.4 - 4)**
25. Find the condition on a, b and c so that the following system of linear equations has one parameter family of solutions: $x + y + z = a,\ x + 2y + 3z = b, 3x + 5y + 7z = c$. **(EG 1.33)**
26. Test for consistency $(iv) 2x - y + z = 2, 6x - 3y + 3z = 6, 4x - 2y + 2z = 4$. **(EX 1.6 - 1)**

5 MARKS

1. If $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$,verify that $A(adj\ A) = (adj\ A)A = |A|I_3$ **(Eg.1.1)**
2. If $= \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, find x and y such that $A^2 + xA + yI_2 = 0_2$. Hence, find A^{-1}. **(Eg.1.10)**
3. If $A = \frac{1}{7}\begin{bmatrix} 6 & -3 & a \\ b & -2 & 6 \\ 2 & c & 3 \end{bmatrix}$ is orthogonal, find $a, b$and c, and hence A^{-1}. **(Eg.1.12)**
4. Solve the following system of equations, using matrix inversion method:
   $2x_1 + 3x_2 + 3x_3 = 5, x_1 - 2x_2 + x_3 = -4, 3x_1 - x_2 - 2x_3 = 3$. **(Eg.1.23)**
5. If $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ find the products AB and BA and hence solve the system of equations $x - y + z = 4,\quad x - 2y - 2z = 9, 2x + y + 3z = 1$. **(Eg.1.24)**
6. Solve the system of linear equations by matrix inversion method:
   $(iv)\ x + y + z - 2 = 0, 6x - 4y + 5z - 31 = 0, 5x + 2y + 2z = 13$ **(EX 1.3 - 1)**
7. The prices of three commodities A, B and C are ₹xy , and z per units respectively. A person P purchases 4 units of B and sells two units of A and 5 units of C . Person Q purchases 2 units of C and sells 3 units of A and one unit of B. Person Rpurchases one unit of $A$and sells 3 unit of $B$and one unit of C . In the process, P, Q and R earn ₹15,000, ₹1,000 and ₹4,000 respectively. Find the prices per unit of A, B and C. (Use matrix inversion method to solve the problem.) **(EX 1.3 - 5)**
8. Solve, by Cramer's rule,system of equations
   $x_1 - x_2 = 3, 2x_1 + 3x_2 + 4x_3 = 17, x_2 + 2x_3 = 7$. **(Eg.1.25)**
9. Solve the systems of linear equations by Cramer's rule: $3x + 3y - z = 11, 2x - y + 2z = 9, 4x + 3y + 2z = 25$ **(Ex. 1.4 1(iii)**
10. Solve $\frac{3}{x} - \frac{4}{y} - \frac{2}{z} - 1 = 0, \frac{1}{x} + \frac{2}{y} + \frac{1}{z} - 2 = 0, \frac{2}{x} - \frac{5}{y} - \frac{4}{z} + 1 = 0$**(Ex. 1.4 1(iv)**
11. Solve the system of linear equations, by Gaussian elimination method
    $4x + 3y + 6z = 25, x + 5y + 7z = 13, 2x + 9y + z = 1$. **(Eg.1.27)**
12. The upward speed $v(t)$ of a rocket at time t is approximated by $v(t) = at^2 + bt + c, 0 \le t \le 100$,where a, b and c are constants. It has been found that the speed at times $t = 3, t = 6$ and $t = 9$ seconds are respectively 64, 133, and 208 miles per second respectively. Find the speed at time $t = 15$ seconds. (Use Gaussian elimination method.) **(EG 1.28)**
13. If $ax^2 + bx + c$is divided by $x + 3, x - 5$ and $x - 1$,the remainders are 21,61 and 9 respectively. Find a, b and c. (Use Gaussian elimination method.) **(Ex. 1.5- 2)**
14. A boy is walking along the path $y = ax^2 + bx + c$ through the points $(-6,8), (-2,-12)$ and $(3,8)$. He wants to meet his friend at $P(7,60)$. Will he meet his friend? (Use Gaussian elimination method.) **(Ex. 1.5- 4)**
15. Test for consistency of the following system of linear equations and if possible solve: $x + 2y - z = 3, 3x - y + 2z = 1, x - 2y + 3z = 3,\ x - y + z + 1 = 0$. **(EG 1.29)**
16. Test for consistency of the following system of linear equations and if possible solve: $4x - 2y + 6z = 8, x + y - 3z = -1, 15x - 3y + 9z = 21$. **(EG 1.30)**
17. Investigate for what values of λ and μ the system of linear equations $x + 2y + z = 7, x + y + \lambda z = \mu, x + 3y - 5z = 5$has (i) no solution (ii) a unique solution (iii) an infinite number of solutions. **(EG 1.34)**

18. Find the value of k for which the equations $kx - 2y + z = 1, x - 2ky + z = -2, x - 2y + kz = 1$ have (i) no solution (ii) unique solution (iii) infinitely many solution **(EX 1.6 - 2)**
19. Investigate the values of λ and μ the system of linear equations $2x + 3y + 5z = 9, 7x + 3y - 5z = 8, 2x + 3y + \lambda z = \mu$,have (i) no solution (ii) a unique solution (iii) an infinite number of solutions. **(EX 1.6 - 3)**
20. By using Gaussian elimination method, balance the chemical reaction equation: $C_5H_8 + O_2 \rightarrow CO_2 + H_2O$. (The above is the reaction that is taking place in the burning of organic compound called isoprene.) **(EG 1.39)**
21. Determine the values of λ for which the following system of equations $x + y + 3z = 0, 4x + 3y + \lambda z = 0, 2x + y + 2z = 0$, has (i)a unique solution (ii) a non-trivial solution. **(EX 1.7 - 2)**
22. By using Gaussian elimination method, balance the chemical reaction equation: $C_2H_6 + O_2 \rightarrow H_2O + CO_2$. **(EX 1.7 - 3)**

2.COMPLEX NUMBERS.

2 Marks

1. Simplify $(iii) i^{-1924} + i^{2018} (iv) \sum_{n=1}^{102} i^n \; (v)\; i.i^2.i^3 \ldots\ldots i^{40}$. **(EG 2.1)**
2. Simplify (i) $i^{59} + \frac{1}{i^{59}}$ **(EX 2.1 - 3)** (ii) $\sum_{n=1}^{10} i^{n+50}$ **(EX 2.1 - 6)**
3. Evaluate if $z = 5 - 2i$ and $w = -1 + 3i$ $(i) z - iw$ $(ii) z^2 + 2zw + w^2$ **(EX 2.2 - 1)**
4. Given the complex number $z = 2 + 3i$, represent the complex numbers in Argand diagram. $(ii) z, -iz$ and $z - iz$. **(EX 2.2 - 2)**
5. If $z_1 = 2 + 5i$ find the additive and multiplicative inverse of z_1. **(EX 2.3 - 3)**
6. For any two complex numbers z_1 and z_2 prove that $\overline{z_1 + z_2} = \bar{z_1} + \bar{z_2}$ **(Property)**
7. z is purely imaginary if and only if $z = \bar{z}$. **(Property)**
8. Write $\frac{3+4i}{5-12i}$ in the $x + iy$form, hence find its real and imaginary parts. **(EG 2.3)**
9. Simplify $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$. **(EG 2.4)**
10. If $\frac{z+3}{z-5i} = \frac{1+4i}{2}$, find the complex number z. **(EG 2.5)**
11. Show that (i) $(2 + i\sqrt{3})^{10} + (2 - i\sqrt{3})^{10}$is real **(EG 2.8)**.
12. $\frac{z+3}{z-5i} = \frac{1+4i}{2}$, find the complex number z. **(EG 2.5)**
13. If $z = x + iy$, find the following in rectangular form. (i) $Re\left(\frac{1}{z}\right)$ $(ii) Re(i\bar{z})$ (iii) $Im(3z + 4\bar{z} - 4i)$ **(EX 2.4 - 2)**
14. Find the following (i) $\left|\frac{2+i}{-1+2i}\right|$ (ii) $\left|\overline{(1+i)}(2+3i)(4i-3)\right|$ (iii) $\left|\frac{i(2+i)^3}{(1+i)^2}\right|$ **(EG 2.10)**
15. Which one of the points $i, -2 + i$,and 3 is farthest from the origin? **(EG 2.11)**
16. Show that the equation $z^2 = \bar{z}$ has four solutions. **(EG 2.16)**
17. Find the square root of $6 - 8i$. **(EG 2.17)**
18. If the area of the triangle formed by the vertices z, iz and $z + iz$is 50square units, find the value of $|z|$. **(EX 2.5 - 8)**
19. Show that $|3z - 5 + i|$ represents a circle, and, find its centre and radius. **(EG 2.19)**

20. Show that the following equations represent a circle, and, find its centre and radius. $|z-2-i| = 3$ **(EG 2.20)**
21. Obtain the Cartesian form of the locus of z in each of the following cases. $(i)\ |z| = |z-i|$ $(ii)\ |2z-3-i| = 3$. **(EG 2.21)**
22. If $\omega \neq 1$ is a cube root of unity, then show that $\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2} + \frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2} = -1$. **(EX 2.8 - 1)**
23. Show that $\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5 = -\sqrt{3}$. **(EX 2.8 - 2)**
24. If $\omega \neq 1$ is a cube root of unity, show that
    $(i)(1-\omega+\omega^2)^6(1+\omega-\omega^2)^6 = 128$
    $(ii)(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)\ldots\ldots(1+\omega^{2^{11}}) = 1$. **(EX 2.8 - 8)**

3 Marks

1. Find the value of the real numbers x and y, if the complex number $(2+i)x+(1-i)y+2i-3$ and $x+(-1+2i)y+1+i$ are equal. **(EG 2.2)**
2. For any two complex numbers z_1 and z_2 prove that $\overline{z_1+z_2} = \bar{z_1}+\bar{z_2}$ **(Property)**
3. The complex numbers u, v and w are related by $\frac{1}{u} = \frac{1}{v}+\frac{1}{w}$. If $v = 3-4i$ and $w = 4+3i$, find u in rectangular form. **(EX 2.4 - 4)**
4. Prove the following properties:
   (i) z is real if and only if $z = \bar{z}$ (ii) $Re(z) = \frac{z+\bar{z}}{2}$ and $Im(z) = \frac{z-\bar{z}}{2i}$ **(EX 2.4 - 5)**
5. Find the least value of the positive integer n for which $\left(\sqrt{3}+i\right)^n$ (i)real(ii) purely imaginary. **(EX 2.4 - 6)**
6. **To Prove Triangle inequality:** For any two complex numbers z_1 and z_2, prove that $|z_1+z_2| \leq |z_1|+|z_2|$. **(property)**
7. If z_1, z_2 and z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = |z_1+z_2+z_3| = 1$, find the value of $\left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right|$. **(EG 2.12)**
8. If $|z| = 2$ show that $3 \leq |z+3+4i| \leq 7$. **(EG 2.13)**
9. Show that the points $1, \frac{-1}{2}+i\frac{\sqrt{3}}{2}$ and $\frac{-1}{2}-i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle. **(EG 2.14)**
10. For any two complex numbers z_1 and z_2, such that $|z_1| = |z_2| = 1$ and $z_1z_2 \neq -1$, then show that $\frac{z_1+z_2}{1+z_1z_2}$ is a real number. **(EX 2.5 - 2)**
11. Which one of the points $10-8i$, $11+6i$ is closest to $1+i$. **(EX 2.5 - 3)**
12. If $|z| = 1$, show that $2 \leq |z^2-3| \leq 4$. **(EX 2.5 - 5)**
13. Show that the equation $z^3+2\bar{z} = 0$ has five solutions. **(EX 2.5 - 9)**
14. Given the complex number $z = 3+2i$, represent the complex numbers z, iz and $z+iz$ in one Argand diagram. Show that these complex numbers form the vertices of an isosceles right triangle. **(EG 2.18)**
15. If $z = x+iy$ is a complex number such that $\left|\frac{z-4i}{z+4i}\right| = 1$. Show that locus of z is real axis **(EX 2.6 - 1)**
16. Obtain the Cartesian form of the locus of $z = x+iy$ in each of the following cases: $(i)[Re(iz)]^2 = 3$ $(ii) Im[(1-i)z+1] = 0$ **(EX 2.6 - 3)**

17. Obtain the Cartesian equation for the locus of $z = x + iy$ in each of the following cases: $(ii)|z-4|^2 - |z-1|^2 = 16$. **(EX 2.6 - 5)**
18. Find the principal argument $Arg\ z$, when $z = \frac{-2}{1+i\sqrt{3}}$. **(EG 2.24)**
19. Find the quotient $\frac{2\left(\cos\frac{9\pi}{4}+i\sin\frac{9\pi}{4}\right)}{4\left(\cos\left(\frac{-3\pi}{2}\right)+i\sin\left(\frac{-3\pi}{2}\right)\right)}$ in rectangular form. **(EG 2.26)**
20. Write in polar form of the following complex numbers
    $(i)\ 2 + i2\sqrt{3}$ $(iv)\ \frac{1-i}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$. **(EX 2.7 - 1)**
21. If $(x_1+iy_1)(x_2+iy_2)(x_3+iy_3)\dots\dots(x_n+iy_n) = a+ib$, show that $(i)(x_1^2+y_1^2)(x_2^2+y_2^2)(x_3^2+y_3^2)\dots\dots(x_n^2+y_n^2) = a^2+b^2$ $(ii)\sum_{r=1}^{n}\tan^{-1}\left(\frac{y_r}{x_r}\right) = \tan^{-1}\left(\frac{b}{a}\right) + 2k\pi, k \in Z$. **(EX 2.7 - 3)**
22. If $\frac{1+z}{1-z} = \cos 2\theta + i\sin 2\theta$, show that $z = i\tan\theta$. **(EX 2.7 - 4)**
23. If $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$, then show that $(i)\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha+\beta+\gamma)$ and $(ii)\ \sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha+\beta+\gamma)$. **(EX 2.7 - 5)**
24. If $z = (\cos\theta + i\sin\theta)$, show that $z^n + \frac{1}{z^n} = 2\cos n\theta$ and $z^n - \frac{1}{z^n} = 2i\sin n\theta$. **(EG 2.28)**
25. Simplify $\left(\sin\frac{\pi}{6} + i\cos\frac{\pi}{6}\right)^{18}$ **(EG 2.29)**
26. Simplify $\left(\frac{1+\cos 2\theta+i\sin 2\theta}{1+\cos 2\theta-i\sin 2\theta}\right)^{30}$. **(EG 2.30)**
27. Simplify $(i)\ (1+i)^n$ $(ii)\ (-\sqrt{3}+3i)^{31}$. **(EG 2.31)**
28. Find the value of $\left(\frac{1+\sin\frac{\pi}{10}+i\cos\frac{\pi}{10}}{1+\sin\frac{\pi}{10}-i\cos\frac{\pi}{10}}\right)^{10}$. **(EX 2.8 - 3)**
29. Find the value of $\sum_{k=1}^{8}\left(\cos\frac{2k\pi}{9} + i\sin\frac{2k\pi}{9}\right)$. **(EX 2.8 - 7)**
30. If $z = 2 - 2i$, find the rotation of z by θ radians in the counter clockwise direction about the origin when
    $(i)\ \theta = \frac{\pi}{3}$ $(ii)\ \theta = \frac{2\pi}{3}$ $(iii)\ \theta = \frac{3\pi}{2}$. **(EX 2.8 - 9)**

<u>5 Marks</u>

1. Show that $(i)\ (2+i\sqrt{3})^{10} + (2-i\sqrt{3})^{10}$ is real and $(ii)\ \left(\frac{19+9i}{5-3i}\right)^{15} - \left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary. **(Eg.2.8)**
2. Show that $(i)\ (2+i\sqrt{3})^{10} + (2-i\sqrt{3})^{10}$ is real and $-\left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary. **(Ex.2.4 - 7)**
3. Show that the points $1, \frac{-1}{2} + i\frac{\sqrt{3}}{2}$ and $\frac{-1}{2} - i\frac{\sqrt{3}}{2}$ are vertices of an equilateral triangle. **(Eg.2.14)**
4. Let z_1, z_2 and z_3 be complex numbers such that $|z_1| = |z_2| = |z_3| = r > 0$ and $z_1+z_2+z_3 \neq 0$, P.T. $\left|\frac{z_1z_2+z_2z_3+z_3z_1}{z_1+z_2+z_3}\right| = r$. **(Eg.2.15)**
5. If $z = x + iy$ is a complex number such that $Im\left(\frac{2z+1}{iz+1}\right) = 0$. Show that the locus of z is $2x^2 + 2y^2 + x - 2y = 0$. **(Ex.2.6 - 2)**
6. If $z = x + iy$ and $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$, then show that $x^2 + y^2 = 1$. **(EG 2.27)**
7. If $z = x + iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, then show that $x^2 + y^2 + 3x - 3y + 2 = 0$. **(EX 2.7 - 6)**

8. If $2\cos\alpha = x + \frac{1}{x}$ and $2\cos\beta = y + \frac{1}{y}$ show that

   (i) $\frac{x}{y} + \frac{y}{x} = 2\cos(\alpha - \beta)$ (ii) $xy - \frac{1}{xy} = 2i\sin(\alpha + \beta)$

   (iii) $\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i\sin(m\alpha - n\beta)$ (iv) $x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)$. **(EX 2.8 - 4)**
9. Solve the equation $z^3 + 27 = 0$. **(EX 2.8 - 5)**
10. If $\omega \neq 1$ is a cube root of unity, show that the roots of the equation $(z-1)^3 + 8 = 0$ are $-1, 1 - 2\omega, 1 - 2\omega^2$. **(EX 2.8 - 6)**

3. THEORY OF EQUATIONS

2 Marks

1. If α and β are the roots of the quadratic equation $2x^2 - 7x + 13 = 0$, construct a quadratic equation whose roots are α^2 and β^2. **(EG 3.2)**
2. If α, β and γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\sum \frac{1}{\beta\gamma}$ in terms of the coefficients. **(EG 3.3)**
3. Construct a cubic equation with roots 1,1, and -2 **(EX 3.1 - 2)**
4. If α, β, γ and δ are the roots of the polynomial equation $2x^4 + 5x^3 - 7x^2 + 8 = 0$, find a quadratic equation with integer coefficients whose roots are $\alpha + \beta + \gamma + \delta$ and $\alpha\beta\gamma\delta$. **(EX 3.1 - 8)**
5. Formulate into a mathematical problem to find a number such that when its cube root is added to it, the result is 6 . **(EX 3.1 - 11)**
6. A 12 metre tall tree was broken into two parts. It was found that the height of the part which was left standing was the cube root of the length of the part that was cut away. Formulate this into a mathematical problem to find the height of the part which was cut away. **(EX 3.1 - 12)**
7. Find a polynomial equation of minimum degree with rational coefficients, having $2 - \sqrt{3}i$ as a root. **(EG 3.8)**
8. Show that the equation $2x^2 - 6x + 7 = 0$ cannot be satisfied by any real values of x. **(EG 3.11)**
9. If $x^2 + 2(k+2)x + 9k = 0$ has equal roots, find k. **(EG 3.12)**
10. Show that, if p, q, r are rational, the roots of the equation $x^2 - 2px + p^2 - q^2 + 2qr - r^2 = 0$ are rational. **(EG 3.13)**
11. Prove that a line cannot intersect a circle at more than two points. **(EG 3.14)**
12. Prove that a line cannot intersect a circle at more than two points. **(EX 3.2 - 5)**
13. Find a polynomial equation of minimum degree with rational coefficients, having $2 + \sqrt{3}i$ as a root. **(EX 3.2 - 1)**
14. Find a polynomial equation of minimum degree with rational coefficients, having $2i + 3$ as a root. **(EX 3.2 - 3)**
15. Solve the equation $x^4 - 9x^2 + 20 = 0$. **(EG 3.16)**
16. Solve the equation $x^3 - 3x^2 - 33x + 35 = 0$. **(EG 3.17)**
17. Solve the equation $2x^3 + 11x^2 - 9x - 18 = 0$. **(EG 3.18)**
18. Show that the polynomial $9x^9 + 2x^5 - x^4 - 7x^2 + 2$ has at least six imaginary roots. **(EG 3.30)**

3 Marks

1. Find the sum of the squares of the roots of $ax^4 + bx^3 + cx^2 + dx + e = 0$. **(EG 3.4)**
2. Find the condition that the roots of $x^3 + ax^2 + bx + c = 0$, are in the ratio $p:q:r$. **(EG 3.5)**

3. Form the equation whose roots are the squares of the roots of the cubic equation $x^3 + ax^2 + bx + c = 0$. **(EG 3.6)**
4. If p is real, discuss the nature of the roots of the equation $4x^2 + 4px + p + 2 = 0$, in terms of p. **(EG 3.7)**
5. If α, β and γ are the roots of the polynomial equation $ax^3 + bx^2 + cx + d = 0$, find the value of $\sum \frac{\alpha}{\beta\gamma}$ in terms of the coefficients. **(EX 3.1 - 7)**
6. If p and q are the roots of the $lx^2 + nx + n = 0$, show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$. **(EX 3.1 - 9)**
7. If the equations $x^2 + px + q = 0$, and $x^2 + p'x + q' = 0$, have a common root, show that it must be equal to $\frac{pq' - p'q}{q - q'}$ or $\frac{q - q'}{p' - p}$. **(EX 3.1 - 10)**
8. Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root. **(EG 3.10)**
9. Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5} - \sqrt{3}$ as a root. **(EX 3.2 - 4)**
10. Solve the cubic equation : $2x^3 - x^2 - 18x + 9 = 0$ if sum of two of its roots vanishes. **(EX 3.3 - 1)**
11. Solve the equation $9x^3 - 36x^2 + 44x - 16 = 0$ if the roots form an arithmetic progression. **(EX 3.3 - 2)**
12. Solve the equation $3x^3 - 26x^2 + 52x - 24 = 0$ if its roots form a geometric progression. **(EX 3.3 - 3)**
13. Determine k and solve the equation $2x^3 - 6x^2 + 3x + k = 0$ if one of its roots is twice the sum of the other two roots. **(EX 3.3 - 4)**
14. Solve the equation $x^3 - 5x^2 - 4x + 20 = 0$. **(EG 3.25)**
15. Solve the equation $7x^3 - 43x^2 - 43x + 7 = 0$ **(EG 3.27)**
16. Solve the following equations (i) $\sin^2 x - 5 \sin x + 4 = 0$ **(EX 3.5 - 1)**
17. Solve : $8x^{\frac{3}{2n}} - 8x^{\frac{-3}{2n}} = 63$. **(EX 3.5 - 3)**
18. Solve : $2\sqrt{\frac{x}{a}} + 3\sqrt{\frac{a}{x}} = \frac{b}{a} + \frac{6a}{b}$. **(EX 3.5 - 4)**
19. Find all real numbers satisfying $4^x - 3(2^{x+2}) + 2^5 = 0$. **(EX 3.5 - 6)**
20. Discuss the maximum possible number of positive and negative roots of the polynomial equations $x^2 - 5x + 6$ and $x^2 - 5x + 16$. Also draw rough sketch of the graphs. **(EX 3.6 - 2)**

5 Marks

1. If $2 + i$ and $3 - \sqrt{2}$ are roots of the equation $x^6 - 13x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140 = 0$, find all roots. **(EG 3.15)**
2. Find all zeros of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$, if it is known that $1 + 2i$ and $\sqrt{3}$ are two of its zeros. **(EX 3.3 - 5)**
3. Solve the equation $(x - 2)(x - 7)(x - 3)(x + 2) + 19 = 0$. **(EG 3.23)**
4. Solve the equation $(2x - 3)(6x - 1)(3x - 2)$**(EG 3.24)**
5. Solve the following equation: $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$. **(EG 3.28)**
6. Solve the equations (i) $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$, **(EX 3.5 - 5)**
7. Solve the equations $(ii) x^4 + 3x^3 - 3x - 1 = 0$. **(EX 3.5 - 5)**
8. Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution. **(EX 3.5 - 7)**

CHAPTER 4
INVERSE TRIGONOMETRIC FUNCTIONS

2 Marks

1. Find the principal value of $\sin^{-1}(2)$, if it exists **(EG 4.2)**
2. Find the principal value of $\sin^{-1}\left(\sin\left(\frac{5\pi}{6}\right)\right)$. **(EG 4.3)**
3. Find all the values of x such that $-10\pi \le x \le 10\pi$ and $\sin x = 0$ **(EX 4.1 - 1)**
4. Find the period and amplitude of
   $(i)\ y = \sin 7x\ \ (ii)\ y = -\sin\left(\frac{1}{3}x\right)\ \ (iii) y = 4\sin(-2x)$. **(EX 4.1 - 2)**
5. Sketch the graph of $y = sin\left(\frac{1}{3}x\right)$ for $0 \le x < 6\pi$. **(EX 4.1 - 3)**
6. For what value of x does $\sin x = \sin^{-1} x$? **(EX 4.1 - 5)**
7. Find the value of $\sin^{-1}\left(sin\frac{5\pi}{9}cos\frac{\pi}{9} + cos\frac{5\pi}{9}sin\frac{\pi}{9}\right)$ **(EX 4.1 - 7)**
8. Find $(iii)\ cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$ **(EG 4.6)**
9. State the reason for $cos^{-1}\left(\cos\left(\frac{-\pi}{6}\right)\right) \neq \frac{-\pi}{6}$ **(EX 4.2 - 2)**
10. Is $cos^{-1}(-x) = \pi - cos^{-1}(x)$ true? Justify your answer. **(EX 4.2 - 3)**
11. Find the value of $(i)\ 2\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)$ **(EX 4.2 - 5)**
12. Find the domain of $(ii)\ g(x) = \sin^{-1} x + \cos^{-1} x$. **(EX 4.2 - 6)**
13. Find the value of $(i)\ \cos\left(cos^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{4}{5}\right)\right)$ **(EX 4.2 - 8)**
14. Find the value of $(ii)\ tan^{-1}\left(\tan\left(\frac{-\pi}{6}\right)\right)$. **(EX 4.3 - 2)**
15. Find the value of $(i)\ tan\left(tan^{-1}\left(\frac{7\pi}{4}\right)\right)$ $(ii)\ tan(tan^{-1}(1947))$ (iii) **(EX 4.3 - 3)**
16. Find the value of $(i) \tan\left(\cos^{-1}\left(\frac{1}{2}\right) - \sin^{-1}\left(\frac{-1}{2}\right)\right)$
17. Find the value of $sec^{-1}\left(\frac{-2\sqrt{3}}{3}\right)$. **(EG 4.13)**
18. If $\cot^{-1}\left(\frac{1}{7}\right) = \theta$, find the value of $\cos\theta$. **(EG 4.14)**
19. Find the value of $(i) \tan^{-1}(\sqrt{3}) - sec^{-1}(-2)$ **(EX 4.4 - 2)**
20. Simplify $(iv)\ \sin^{-1}(\sin 10)$ **(EG 4.17)**
21. Find, if it exists. If not, give the reason for non-existence. $(iii)\ \sin^{-1}(\sin 5)$ **(EX 4.5 - 1)**

3.Marks

1. Find the domain of $\sin^{-1}(2 - 3x^2)$ **(EG 4.4)**
2. Find the domain of $cos^{-1}\left(\frac{2+\sin x}{3}\right)$ **(EG 4.7)**
3. Find the domain of the following $g(x) = 2\sin^{-1}(2x - 1) - \frac{\pi}{4}$. **(EX 4.2 - 6)**
4. For what value of x, the inequality $\frac{\pi}{2} < \cos^{-1}(3x - 1) < \pi$ holds? **(EX 4.2 - 7)**
5. Find the value of $(ii)\ cos^{-1}\left(\cos\left(\frac{4\pi}{3}\right)\right) + cos^{-1}\left(\cos\left(\frac{5\pi}{4}\right)\right)$ **(EX 4.2 - 8)**
6. Find the value of $tan^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + sin^{-1}\left(\frac{-1}{2}\right)$ **(EG 4.10)**
7. Prove that $\tan(\sin^{-1} x) = \frac{x}{\sqrt{1-x^2}}, -1 < x < 1$. **(EG 4.11)**
8. Find the domain of the following functions: $tan^{-1}(\sqrt{9 - x^2})$ **(EX 4.3 - 1)**
9. Find the value of $sin\left(tan^{-1}\left(\frac{1}{2}\right) - cos^{-1}\left(\frac{4}{5}\right)\right)$. **(EX 4.3 - 4)**
10. Show that $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \sec^{-1} x, |x| > 1$. **(EG 4.15)**
11. Find the value of $cot^{-1}(1) + \sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) - sec^{-1}(-\sqrt{2})$ **(EX 4.4 - 2)**

12. Prove that $\frac{\pi}{2} \leq \sin^{-1} x + 2\cos^{-1} x \leq \frac{3\pi}{2}$. **(EG 4.16)**
13. Find the value of (ii) $\cos\left(\frac{1}{2}\cos^{-1}\left(\frac{1}{8}\right)\right)$. **(EG 4.18)**
14. Find the value of (iii) $\tan\left(\frac{1}{2}\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \frac{1}{2}\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)\right)$. **(EG 4.18)**
15. Prove that (i) $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}$ **(EG 4.21)**
16. Prove that (ii) $2\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{31}{17}\right)$ **(EG 4.21)**
17. Solve $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1} x$ for $x > 0$. **(EG 4.24)**
18. Solve $\sin^{-1} x > \cos^{-1} x$. **(EG 4.25)**
19. Solve $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$. **(EG 4.28)**
20. Prove that $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$. **(EX 4.5 - 5**

5 Marks

1. Find the domain of (i) $f(x) = \sin^{-1}\left(\frac{|x|-2}{3}\right) + \cos^{-1}\left(\frac{1-|x|}{4}\right)$ **EX. 4.2-6(i)**
2. Find the value of (iii) $\cos\left(\sin^{-1}\left(\frac{4}{5}\right) - \tan^{-1}\left(\frac{3}{4}\right)\right)$ **(EX 4.3 - 4)**
3. Find the value of (ii) $\sin\left(\tan^{-1}\left(\frac{1}{2}\right) - \cos^{-1}\left(\frac{4}{5}\right)\right)$.**(EX 4.3 - 4)**
4. Evaluate $\sin\left[\sin^{-1}\left(\frac{3}{5}\right) + \sec^{-1}\left(\frac{5}{4}\right)\right]$. **(EG 4.20)**
5. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$ and $0 < x, y, z < 1$, show that $x^2 + y^2 + z^2 + 2xyz = 1$. **(EG 4.22)**
6. If $a_1, a_2, a_3, \ldots, a_n$ is an arithmetic progression with common difference d, P.T. $\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + .. + \tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right] = \frac{a_n - a_1}{1+a_1a_n}$. **(EG 4.23)**
7. Solve $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$, if $6x^2 < 1$. **(EG 4.27)**
8. Prove that (ii) $\sin^{-1}\frac{3}{5} + \cos^{-1}\frac{12}{13} = \sin^{-1}\frac{16}{65}$ **(EX 4.5 - 4)**
9. If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, show that $x + y + z = xyz$. **(EX 4.5 - 6)**
10. Prove that $\tan^{-1} x + \tan^{-1}\frac{2x}{1-x^2} = \tan^{-1}\frac{3x - x^3}{1-3x^2}$, $|x| < \frac{1}{\sqrt{3}}$. **(EX 4.5 - 7)**
11. Solve: (i) $\sin^{-1}\frac{5}{x} + \sin^{-1}\frac{12}{x} = \frac{\pi}{2}$
    (ii) $2\tan^{-1} x = \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) - \cos^{-1}\left(\frac{1-b^2}{1+b^2}\right)$, $a > 0, b > 0$. **(EX 4.5 - 9)**
12. Solve: (iii) $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$
    (iv) $\cot^{-1} x - \cot^{-1}(x+2) = \frac{\pi}{12}$, $x > 0$. **(EX 4.5 - 9)**
13. Find the number of solution of the equation $\tan^{-1}(x-1) + \tan^{-1} x + \tan^{-1}(x+1) = \tan^{-1} 3x$. **(EX 4.5 - 10)**

CHAPTER 5
TWO DIMENSIONAL ANALYTICAL GEOMETRY-II

1. Find the general equation of a circle with centre $(-3, -4)$ and radius 3 units. **(EG 5.1)**
2. Determine whether $x + y - 1 = 0$ is the equation of a diameter of the circle $x^2 + y^2 - 6x + 4y + c = 0$, for all possible values of c. **(EG 5.3)**
3. Find the general equation of the circle whose diameter is the line segment joining the points $(-4, -2)$ and $(1, 1)$. **(EG 5.4)**
4. Examine the position of the point $(2, 3)$ with respect to the circle $x^2 + y^2 - 6x - 8y + 12 = 0$. **(EG 5.5)**

5. A circle of radius 3 units touches both the axes. Find the equations of all possible circles formed in the general form. **(EG 5.8)**
6. If $y = 4x + c$ is a tangent to the circle $x^2 + y^2 = 9$, find c. **(EG 5.12)**
7. Obtain the equation of the circles with radius 5 cm and touching x −axis at the origin in general form. **(EX 5.1 - 1)**
8. Find the equation of the circle with centre $(2, -1)$ and passing through the point $(3,6)$ in standard form. **(EX 5.1 - 2)**
9. Obtain the equation of the circle for which $(3,4)$ and $(2, -7)$ are the ends of a diameter. **(EX 5.1 - 5)**
10. If $y = 2\sqrt{2}x + c$ is a tangent to the circle $x^2 + y^2 = 16$, find the value of c. **(EX 5.1 - 8)**
11. Find the equation of the tangent and normal to the circle $x^2 + y^2 - 6x + 6y - 8 = 0$ at $(2,2)$. **(EX 5.1 - 9)**
12. Find centre and radius of the following circles. $x^2 + (y + 2)^2 = 0$. **(EX 5.1 - 11)**
13. Find the length of Latus rectum of the parabola $y^2 = 4ax$. **(EG 5.14)**
14. Find the length of Latus rectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. **(EG 5.15)**
15. Find the equation of the parabola with focus $(-\sqrt{2}, 0)$ and directrix $x = \sqrt{2}$. **(EG 5.14)**
16. Find the equation of the parabola whose vertex is $(5, -2)$ and focus $(2, -2)$. **(EG 5.15)**
17. Find the equation of the parabola with vertex $(-1, -2)$, axis parallel to y − axis and passing through $(3,6)$. **(EG 5.16)**
18. Find the equation of the ellipse with foci $(\pm 2,0)$ vertices $(\pm 3,0)$. **(EG 5.18)**
19. Find the vertices, foci for the hyperbola $9x^2 - 16y^2 = 144$. **(EG 5.23)**
20. Find the equation of the parabola in each of the cases given below:
    (i) focus $(4,0)$ and directrix $x = -4$.
    (ii) passes through $(2, -3)$ and symmetric about y −axis.
    (iii) vertex$(1, -2)$ and focus $(4, -2)$.
    (iv) end points of latus rectum$(4, -8)$ and $(4,8)$. **(EX 5.2 - 1)**
21. Identify the type of the conic for the following equations:
    (1) $16y^2 = -4x^2 + 64$ (2) $x^2 + y^2 = -4x - y - 4$
    (3) $x^2 - 2y = x + 3$ (4) $4x^2 - 9y^2 - 16x + 18y - 29 = 0$ **(EG 5.26)**

3Marks

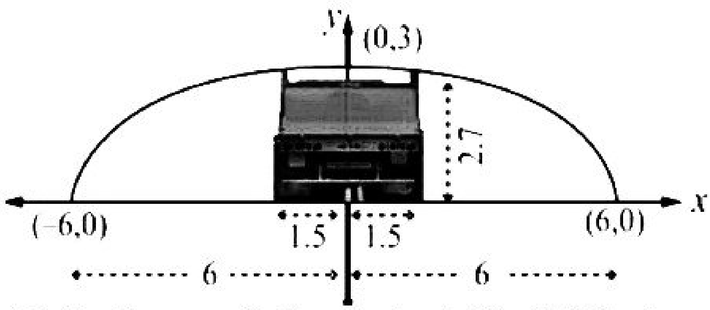
1. Find the equation of the circle described on the chord $3x + y + 5 = 0$ of the circle $x^2 + y^2 = 16$ as diameter. **(EG 5.2)**
2. The line $3x + 4y - 12 = 0$ meets the coordinate axes at A and B. Find the equation of the circle drawn on AB as diameter. **(EG 5.6)**
3. A line $3x + 4y + 10 = 0$ cuts a chord of length 6 units on a circle with centre of the circle $(2,1)$. Find the equation of the circle in general form **(EG 5.7)**
4. Find the centre and radius of the circle $3x^2 + (a + 1)y^2 + 6x - 9y + a + 4 = 0$. **(EG 5.9)**
5. A road bridge over an irrigation canal have two semi circular vents each with a span of $20m$ and the supporting pillars of width $2m$. to write the equations that model the arches. **(EG 5.13)**
6. Find the equation of circles that touch both the axes and pass through $(-4, -2)$ in general form. **(EX 5.1 - 3)**
7. Find the equation of the circle with centre $(2,3)$ and passing through the intersection of the lines $3x - 2y - 1 = 0$ and $4x + y - 27 = 0$. **(EX 5.1 - 4)**
8. Find the equation of the circle through the points $(1,0), (-1,0)$ and $(0,1)$. **(EX 5.1 - 6)**
9. A circle of area 9π square units has two of its diameters along the lines $x + y = 5$ and $x - y = 1$. Find the equation of the circle. **(EX 5.1 - 7)**
10. Determine whether the points $(-2,1), (0,0)$ and $(-4, -3)$ lie outside, on or inside the circle $x^2 + y^2 - 5x + 2y - 5 = 0$. **(EX 5.1 - 10)**

11. If the equation $3x^2 + (3-p)xy + qy^2 - 2px = 8pq$ represents a circle, Find p and q . Also determine the centre and radius of the circle. **(EX 5.1 - 12)**
12. The orbit of Halley's Comet is an ellipse 36.18 .astronomical units long and by 9.12 astronomical units wide. Find its eccentricity. **(EG 5.25)**
13. Find the equation of the ellipse in each of the cases given below:
    (i) foci $(\pm 3,0)$, $e = \frac{1}{2}$.
    (ii) foci $(0, \pm 4)$ and end points of major axis are $(0, \pm 5)$.
    (iii) length of latus rectum 8, eccentricity $= \frac{3}{5}$ and major axis on x –axis.
    (iv) length of latus rectum 4, distance between foci $4\sqrt{2}$ and major axis as y – axis. **(EX 5.2 - 2)**
14. Find the equation of the hyperbola in each of the cases given below:
    (i) foci $(\pm 2,0)$, eccentricity $= \frac{3}{2}$.
    (ii) Centre $(2,1)$, one of the foci $(8,1)$ and corresponding directrix $x = 4$.
    (iii) passing through $(5,-2)$ and length of the transverse axis along x axis and of length 8 units. **(EX 5.2 - 3)**
15. Prove that the length of the latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{2b^2}{a}$ **(EX 5.2 - 6)**
16. Show that the absolute value of difference of the focal distances of any point P on the hyperbola is the length of its transverse axis. **(EX 5.2 - 7)**
17. Find the equations of tangent and normal to the ellipse $x^2 + 4y^2 = 32$ when $\theta = \frac{\pi}{4}$. **(EG 5.28)**
18. Find the equations of the two tangents that can be drawn from $(5,2)$ to the ellipse $2x^2 + 7y^2 = 14$. **(EX 5.4 - 1)**
19. Find the equations of tangents to the hyperbola $\frac{x^2}{16} - \frac{y^2}{64} = 1$ which are parallel to$10x - 3y + 9 = 0$. **(EX 5.4 - 2)**
20. Find the equation of the tangent to the parabola $y^2 = 16x$ perpendicular to $2x + 2y + 3 = 0$. **(EX 5.4 - 4)**
21. Find the equation of the tangent at $t = 2$to the parabola $y^2 = 8x$. (Hint: use parametric form) **(EX 5.4 - 5)**
22. Prove that the point of intersection of the tangents at t_1 and t_2 on the parabola$y^2 = 4ax$ is $[at_1t_2, a(t_1 + t_2)]$. **(EX 5.4 - 7)**
23. If the normal at the point t_1 on the parabola $y^2 = 4ax$ meets the parabola again at the point t_2 then prove that $t_2 = -\left(t_1 + \frac{2}{t_1}\right)$. **(EX 5.4 - 8)**
24. The maximum and minimum distances of the Earth from the Sun respectively are $152 \times 10^6 km$ and $94.5 \times 10^6 km$. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus. **(EG 5.31)**
25. A concrete bridge is designed as a parabolic arch. The road over bridge is $40m$ long and the maximum height of the arch is $15m$. Write the equation of the parabolic arch. **(EG 5.32)**
26. The parabolic communication antenna has a focus at $2m$ distance from the vertex of the antenna. Find the width of the antenna $3m$ from the vertex **(EG 5.33)**
27. The equation $y = \frac{1}{32}x^2$ models cross sections of parabolic mirrors that are used for solar energy. There is a heating tube located at the focus of each parabola; how high is this tube located above the vertex of the parabola? **(EG 5.34)**
28. A search light has a parabolic reflector (has a cross section that forms a bowl). The parabolic bowl is $40cm$ wide from rim to rim and $30cm$ deep. The bulb is located at the focus .(a) What is the equation of the parabola used for reflector?
    (b) How far from the vertex is the bulb to be placed so that the maximum distance covered? **(EG 5.35)**

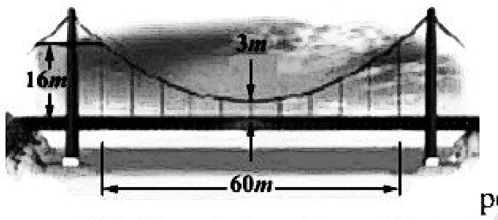
29. An equation of the elliptical part of an optical lens system is $\frac{x^2}{16}+\frac{y^2}{9}=1$. The parabolic part of the system has a focus in common with the right focus of the ellipse .The vertex of the parabola is at the origin and the parabola opens to the right. Determine the equation of the parabola.
30. A room $34m$ long is constructed to be a whispering gallery. The room has an elliptical ceiling, as shown in If the maximum height of the ceiling is $8m$, determine where the foci are located. **(EG 5.36)**
31. If the equation of the ellipse is $\frac{(x-11)^2}{484}+\frac{y^2}{64}=1$ ($x$and y are measured in centimeters) where to the nearest centimeter, should the patient's kidney stone be placed so that the reflected sound hits the kidney stone? **(EG 5.38)**
32. Points A and B are 10 km apart and it is determined from the sound of an explosion heard at those points at different times that the location of the explosion is 6 km closer to A than B . Show that the location of the explosion is restricted to a particular curve and find an equation of it. **(EX 5.5 - 10)**

5 Marks

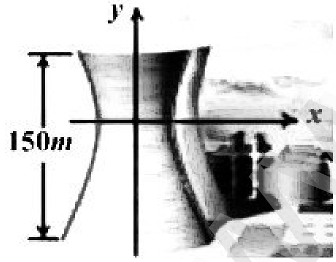
1. Find the equation of the circle passing through the points $(1,1),(2,-1)$ and $(3,2)$. **Eg.5.10**
2. A road bridge over an irrigation canal have two semi circular vents each with a span of $20m$ and the supporting pillars of width $2m$. to write the equations that model the arches. **Eg.5.13**
3. Find the vertex, focus, directrix, and length of the latus rectum of the parabola $x^2-4x-5y-1=0$. **Eg.5.17**
4. Find the vertex, focus, equation of directrix and length of the latus rectum of the $y^2-4y-8x+12=0$ **EX. 5.2-4(ii)**
5. Identify the type of conic and find centre, foci, vertices, and directrices of each of the f $9x^2-y^2-36x-6y+18=0$ **EX. 5.2-8(vi)**
6. Find the foci, vertices and length of major and minor axis of the conic $4x^2+36y^2+40x-288y+532=0$. **Eg.5.20**
7. For the ellipse $4x^2+y^2+24x-2y+21=0$, find the centre, vertices, and the foci. Also prove that the length of latus rectum is 2. **Eg.5.21**
8. Find the centre, foci, and eccentricity of the hyperbola $11x^2-25y^2-44x+50y-256=0$. **Eg.5.24**
9. Find the vertex, focus, equation of directrix and length of the latus rectum of the following: $(iv)\ x^2-2x+8+17=0y$ $(v)\ y^2-4y-8x+12=0$ **(EX 5.2 - 4)**
10. Identify the type of conic and find centre, foci, vertices, and directrices of each of the following: $(v)\ 18x^2+12y^2-144x+48y+120=0$ $(vi)\ 9x^2-y^2-36x-6y+18=0$ **(EX 5.2 - 8)**
11. Show that the line $x-y+4=0$ is a tangent to the ellipse $x^2+3y^2=12$. Also find the coordinates of the point of contact. **(EX 5.4 - 3)**
12. Find the equations of the tangent and normal to hyperbola $12x^2-9y^2=108$ at $\theta=\frac{\pi}{3}$. (Hint: use parametric form) **(EX 5.4 - 6)**
13. A semielliptical archway over a one-way road has a height of $3m$ and a width of $12m$. The truck has a width of $3m$ and a height of $27.m$. Will the truck clear the opening of the archway? $(Fig.5.6)$ **Eg.5.30**



14. A bridge has a parabolic arch that is $10m$ high in the centre and $30m$ wide at the bottom. Find the height of the arch $6m$ from the centre, on either sides. **EX. 5.5-1**
15. A tunnel through a mountain for a four lane highway is to have a elliptical opening. The total width of the highway (not the opening) is to be $16m$, and the height at the edge of the road must be sufficient for a truck $4m$ high to clear if the highest point of the opening is to be $5m$ approximately . How wide must the opening be? **EX. 5.5-2**
16. At a water fountain, water attains a maximum height of $4m$ at horizontal distance of $0.5\ m$ from its origin. If the path of water is a parabola, find the height of water at a horizontal distance of $0.75\ m$ from the point of origin. **EX. 5.5-3**
17. An engineer designs a satellite dish with a parabolic cross section. The dish is $5m$ wide at the opening, and the focus is placed $1.2\ m$ from the vertex (a) Position a coordinate system with the origin at the vertex and the x −axis on the parabola's axis of symmetry and find an equation of the parabola. (b) Find the depth of the satellite dish at the vertex. **EX. 5.5-4**
18. Parabolic cable of a $60m$ portion of the roadbed of a suspension bridge are



positioned as shown below. Vertical Cables are to be spaced every $6m$ along this portion of the roadbed. Calculate the lengths of first two of these vertical cables from the vertex. **EX. 5.5-5**
19. Cross section of a Nuclear cooling tower is in the shape of a hyperbola with equation $\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$.



The tower is $150\ m$ tall and the distance from the top of the tower to the centre of the hyperbola is half the distance from the base of the tower to the centre of the hyperbola. Find the diameter of the top and base of the tower. **EX. 5.5-6**
20. A rod of length $1.2\ m$ moves with its ends always touching the coordinate axes. The locus of a point P on the rod, which is $0.3\ m$ from the end in contact with x −axis is an ellipse. Find the eccentricity. **EX. 5.5-7**
21. Assume that water issuing from the end of a horizontal pipe, $7.5\ m$above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position $2.5\ m$below the line of the pipe, the flow of water has curved outward $3m$ beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground? **Eg.,6.5**

22. On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of $4\,m$ when it is $6\,m$ away from the point of projection. Finally it reaches the ground $12\,m$ away from the starting point. Find the angle of projection. **EX. 5.5-9**

CHAPTER 6
APPLICATIONS OF VECTOR ALGEBRA

2 Marks

1. A particle acted upon by constant forces $2\vec{i}+5\vec{j}+6\vec{k}$ and $-\vec{i}-2\vec{j}-\vec{k}$ is displaced from the point $(4,-3,-2)$ to the point $(6,1,-3)$. Find the total work done by the forces. **(EG 6.9)**
2. Find the magnitude and the direction cosines of the torque about the point $(2,0,-1)$ of a force $2\vec{i}+\vec{j}-\vec{k}$ whose line of action passes through the origin. **(EG 6.11)**
3. If $\vec{a}=-3\vec{i}-\vec{j}+5\vec{k},\ \vec{b}=\vec{i}-2\vec{j}+\vec{k},\ \vec{c}=4\vec{j}-5\vec{k}$, find $\vec{a}.(\vec{b}\times\vec{c})$. **(EG 6.12)**
4. Find the volume of the parallelepiped whose coterminus edges are given by the vectors $2\vec{i}-3\vec{j}+4\vec{k},\ \vec{i}+2\vec{j}-\vec{k}$ and $3\vec{i}-\vec{j}+2\vec{k}$. **(EG 6.13)**
5. Show that the vectors $\vec{i}+2\vec{j}-3\vec{k},\ 2\vec{i}-\vec{j}+2\vec{k}$ and $3\vec{i}+\vec{j}-\vec{k}$ are coplanar. **(EG 6.14)**
6. If $2\vec{i}-\vec{j}+3\vec{k},\ 3\vec{i}+2\vec{j}+\vec{k},\vec{i}+m\vec{j}+4\vec{k}$ are coplanar, find the value of m. **(EG 6.15)**
7. If $\vec{a},\vec{b},\vec{c}$ are three vectors, prove that $[\vec{a}+\vec{c},\vec{a}+\vec{b},\vec{a}+\vec{b}+\vec{c}]=-[\vec{a},\vec{b},\vec{c}]$**(EG 6.18)**
8. Find the volume of the parallelepiped whose coterminous edges are represented by the vectors $-6\vec{i}+14\vec{j}+10\vec{k},\ 14\vec{i}-10\vec{j}-6\vec{k}$ and $2\vec{i}+4\vec{j}-2\vec{k}$. **(EX 6.2 - 2)**
9. Let $\vec{a}=\vec{i}+\vec{j}+\vec{k},\ \vec{b}=\vec{i}$ and $\vec{c}=c_1\vec{i}+c_2\vec{j}+c_3\vec{k}$. If $c_1=1$ and $c_2=2$, find c_3 such that $\vec{a},\vec{b}$ and $\vec{c}$ are coplanar. **(EX 6.2 - 7)**
10. If $\vec{a}=\vec{i}-\vec{k},\ \vec{b}=x\vec{i}+\vec{j}+(1-x)\vec{k},\ \vec{c}=y\vec{i}+x\vec{j}+(1+x-y)\vec{k}$show that $[\vec{a},\vec{b},\vec{c}]$ depends on neither x nor y. **(EX 6.2 - 8)**
11. If the vectors $\vec{a}=a\vec{i}+a\vec{j}+a\vec{k},\ \vec{i}+\vec{k},\ c\vec{i}+c\vec{j}+b\vec{k}$ are coplanar, prove that c is the geometric mean of a and b. **(EX 6.2 - 9)**
12. Let $\vec{a},\vec{b},\vec{c}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, show that $[\vec{a},\vec{b},\vec{c}]^2=\frac{1}{4}|\vec{a}|^2|\vec{b}|^2$ **(EX 6.2 - 10)**
13. For any vector $\vec{a}$, prove that $\vec{i}\times(\vec{a}\times\vec{i})+\vec{j}\times(\vec{a}\times\vec{j})+\vec{k}\times(\vec{a}\times\vec{k})=2\vec{a}$. **(EX 6.3 - 2)**
14. Prove that $[\vec{a}-\vec{b},\vec{b}-\vec{c},\vec{c}-\vec{a}]=0$. **(EX 6.3 - 3)**
15. $\vec{a}=2\vec{i}+3\vec{j}-\vec{k},\ \vec{b}=-\vec{i}+2\vec{j}-4\vec{k},\ \vec{c}=\vec{i}+\vec{j}+\vec{k}$, then find the value of $(\vec{a}\times\vec{b}).(\vec{a}\times\vec{c})$
16. If $\vec{a},\vec{b},\vec{c},\vec{d}$ are coplanar vectors, then show that $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=\vec{0}$ **(EX 6.3 - 6)**
17. If $\vec{a},\vec{b},\vec{c}$ are three unit vectors such that $\vec{b}$ and $\vec{c}$ are non-parallel and $\vec{a}\times(\vec{b}\times\vec{c})=\frac{1}{2}\vec{b}$, find the angle between $\vec{a}$ and $\vec{c}$. **(EX 6.3 - 8)**
18. Show that the lines $\frac{x-1}{4}=\frac{2-y}{6}=\frac{z-4}{12}$ and $\frac{x-3}{-2}=\frac{y-3}{3}=\frac{5-z}{6}$ are parallel. **(EG 6.32)**
19. Find the angle between the following lines.
    $(i)\ \vec{r}=(4\vec{i}-\vec{j})+t(\vec{i}+2\vec{j}-2\vec{k}),\ \vec{r}=(\vec{i}-2\vec{j}+4\vec{k})+s(-\vec{i}-2\vec{j}+2\vec{k})$. $(ii)\ \frac{x+4}{3}=\frac{y-7}{4}=\frac{z+5}{5}$, $\vec{r}=4\vec{k}+t(2\vec{i}+\vec{j}+\vec{k})$
    $(iii)\ 2x=3y=-z$ and $6x=-y=-4z$. **(EX 6.4 - 5)**
20. The vertices of ΔABC are $A(7,2,1),\ B(6,0,3)$, and $C(4,2,4)$. Find $\angle ABC$. **(EX 6.4 - 6)**
21. Show that the points $(2,3,4),(-1,4,5)$ –and $(8,1,2)$ are collinear. **(EX 6.4 - 9)**
22. Fnid the point of intersection of the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$. **(EG 6.33)**
23. If the two lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-m}{2}=z$ intersect at a point, find the value of m. **(EX 6.5 - 3)**

24. If the cartesian equation of a plane is $3x - 4y - 3z = -8$, find the vector equation of the plane in the standard form. **(EG 6.39)**
25. Verify whether the line $\frac{x-3}{-4} = \frac{y-4}{-7} = \frac{z+3}{12}$ lies in the plane $5x - y + z = 8$. **(EG 6.45)**
26. Find the acute angle between the planes $\vec{r}.(2\vec{i} + 2\vec{j} + 2\vec{k}) = 11$ and $4x - 2y + 2z = 15$. **(EG 6.47)**
27. Find the distance of a point $(2,5,-3)$ from the plane $\vec{r}.(6\vec{i} - 3\vec{j} + 2\vec{k}) = 5$. **(EG 6.49)**
28. Find the distance between the parallel planes $x + 2y - 2z + 1 = 0$ and $2x + 4y - 4z + 5 = 0$. **(EG 6.51)**
29. Find the angle between the line $\vec{r} = (2\vec{i} - \vec{j} + \vec{k}) + t(\vec{i} + 2\vec{j} - 2\vec{k})$ and the plane $\vec{r}.(6\vec{i} + 3\vec{j} + 2\vec{k}) = 8$. **(EX 6.9 – 3)**
30. Find the length of the perpendicular from the point $(1,-2,3)$ to the plane $x - y + z = 5$. **(EX 6.9 – 6)**

3 Marks

1. **(Cosine formulae):** With usual notations, in any triangle ABC, prove the following by vector method. $(i)\ a^2 = b^2 + c^2 - 2bc\cos A$ $(ii) b^2 = a^2 + c^2 - 2ac\cos B$ $(iii) c^2 = a^2 + b^2 - 2ab\cos C$. **(EG 6.1)**
2. With usual notations, in any triangle ABC, prove the following by vector method. $(i)\ a = b\cos C + c\cos B$ $(ii)\ b = c\cos A + a\cos C$ $(iii)\ c = a\cos B + b\cos A$**(EG 6.2)**
3. With usual notations, in any triangle ABC, prove by vector method that $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$. **(EG 6.4)**
4. In triangle ABC the points D, E, F are the midpoints of the sides BC, CA and AB respectively. Using vector method, show that the area of ΔDEF is equal to $\frac{1}{4}$ (area of ΔABC). **(EG 6.8)**
5. Prove by vector method that if a line is drawn from the centre of a circle to the midpoint of a chord, then the line is perpendicular to the chord. **(EX 6.1 - 1)**
6. Prove by vector method that the median to the base of an isosceles triangle is perpendicular to the base. **(EX 6.1 - 2)**
7. Prove by vector method that an angle in a semi-circle is a right angle. **(EX 6.1 - 3)**
8. Prove by vector method that the diagonals of a rhombus bisect each other at right angles. **(EX 6.1 - 4)**
9. Using vector method, prove that if the diagonals of a parallelogram are equal, then it is a rectangle. **(EX 6.1 - 5)**
10. Prove by vector method that the area of the quadrilateral $ABCD$ having diagonals AC and BD is $\frac{1}{2}|\overrightarrow{AC} \times \overrightarrow{BD}|$. **(EX 6.1 - 6)**
11. Prove by vector method that the parallelograms on the same base and between the same parallels are equal in area**(EX 6.1 - 7)**
12. Forces of magnitudes $5\sqrt{2}$ and $10\sqrt{2}$ units acting in the directions $3\vec{i} + 4\vec{j} + 5\vec{k}$ and $10\vec{i} + 6\vec{j} - 8\vec{k}$ respectively, act on a particle which is displaced from the point with position vector $4\vec{i} - 3\vec{j} - 2\vec{k}$ to the point with position vector $6\vec{i} + \vec{j} - 3\vec{k}$. Find the work done by the forces. **(EX 6.1 - 12)**
13. Find the magnitude and direction cosines of the torque of a force represented by $3\vec{i} + 4\vec{j} - 5\vec{k}$ about the point with position vector $2\vec{i} - 3\vec{j} + 4\vec{k}$ acting through a point whose position vector is $4\vec{i} + 2\vec{j} - 3\vec{k}$. **(EX 6.1 - 13)**
14. Find the torque of the resultant of the three forces represented by $-3\vec{i} + 6\vec{j} - 3\vec{k}, 4\vec{i} - 10\vec{j} + 12\vec{k}$ and $4\vec{i} + 7\vec{j}$ acting at the point with position vector $8\vec{i} - 6\vec{j} - 4\vec{k}$, about the point with position vector $18\vec{i} + 3\vec{j} - 9\vec{k}$. **(EX 6.1 - 14)**
15. If G is the centroid of a ΔABC, prove that (area of ΔGAB) = (area of ΔGBC) = (area of ΔGCA) = $\frac{1}{3}$(area of ΔABC). **(EX 6.1 - 8)**

16. Show that the four points $(6,-7,0),(16,-19,-4),(0,3,-6),(2,-5,10)$ Lie on a same plane. **(EG 6.16)**
17. If the vectors $\vec{a},\vec{b},\vec{c}$ are coplanar, then prove that the vectors $\vec{a}+\vec{b},\vec{b}+\vec{c},\vec{c}+\vec{a}$ are also coplanar. **(EG 6.17)**
18. If $\vec{a},\vec{b},\vec{c}$ are three non-coplanar vectors represented by concurrent edges of a parallelepiped of volume 4 cubic units, find the value of $(\vec{a}+\vec{b}).(\vec{b}\times\vec{c})+(\vec{b}+\vec{c}).(\vec{c}\times\vec{a})+(\vec{c}+\vec{a}).(\vec{a}\times\vec{b})$. **(EX 6.2 - 4)**
19. Find the altitude of a parallelepiped determined by the vectors $\vec{a}=-2\vec{i}+5\vec{j}+3\vec{k}$, $\vec{b}=\vec{i}+3\vec{j}-2\vec{k}$ and $\vec{c}=-3\vec{i}+\vec{j}+4\vec{k}$ if the base is taken as the parallelogram determined by $\vec{b}$ and $\vec{c}$. **(EX 6.2 - 5)**
20. Prove that $[\vec{a}\times\vec{b},\vec{b}\times\vec{c},\vec{c}\times\vec{a}]=[\vec{a},\vec{b},\vec{c}]^2$. **(EG 6.19)**
21. For any four vectors $\vec{a},\vec{b},\vec{c},\vec{d}$ we have $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=[\vec{a},\vec{b},\vec{d}]\vec{c}-[\vec{a},\vec{b},\vec{c}]\vec{d}=[\vec{a},\vec{c},\vec{d}]\vec{b}-[\vec{b},\vec{c},\vec{d}]\vec{a}$ **(EG 6.19)**
22. Prove that $\left(\vec{a}.(\vec{b}\times\vec{c})\right)\vec{a}=(\vec{a}\times\vec{b})\times(\vec{a}\times\vec{c})$. **(EG 6.20)**
23. If $\vec{a}=\vec{i}+2\vec{j}+3\vec{k}$, $\vec{b}=2\vec{i}-\vec{j}+\vec{k}$, $\vec{c}=3\vec{i}+2\vec{j}+\vec{k}$, and $\vec{a}\times(\vec{b}\times\vec{c})=l\vec{a}+m\vec{b}+n\vec{c}$, find the values of l,m,n. **(EX 6.3 - 7)**
24. Find the vector equation in parametric form and Cartesian equations of the line passing through $(-4,2,-3)$ and is parallel to the line $\frac{-x-2}{4}=\frac{y+3}{-2}=\frac{2z-6}{3}$. **(EG 6.26)**
25. Find the vector equation in parametric form and Cartesian equations of a straight passing through the points $(-5,7,-4)$ and $(13,-5,2)$. Find the point where the straight line crosses the xy −plane. **(EG 6.27)**
26. Find the angle between the straight line $\frac{x+3}{2}=\frac{y-1}{2}=-z$ with coordinate axes. **(EG 6.28)**
27. Find the angle between the straight lines $\frac{x-4}{2}=\frac{y}{1}=\frac{z+1}{-2}$ and $\frac{x-1}{4}=\frac{y+1}{-4}=\frac{z-2}{2}$ and state whether they are parallel or perpendicular. **(EG 6.30)**
28. Show that the straight line passing through the points $A(6,7,5)$ and $B(8,10,6)$ is perpendicular to the straight line passing through the points $C(10,2,-5)$ and $D(8,3,-4)$. **(EG 6.31)**
29. If the straight line joining the points $(2,1,4)$ and $(a-1,4,-1)$ is parallel to the line joining the points $(0,2,b-1)$ and $(5,3,-2)$, find the values of a and b **(EX 6.4 - 7)**
30. If the straight lines $\frac{x-5}{5m+2}=\frac{2-y}{5}=\frac{1-z}{-1}$ and $x=\frac{2y+1}{4m}=\frac{1-z}{-3}$ are perpendicular to each other, find the value of m. **(EX 6.4 - 8)**
31. Determine whether the pair of straight lines $\vec{r}=(2\vec{i}+6\vec{j}+3\vec{k})+t(2\vec{i}+3\vec{j}+4\vec{k})$, $\vec{r}=(2\vec{j}-3\vec{k})+s(\vec{i}+2\vec{j}+3\vec{k})$ are parallel. Find the shortest distance between them. **(EG 6.35)**
32. Find the shortest distance between the two lines $\vec{r}=(2\vec{i}+3\vec{j}+4\vec{k})+t(-2\vec{i}+\vec{j}-2\vec{k})$ and $\frac{x-3}{2}=\frac{y}{-1}=\frac{z+2}{2}$. **(EG 6.36)**
33. Find the coordinates of the foot of the perpendicular drawn from the point $(-1,2,3)$ to the straight line $\vec{r}=(\vec{i}-4\vec{j}+3\vec{k})+t(2\vec{i}+3\vec{j}+\vec{k})$. Also, find the shortest distance from the point to the straight line. **(EG 6.37)**
34. Find the vector and Cartesian form of the equations of a plane which is at a distance of 12 units from the origin and perpendicular to $6\vec{i}+2\vec{j}-3\vec{k}$. **(EG 6.38)**
35. Find the direction cosines and length of the perpendicular from the origin to the plane $\vec{r}.(3\vec{i}-4\vec{j}+12\vec{k})=5$. **(EG 6.40)**
36. A variable plane moves in such a way that the sum of the reciprocals of its intercepts on the coordinate axes is a constant. Show that the plane passes through a fixed point **(EG 6.42)**

37. Find the direction cosines of the normal to the plane $12x + 3y - 4z = 65$. Also, find the non-parametric form of vector equation of a plane and the length of the perpendicular to the plane from the origin. **(EX 6.6 - 2)**
38. Find the vector and Cartesian equations of the plane passing through the point with position vector $2\vec{i} + 6\vec{j} + 3\vec{k}$ and normal to the vector $\vec{i} + 3\vec{j} + 5\vec{k}$. **(EX 6.6 - 3)**
39. A plane passes through the point $(-1,1,2)$ and the normal to the plane of magnitude $3\sqrt{3}$ makes equal acute angles with the coordinate axes. Find the equation of the plane. **(EX 6.6 - 4)**
40. If a plane meets the coordinate axes at A, B, C such that the centriod of the triangle ABC is the point (u, v, w), find the equation of the plane. **(EX 6.6 - 6)**
41. Show that the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{m^2}$ and $\frac{x-3}{1} = \frac{y-2}{m^2} = \frac{z-1}{2}$ are coplanar, find the distinct real values of m. **(EX 6.8 – 3)**
42. Find the distance of a point $(2,5,-3)$ from the plane $\vec{r}.(6\vec{i} - 3\vec{j} + 2\vec{k}) = 5$. **(EG 6.49)**
43. Find the distance of the point $(5,-5,-10)$ from the point of intersection
44. of a straight line passing through the points $A(4,1,2)$ and $B(7,5,4)$ with the plane $x - y + z = 5$. **(EG 6.50)**
45. Find the equation of plane passing through the intersection of the planes $\vec{r}.(\vec{i} + \vec{j} + \vec{k}) + 1 = 0$ and $\vec{r}.(2\vec{i} - 3\vec{j} + 5\vec{k}) = 2$ and the point $(-1,2,1)$. **(EG 6.53)**
46. Find the equation of the plane passing through intersection of the planes $2x + 3y - z + 7 = 0$ and $x + y - 2z + 5 = 0$ and is perpendicular to the plane $x + y - 3z - 5 = 0$. **(EG 6.54)**
47. Find the point of intersection of the line $x - 1 = \frac{y}{2} = z + 1$ with the plane $2x - y + 2z = 2$. Also, find the angle between the line and the plane. **(EX 6.9 – 7)**

5 Marks

1. By vector method, prove that $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$. **Eg.,6.3**
2. Prove by vector method that $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$. **Eg.,6.5**
3. Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent. **Eg.,6.7**
4. Using vector method, prove that $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$. **EX. 6. 1 – 9**
5. Prove by vector method that $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$. **EX. 6. 1 – 10**
6. If $\vec{a} = -2\vec{i} + 3\vec{j} - 2\vec{k}$, $\vec{b} = 3\vec{i} - \vec{j} + 3\vec{k}$, $\vec{c} = 2\vec{i} - 5\vec{j} + \vec{k}$, find $(\vec{a} \times \vec{b}) \times \vec{c}$ and $\vec{a} \times (\vec{b} \times \vec{c})$ State whether they are equal. **Eg.,6.22**
7. If $\vec{a} = \vec{i} - \vec{j}$, $\vec{b} = \vec{i} - \vec{j} - 4\vec{k}$, $\vec{c} = 3\vec{j} - \vec{k}$ and $\vec{d} = 2\vec{i} + 5\vec{j} + \vec{k}$, verify that (i) $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}, \vec{b}, \vec{d}]\vec{c} - [\vec{a}, \vec{b}, \vec{c}]\vec{d}$ (ii) $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}, \vec{c}, \vec{d}]\vec{b} - [\vec{b}, \vec{c}, \vec{d}]\vec{a}$ **Eg.,623**
8. If $\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k}$, $\vec{b} = 3\vec{i} + 5\vec{j} + 2\vec{k}$, $\vec{c} = -\vec{i} - 2\vec{j} + 3\vec{k}$, verify that (i) $(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a}.\vec{c})\vec{b} - (\vec{b}.\vec{c})\vec{a}$ (ii) $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$. **EX. 6. 3 – 4**
9. Find the equation of a straight line passing through the point of intersection of the straight lines $\vec{r} = (\vec{i} + 3\vec{j} - \vec{k}) + t(2\vec{i} + 3\vec{j} + 2\vec{k})$ and $\frac{x-2}{1} = \frac{y-4}{2} = \frac{z+3}{4}$, and perpendicular to both straight lines. **Eg.634**
10. Find the coordinates of the foot of the perpendicular drawn from the point $(-1,2,3)$ to the straight line $\vec{r} = (\vec{i} - 4\vec{j} + 3\vec{k}) + t(2\vec{i} + 3\vec{j} + \vec{k})$. Also, find the shortest distance from the point to the straight line. **(EG 6.37)**
11. Show that the lines $\frac{x-3}{3} = \frac{y-3}{-1}, z - 1 = 0$ and $\frac{x-6}{2} = \frac{z-1}{3}, y - 2 = 0$ intersect. Also find the point of intersection. **EX. 6. 5 – 4**

12. Find the foot of the perpendicular drawn from the point $(5,4,2)$ to the line $\frac{x+1}{2}=\frac{y-3}{3}=\frac{z-1}{-1}$. Also, find the equation of the perpendicular. **EX. 6.5 – 7**
13. Find the parametric form of vector equation of the straight line passing through $(-1,2,1)$ and parallel to the straight line $\vec{r}=(2\vec{i}+3\vec{j}-\vec{k})+t(\vec{i}-2\vec{j}+\vec{k})$ and hence find the shortest distance between the lines. **EX. 6.5 – 6**
14. Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point $(0,1,-5)$ and parallel to the straight lines $\vec{r}=(\vec{i}+2\vec{j}-4\vec{k})+s(2\vec{i}+3\vec{j}+6\vec{k})$ and $\vec{r}=(\vec{i}-3\vec{j}+5\vec{k})+t(\vec{i}+\vec{j}-\vec{k})$. **Eg. 6.43**
15. Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point $(0,1,-5)$ and parallel to the straight lines $\vec{r}=(\vec{i}+2\vec{j}-4\vec{k})+s(2\vec{i}+3\vec{j}+6\vec{k})$ and $\vec{r}=(\vec{i}-3\vec{j}+5\vec{k})+t(\vec{i}+\vec{j}-\vec{k})$. **Eg. 6.44**
16. Find the non-parametric form of vector equation, and cartesian equation of the plane passing through the point $(2,3,6)$ and parallel to the straight lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-3}{1}$ and $\frac{x+3}{2}=\frac{y-3}{-5}=\frac{z+1}{-3}$ **EX. 6.7 – 1**
17. Find the parametric form of vector equation, and Cartesian equations of the plane passing through the points $(2,2,1),(9,3,6)$ and perpendicular to the plane $2x+6y+6z=9$. **EX. 6.7 – 2**
18. Find parametric form of vector equation and Cartesian equations of the plane passing through the points $(2,2,1),(1,-2,3)$ and parallel to the straight line passing through the points $(2,1,-3)$ and $(-1,5,-8)$. **EX. 6.7 – 3**
19. Find the non-parametric form of vector equation of the plane passing through the point $(1,-2,4)$ and perpendicular to the plane $x+2y-3z=11$ and parallel to the line $\frac{x+7}{3}=\frac{y+3}{-1}=\frac{z}{1}$. **EX. 6.5 – 4**
20. Find the parametric form of vector equation, and Cartesian equations of the plane containing the line $\vec{r}=(\vec{i}-\vec{j}+3\vec{k})+t(2\vec{i}-\vec{j}+4\vec{k})$ and perpendicular to plane $\vec{r}.(\vec{i}+2\vec{j}+\vec{k})=8$. **EX. 6.5 – 5**
21. Find the parametric vector, non-parametric vector and Cartesian form of the equations of the plane passing through the points $(3,6,-2),(-1,-2,6)$ and $(6,-4,-2)$. **EX. 6.5 – 6**
22. Find the non-parametric form of vector equation, and Cartesian equations of the plane $\vec{r}=(6\vec{i}-\vec{j}+\vec{k})+s(-\vec{i}+2\vec{j}+\vec{k})+t(-5\vec{i}-4\vec{j}-5\vec{k})$ **EX. 6.5 – 7**
23. Show that the lines $\vec{r}=(-\vec{i}-3\vec{j}-5\vec{k})+s(3\vec{i}+5\vec{j}+7\vec{k})$ and $\vec{r}=(2\vec{i}+4\vec{j}+6\vec{k})+t(\vec{i}+4\vec{j}+7\vec{k})$ are coplanar. Also, find the non-parametric form of vector equation of the plane containing these lines. **(EG 6.46)**
24. Show that the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{m^2}$ and $\frac{x-3}{1}=\frac{y-2}{m^2}=\frac{z-1}{2}$ are coplanar, find the distinct real values of m. **(EX 6.8 – 3)**
25. If the straight lines $\frac{x-1}{2}=\frac{y+1}{\lambda}=\frac{z}{2}$ and $\frac{x+1}{5}=\frac{y+1}{2}=\frac{z}{\lambda}$ are coplanar, find λ and equations of the planes containing these two lines. **(EX 6.8 – 4)**
26. Find the equation of te plane passing through the line of intersection of the planes $x+2y+3z=2$ and $x-y+z+11=3$ and at a distance $\frac{2}{\sqrt{3}}$ from the point $(3,1,-1)$. **(EX 6.9 – 2)**

CHAPTER 7
APPLICATION OF DIFFERENTIAL CALCULUS

2 Marks

1. For the function $f(x) = x^2, x \in [0,2]$ compute the average rate of changes in the subintervals $[0,0.5], [0.5,1], [1,1.5], [1.5,2]$ and the instantaneous rate of changes at the points $x = 0.5, 1, 1.5, 2$. **(EG 7.1)**
2. The temperature in celsius in a long rod of length $10\ m$, insulated at both ends, is a function of length x given by $T = x(10 - x)$. Prove that the rate of change of temperature at the midpoint of the rod is zero **(Eg.7.2).**
3. A person learnt 100 words for an English test. The number of words the person remembers in t days after learning is given by $W(t) = 100 \times (1 - 0.1t^2), 0 \leq t \leq 10$. What is the rate at which the person forgets the words 2 days after learning? **(Eg.7.3).**
4. A particle moves so that the distance moved is according to the law $s(t) = \frac{t^3}{3} - t^2 + 3$. At what time the velocity and acceleration are zero respectively? **Example 7.4**
5. If the volume of a cube of side length x is $v = x^3$. Find the rate of change of the volume with respect to x when $x = 5$ units. **Ex 7.1- 4**
6. If the mass $m(x)$ (in kilograms) of a thin rod of length x (in metres) is given by, $m(x) = \sqrt{3}x$ then what is the rate of change of mass with respect to the length when it is $x = 3$ and $x = 27$ metres. **Ex 7.1- 5**
7. Find the equations of tangent and normal to the curve $y = x^2 + 3x - 2$ at the point $(1, 2)$. **Eg 7.11**
8. For what value of x the tangent of the curve $y = x^3 - 5x^2 + x - 2$ is parallel to the line $y = x$. **Eg 7.12**
9. Find the angle of intersection of the curve $y = \sin x$ with the positive x −axis. **Eg 7.16**
10. Find the slope of the tangent to the curves at the respective given points. (i) $y = x^4 + x^2 - x$ at $x = 1$ **EX 7.2 – 1)**
11. Compute the value of $'c'$ satisfied by the Rolle's theorem for the function $f(x) = x^2(1 - x)^2, x \in [0,1]$. **(EG 7.19)**
12. Find the values in the interval $\left(\frac{1}{2}, 2\right)$ satisfied by the Rolle's theorem for the function $f(x) = x + \frac{1}{x}, x \in \left[\frac{1}{2}, 2\right]$. **(EG 7.20)**
13. Explain why Rolle's theorem is not applicable to the following functions in the respective intervals. (iii) $f(x) = x - 2\log x, x \in [2,7]$. **(EX 7.3 – 1)**
14. Write the Maclaurin series expansion of e^x
15. Expand the polynomial $f(x) = x^2 - 3x + 2$ in powers of $x - 1$. **(EX 7.4 – 4)**
16. Evaluate: $\lim_{x \to 1} \left(\frac{x^2 - 3x + 2}{x^2 - 4x + 3}\right)$. **(EG 7.33)**
17. Compute the limit $\lim_{x \to a} \left(\frac{x^n - a^n}{x - a}\right)$. **Eg 7.34**
18. Evaluate the limit $\lim_{x \to 0} \left(\frac{\sin mx}{x}\right)$. **(EG 7.35)**
19. Evaluate the limit $\lim_{x \to 0} \left(\frac{\sin x}{x^2}\right)$. **(EG 7.36)**
20. Evaluate: $\lim_{x \to 0^+} (x \log x)$. **Eg 7.40**
21. Evaluate: $\lim_{x \to \infty} \left(\frac{e^x}{x^m}\right), m \in \mathbb{N}$. **Eg 7.42**
22. $\lim_{x \to \infty} \left(\frac{x}{\log x}\right)$ **(EX 7.5 – 3)**
23. $\lim_{x \to \infty} e^{-x}\sqrt{x}$ **(EX 7.5 – 5)**

24. Prove that the function $f(x) = x^2 + 2$ is strictly increasing in the interval $(2,7)$ and strictly decreasing in the interval $(-2,0)$. **Eg. 7.46**
25. Find the absolute extrema on the given closed interval $f(x) = x^2 - 12x + 10$; $[1,2]$ **Ex. 7.6 – 1(i)**
26. Find the stationary point and point of inflection $f(x) = 6x^{\frac{4}{3}} - 3x^{\frac{1}{3}}$; $[-1,1]$ **(EX 7.6 – 1 related)**
27. Find the local extremum of the function $f(x) = x^4 + 32x$. **Eg. 7.59**
28. Find intervals of concavity and points of inflexion for the $f(x) = x(x-4)^3$
29. Find the slant (oblique) asymptote for the function $f(x) = \frac{x^2-6x+7}{x+5}$. **(EG 7.67)**

3 Marks

1. A particle is fired straight up from the ground to reach a height of s feet in t seconds, where $s(t) = 128t - 16t^2$. (a) Compute the maximum height of the particle reached (b) What is the velocity when the particle hits the ground? **Eg. 7.5**
2. If we blow air into a balloon of spherical shape at a rate of 1000 cm^3 per second. At what rate the radius of the balloon changes when the radius is $7cm$? Also compute the rate at which the surface area changes. **Eg. 7.7**
3. Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is 10 metre high **Eg. 7.9**
4. A stone is dropped into a pond causing ripples in the form of concentric circles. The radius r of the outer ripple is increasing at a constant rate at 2 cm per second. When the radius is 5 cm find the rate of changing of the total area of the disturbed water? **(EX 7.1 – 6)**
5. A beacon makes one revolution every 10 seconds. It is located on a ship which is anchored 5 km from a straight shore line. How fast is the beam moving along the shore line when it makes an angle of 45° with the shore? **Ex. 7.1 - 7**
6. Find the equation of the tangent and normal to the Lissajous curve given by $x = 2\cos 3t$ and $y = 3\sin 2t$, $t \in \mathbb{R}$. **Eg. 7.13**
7. Find the acute angle between the curves $y = x^2$ and $x = y^2$ at their points of intersection $(0,0), (1,1)$. **Eg. 7.15**
8. Find the point on the curve $y = x^2 - 5x + 4$ at which the tangent is parallel to the line $3x + y = 7$. **(EX 7.2 – 2)**
9. Find the points on the curve $y = x^3 - 6x^2 + x + 3$ where the normal is parallel to the line $x + y = 1729$. **(EX 7.2 – 3)**
10. Find the angle between the rectangular hyperbola $xy = 2$ and the parabola $x^2 + 4y = 0$. **(EX 7.2 – 9)**
11. Show that the two curves $x^2 - y^2 = r^2$ and $xy = c^2$ where c, r are constants, cut orthogonally. **Ex. 7.2 - 10**
12. Without actually solving show that the equation $x^4 + 2x^3 - 2 = 0$ has only one real root in the interval $(0,1)$. **(EG 7.22)**
13. Prove using the Rolle's theorem that between any two distinct real zeros of the polynomial $a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots\ldots\ldots\ldots + a_1x + a_0$ there is a zero of the polynomial $na_nx^{n-1} + (n-1)a_{n-1}x^{n-2} + \ldots\ldots + a_1$. **(EG 7.23)**
14. Prove that there is a zero of the polynomial, $2x^3 - 9x^2 - 11x + 12$ in the interval $(2,7)$ given that 2 and 7 are the zeros of the polynomial $x^4 - 6x^3 - 11x^2 + 24x + 28$. **(EG 7.24)**
15. Find the values in the interval $(1,2)$ of the mean value theorem satisfied by the function $f(x) = x - x^2$ for $1 \le x \le 2$. **(EG 7.25)**

16. Suppose $f(x)$ is a differentiable function for all x with $f'(x) \leq 29$ and $f(b) = 17$. What the maximum value is of $f(7)$? **(EG 7.27)**
17. Prove, using mean value theorem, that $|\sin\alpha - \sin\beta| \leq |\alpha - \beta|, \alpha, \beta \in \mathbb{R}$. **(EG 7.28)**
18. A thermometer was taken from a freezer and placed in a boiling water. It took 22 seconds for the thermometer to raise from $-10°C$ to $100°C$. Show that the rate of change of temperature at some time t is $5°C$ per second. **(EG 7.29)**
19. Using the Lagrange's mean value theorem determine the values of x at which the tangent is parallel to the secant line at the end points of the given interval: $(i)\ f(x) = x^3 - 3x + 2, x \in [-2,2]$ **(EX 7.3 – 4)**
20. Show that the value in the conclusion of the mean value theorem for
    $(i)\ f(x) = \frac{1}{x}$ on a closed interval of positive numbers $[a,b]$ is $\sqrt{ab}$.
    $(ii)\ f(x) = Ax^2 + Bx + C$ on any interval $[a,b]$ is $\frac{a+b}{2}$. **(EX 7.3 – 5)**
21. A race car driver is racing at 20^{th} km. If his speed never exceeds 150 km/hr, what is the maximum distance he can cover in the next two hours. **(EX 7.3 – 6)**
22. Suppose that for a function $f(x), f'(x) \leq 1$ for all $1 \leq x \leq 4$. Show that $f(4) - f(1) \leq 3$. **(EX 7.3 – 7)**
23. Does there exist a differentiable function $f(x)$ such that $f(0) = -1, f(2) = 4$ and $f'(x) \leq 2$ for all x. Justify your answer. **(EX 7.3 – 8)**
24. Show that there lies a point on the curve $f(x) = x(x+3)e^{\frac{-\pi}{2}}, -3 \leq x \leq 0$ where tangent drawn is parallel to the x –axis. **(EX 7.3 – 9)**
25. Using mean value theorem prove that for, $a > 0, b > 0, |e^{-a} - e^{-b}| < |a - b|$. **(EX 7.3 – 10)**
26. Expand $\log(1+x)$ as a Maclaurin's series upto 4 non-zero terms for $-1 < x \leq 1$. **(EG 7.30)**
27. Write the Taylor series expansion of $\frac{1}{x}$ about $x = 2$ by finding the first three non-zero terms. **(EG 7.32)**
28. Write down the Taylor series expansion, of the function $\log x$ about $x = 1$ upto three non-zero terms for $x > 0$. **(EX 7.4 –2)**
29. If $\lim_{\theta \to 0}\left(\frac{1-\cos m\theta}{1-\cos n\theta}\right) = 1$, then prove that $m = \pm n$. **(EG 7.37)**
30. Using the l'Hôpital Rule prove that, $\lim_{x \to 0^+}(1+x)^{\frac{1}{x}} = e$. **(EG 7.43)**
31. Evaluate: $\lim_{x \to 1} x^{\frac{1}{1-x}}$. **(EG 7.45)**
32. $\lim_{x \to 0^+}(\cos x)^{\frac{1}{x^2}}$ **(EX 7.5 – 11)**
33. If an initial amount A_0 of money is invested at an interest rate r compounded n times a year, the value of the investment after t years is $A = A_0 + \left(1 + \frac{r}{n}\right)^{nt}$ If the interest is compounded continuously, (that is as $n \to \infty$), show that the amount after t years is $A = A_0e^{rt}$. **(EX 7.5 – 12)**
34. Find the absolute maximum and absolute minimum values of the function $f(x) = 2x^3 + 3x^2 - 12x$ on $[-3, 2]$. **Eg. 7.48**
35. Find the absolute extrema of the function $f(x) = 3\cos x$ on the closed interval $[0,2\pi]$. **(EG 7.49)**
36. Discuss the monotonicity and local extrema of the function $f(x) = \log(1+x) - \frac{x}{1+x}, x > -1$ and hence find the domain where, $\log(1+x) > \frac{x}{1+x}$. **(EG 7.53)**
37. Find the intervals of monotonicity and local extrema of the function $f(x) = \frac{x}{1+x^2}$. **(EG 7.56)**
38. Determine the intervals of concavity of the curve $f(x) = (x-1)^3.(x-5), x \in \mathbb{R}$ and, points of inflection if any. **(EG 7.57)**
39. Find the local maximum and minimum of the function x^2y^2 on the line $x + y = 10$. **(EG 7.61)**

40. Prove that among all the rectangles of the given area square has the least perimeter. **Eg. 7.65**
41. Find two positive numbers whose sum is 12 and their product is maximum. **Ex. 7.8- 1**
42. Find two positive numbers whose product is 20 and their sum is minimum. **Ex. 7.8 - 2**
43. Find the smallest possible value of $x^2 + y^2$ given that $x + y = 10$. **(EX 7.8 – 3)**
44. Prove that among all the rectangles of the given perimeter, the square has the maximum area. **Ex. 7.8 - 8**
45. Find the dimensions of the largest rectangle that can be inscribed in a semi circle of radius $r\ cm$. **Ex. 7.8 – 9**
46. The volume of a cylinder is given by the formula $V = \pi r^2 h$. Find the greatest and least values of V if $r + h = 6$. **(EX 7.8 – 11)**
47. Find the asymptotes of the function $f(x) = \frac{1}{x}$. **(EG 7.66)**
48. Sketch the curve $= f(x) = x^2 - x - 6$.. **(EG 7.69)**

5 Marks

1. A particle moves along a horizontal line such that its position at any time $t \geq 0$ is given by $s(t) = t^3 - 6t^2 + 9t + 1$, where s is measured in metres and t in seconds? (a) At what time the particle is at rest? (b) At what time the particle changes direction? (c) Find the total distance travelled by the particle in the first 2 seconds. **Eg. 7. 6**
2. The price of a product is related to the number of units available (supply) by the equation $Px + 3P - 16x = 234$, where P is the price of the product per unit in Rupees(`) and x is the number of units. Find the rate at which the price is changing with respect to time when 90 units are available and the supply is increasing at a rate of 15 units/week. **(EG 7.8)**
3. **(Two variable related rate problem):** A road running north to south crosses a road going east to west at the point P . Car A is driving north along the first road and car B is driving east along the second road. At a particular time car A is 10 kilometres to the north of P and travelling at 80 km/hr, while car B is 15 kilometres to the east of P and travelling at 100 km/hr. How fast is the distance between the two cars changing? **(EG 7.10)**
4. A particle moves along a line according to the law $s(t) = 2t^3 - 9t^2 + 12t - 4$, where $t \geq 0$.
   (i) At what times the particle changes direction?
   (ii) Find the total distance travelled by the particle in the first 4 seconds.
   (iii) Find the particle's acceleration each time the velocity is zero. **Ex. 7.1 - 1**
5. A camera is accidentally knocked off an edge of a cliff 400 ft high. The camera falls a distance of $s = 16t^2$ in t seconds.
   (i) How long does the camera fall before it hits the ground?
   (ii) What is the average velocity with which the camera falls during the last 2 seconds?
   `(iii) What is the instantaneous velocity of the camera when it hits the ground? **Ex. 7.1 - 2**
6. A particle moves along a line according to the law $s(t) = 2t^3 - 9t^2 + 12t - 4$, where $t \geq 0$.
   (i) At what times the particle changes direction?
   (ii) Find the total distance travelled by the particle in the first 4 seconds.
   (iii) Find the particle's acceleration each time the velocity is zero. **(EX 7.1 – 3)**
7. A conical water tank with vertex down of 12 metres height has a radius of 5 metres at the top. If water flows into the tank at a rate 10 $cubic\ m/min$, how fast is the depth of the water increases when the water is 8 metres deep? **Ex. 7.1 - 8**
8. A ladder 17 metre long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of 5 m/s. When the base of the ladder is 8 metres from the wall. (i) How fast is the top of the ladder moving down the wall? **Ex. 7.1 - 9**
   (ii) At what rate, the area of the triangle formed by the ladder, wall, and the floor, is changing?

9. A police jeep, approaching an orthogonal intersection from the northern direction, is chasing a speeding car that has turned and moving straight east. When the jeep is 0.6 km north of the intersection and the car is 0.8 km to the east. The police determine with a radar that the distance between them and the car is increasing at 20 km/hr. If the jeep is moving at 60 km/hr at the instant of measurement, what is the speed of the car? **Ex. 7.1 - 10**
10. Find the equation of tangent and normal to the curve given by $x = 7\cos t$ and $y = 2\sin t$, $t \in \mathbb{R}$ at any point on the curve. **(EX 7.2 – 8)**
11. Find the acute angle between $y = x^2$ and $y = (x - 3)^2$. **Eg. 7.14**
12. If the curves $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$ intersect each other orthogonally then, $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$. **(EG 7.17)**
13. Prove that the ellipse $x^2 + 4y^2 = 8$ and the hyperbola $x^2 - 2y^2 = 4$ intersect orthogonally. **(EG 7.18)**
14. For the function $f(x) = 4x^3 + 3x^2 - 6x + 1$ find the intervals of monotonicity, local extrema, intervals of concavity and points of inflection. **Ex. 7.7 - 3**
15. We have a 12 square unit piece of thin material and want to make an open box by cutting small squares from the corners of our material and folding the sides up. The question is, which cut produces the box of maximum volume? **Eg. 7. 62**
16. Find the points on the unit circle $x^2 + y^2 = 1$ nearest and farthest from (1,1). **Eg. 7. 63**
17. A garden is to be laid out in a rectangular area and protected by wire fence. What is the largest possible area of the fenced garden with 40 metres of wire. **Ex. 7.8 - 4**
18. A rectangular page is to contain 24 cm^2 of print. The margins at the top and bottom of the page are 1.5 cm and the margins at other sides of the page is 1 cm. What should be the dimensions of the page so that the area of the paper used is minimum. **Ex. 7.8 - 5**
19. A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain 1,80,000 $sq.mtrs$ in order to provide enough grass for herds. No fencing is needed along the river. What is the length of the minimum needed fencing material? **Ex. 7.8- 6**
20. Find the dimensions of the rectangle with maximum area that can be inscribed in a circle of radius 10 cm. **Ex. 7.8 - 7**
21. A manufacturer wants to design an open box having a square base and a surface area of 108 $sq.cm$. Determine the dimensions of the box for the maximum volume. **Ex. 7.8 - 10**
22. A hollow cone with base radius a cm and height b cm is placed on a table. Show that the volume of the largest cylinder that can be hidden underneath is $\frac{4}{9}$ times volume of the cone **Ex. 7.8 - 11**.
23. Show that the volume of largest right circular cone that can be inscribed in a spare of radius 'a' is $\frac{8}{27}$(volume of the sphere) **(creative)**
24. Te top and bottom margins of a poster are each 6 cms. and the margins are each 4 cms. If the area of printed material on the poster is fixed at 384 cms^2, find the dimension of the poster with the smallest area. **(creative)**
25. Prove that the sum of intercepts on the co-ordinate axes of any tangent to the curve $x = a\cos^3\theta; y = a\sin^3\theta$ at θ is $x\cos\theta - y\sin\theta = a\cos 2\theta$ **(creative)**
26. If the curve $y^2 = x$ and $xy = k$ are orthogonal then prove that $8k^2 = 1$. **(creative)**

CHAPTER 8
DIFFERENTIALS AND PARTIAL DERIVATIVES

2 Marks

1. Find a linear approximation for indicated points. $(i)\ f(x) = x^3 - 5x + 12, x_0 = 2$ $(ii)\ g(x) = \sqrt{x^2+9}, x_0 = -4$ **(EX 8.1 – 3)**
2. Let $f, g: (a, b) \to \mathbb{R}$ be differentiable functions. Show that $d(fg) = fdg + gdf$. **(EG 8.5)**
3. Let $g(x) = x^2 + \sin x$. Calculate the differential dg. **Eg. 8. 6**
4. If the radius of a sphere, with radius $10\ cm$, has to decrease by $0.1\ cm$, approximately how much will its volume decrease? **Eg. 8. 7**
5. Find differential dy for $(iii)\ y = e^{x^2-5x+7}\cos(x^2-1)$ **(EX 8.2 – 1)**
6. Find df for $f(x) = x^2 + 3x$ and evaluate it for $x = 2$ and $dx = 0.1$ **Ex. 8. 2 – 2 (I)**
7. A sphere is made of ice having radius 10 cm. Its radius decreases from $10\ cm$ to $9.8\ cm$. Find approximations for the following: (i) Change in the volume (ii) change in the surface area **(EX 8.1 – 5)**
8. An egg of a particular bird is very nearly spherical. If the radius to the inside of the shell is $5\ mm$ and radius to the outside of the shell is $5.3\ mm$, find the volume of the shell approximately. **Ex. 8. 2 – 6**
9. Assume that the cross section of the artery of human is circular. A drug is given to a patient to dilate his arteries. If the radius of an artery is increased from $2\ mm$ to $2.1\ mm$, how much is cross-sectional area increased approximately? **Ex. 8. 2 – 7**
10. A circular plate expands uniformly under the influence of heat. If it's radius increases from $10.5\ cm$ to $10.75\ cm$, then find an approximate change in the area and the approximate percentage change in the area. **(EX 8.2 – 10)**
11. Let $f(x,y) = \frac{3x-5y+8}{x^2+y^2+1}$ for all $(x,y) \in \mathbb{R}^2$. Show that f is continuous on $\mathbb{R}^2$. **(EG 8.8)**
12. Evaluate $\lim_{(x,y)\to(0,0)} \cos\left(\frac{e^x \sin y}{y}\right)$, if the limit exists. **(EX 8.3 – 4)**
13. Let $F(x,y) = x^3y + y^2x + 7$ for all $(x,y) \in \mathbb{R}^2$. Calculate $\frac{\partial F}{\partial x}(-1,3)$ and $\frac{\partial F}{\partial y}(-2,1)$. **(EG 8.12)**
14. Find the partial derivatives of the following functions at the indicated points. $(i)\ f(x,y) = 3x^2 - 2xy + y^2 + 5x + 2, (2,-5)$ **(EX 8.4 – 1)**
15. If $U(x,y,z) = \log(x^3 + y^3 + z^3)$, find $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z}$. **(EX 8.4 – 4)**
16. If $w(x,y,z) = x^2y + y^2z + z^2x,\ x,y,z \in \mathbb{R}$, find the differential dw. **(EG 8.16)**
17. If $w(x,y) = x^3 - 3xy + 2y^2, x, y \in \mathbb{R}$, find linear approximation for w at $(1,-1)$. **(EX 8.5 – 1)**
18. If $v(x,y) = x^2 - xy + \frac{1}{4}y^2 + 7, x, y \in \mathbb{R}$, find the differential dv. **(EX 8.5 – 3)**
19. Let $V(x,y,z) = xy + yz + zx, x, y, z \in \mathbb{R}$. Find the differential dV. **(EX 8.5 – 5)**
20. Verify the above theorem for $F(x,y) = x^2 - 2y^2 + 2xy$ and $x(t) = \cos t, y(t) = \sin t, t \in [0, 2\pi]$. **(EG 8.18)**
21. Let $g(x,y) = x^2 - yx + \sin(x+y), x(t) = e^{3t}, y(t) = t^2, t \in \mathbb{R}$. Find $\frac{dg}{dt}$. **(EG 8.19)**
22. If $u(x,y) = x^2y + 3xy^4, x = e^t$ and $y = \sin t$, find $\frac{du}{dt}$ and evaluate it at $t = 0$. **(EX 8.6 – 1)**
23. Show that $F(x,y) = \frac{x^2+5xy-10y^2}{3x+7y}$ is a homogeneous function of degree 1. **(EG 8.21)**
24. Determine whether the homogeneous or not. If it is so, find the degree. $(iv)\ U(x,y,z) = xy + \sin\left(\frac{y^2-2z^2}{xy}\right)$. **(EX 8.7 – 1)**
25. If $u(x,y) = \frac{x^2+y^2}{\sqrt{x+y}}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$. **(EX 8.7 – 4)**

3 Marks

1. Find the linear approximation for $f(x) = \sqrt{1+x}, x \geq -1$, at $x_0 = 3$. Use the linear approximation to estimate $f(3.2)$. **(EG 8.1)**
2. Use linear approximation to find an approximate value of $\sqrt{9.2}$ without using a calculator. **(EG 8.2)**

3. Let us assume that the shape of a soap bubble is a sphere. Use linear approximation to approximate the increase in the surface area of a soap bubble as its radius increases from 5 cm to 5.2 cm. Also, calculate the percentage error. **(EG 8.3)**
4. A right circular cylinder has radius $r = 10\ cm$. and height $h = 20\ cm$. Suppose that the radius of the cylinder is increased from10 cm to 10.1 cm and the height does not change. Estimate the change in the volume of the cylinder. Also, calculate the relative error and percentage error. **(EG 8.4)**
5. The time T , taken for a complete oscillation of a single pendulum with length l , is given by the equation $T = 2\pi\sqrt{\frac{l}{g}}$, where g is a constant. Find the approximate percentage error in the calculated value of T corresponding to an error of 2 percent in the value of l. **(EX 8.1 – 6)**
6. Show that the percentage error in the n^{th} root of a number is approximately $\frac{1}{n}$ times the percentage error in the number. **Ex. 8. 1 – 7**
7. Find Δf and df for the function f for the indicated values of $x, \Delta x$ and compare
   $(i)\ f(x) = x^3 - 2x^2;\ x = 2\ , \Delta x = dx = 0.5$ **(EX 8.2 – 3)**
8. Assuming $\log_{10} e = 0.4343$, find an approximate value of $\log_{10} 1003$. **Ex. 8. 2 – 4**
9. The trunk of a tree has diameter 30 cm. During the following year, circumference grew 6 cm.
   (i) Approximately, how much did the tree's diameter grow?
   (ii) What is the percentage increase in area of the tree's cross-section? **Ex. 8. 2 – 5**
10. Consider $f(x,y) = \frac{xy}{x^2+y^2}$ if $(x,y) \neq (0,0)$ and $f(0,0) = 0$. Show that f is not continuous at $(0,0)$ and continuous at all other points of $\mathbb{R}^2$. **(EG 8.9)**
11. Let $g(x,y) = \frac{x^2y}{x^4+y^2}$ for $(x,y) \neq (0,0)$ and $f(0,0) = 0$.
    (i) Show that $\lim_{(x,y)\to(0,0)} g(x,y) = 0$ along every line $y = mx, m \in \mathbb{R}$.
    (ii) Show that $\lim_{(x,y)\to(0,0)} g(x,y) = \frac{k}{1+k^2}$ along every parabola $y = kx^2, k \in \mathbb{R}\setminus\{0\}$. **(EX 8.3 – 5)**
12. Let $g(x,y) = \frac{e^y \sin x}{x}$, for $x \neq 0$ and $g(0,0) = 1$. Show that g is continuous at $(0,0)$. **(EX 8.3 – 7)**
13. Let for all $w(x,y) = xy + \frac{e^y}{y^2+1}$. for all $(x,y) \in \mathbb{R}^2$. Calculate $\frac{\partial^2 w}{\partial y \partial x}$ and $\frac{\partial^2 w}{\partial x \partial y}$. **(EG 8.14)**
14. Let $u(x,y) = e^{-2y}\cos(2x)$ for all $(x,y) \in \mathbb{R}^2$. Prove that u is a harmonic function in $\mathbb{R}^2$. **(EG 8.15)**
15. If $V(x,y) = e^x(x\cos y - y\sin y)$, then prove that $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$. **(EX 8.4 – 7)**
16. If $w(x,y) = xy + sin(xy)$, then prove that $\frac{\partial^2 w}{\partial y \partial x} = \frac{\partial^2 w}{\partial x \partial y}$ **(EX 8.4 – 8)**
17. A firm produces two types of calculators each week, x number of type A and y number of type B . The weekly revenue and cost functions (in rupees) are $R(x,y) = 80x + 90y + 0.04xy - 0.05x^2 - 0.05y^2$ and $C(x,y) = 8x + 6y + 2000$ respectively. (i) Find the profit function $P(x,y)$, (ii) Find $\frac{\partial P}{\partial x}(1200,1800)$ and $\frac{\partial P}{\partial y}(1200,1800)$ and interpret these results. **(EX 8.4 – 10)**
18. Let $U(x,y,z) = x^2 - xy + \sin z\ , x,y,z \in \mathbb{R}$. Find the linear approximation for U at $(2,-1,0)$. **(EG 8.17)**
19. Let $U(x,y,z) = xyz, x = e^{-t}, y = e^{-t}\cos t\ , z = \sin t\ , t \in \mathbb{R}$. Find $\frac{dU}{dt}$. **(EX 8.6 – 4)**
20. If $z(x,y) = x\tan^{-1}(xy)\ , x = t^2, y = se^t, s,t \in \mathbb{R}$. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$. at $s = t = 1$. **(EX 8.6 – 6)**
21. Let $U(x,y) = e^x \sin y$, where $x = st^2, y = s^2t\ , s,t \in \mathbb{R}$. Find $\frac{\partial U}{\partial s}, \frac{\partial U}{\partial t}$ and evaluate them at $s = t = 1$. **(EX 8.6 – 7)**

22. $W(x,y,z) = xy + yz + zx,\ x = u - v, y = uv, z = u + v, u, v \in \mathbb{R}$. Find $\frac{\partial w}{\partial u}, \frac{\partial w}{\partial v}$ and evaluate them at $\left(\frac{1}{2}, 1\right)$. **(EX 8.6 – 9)**
23. Prove that $f(x,y) = x^3 - 2x^2y + 3xy^2 + y^3$ is homogeneous. What is the degree? Verify Euler's Theorem for f. **(EX 8.7 – 2)**
24. P.T $g(x,y) = x\log\left(\frac{y}{x}\right)$ is homogeneous. What is the degree? Verify Euler's Theorem for g. **(EX 8.7 – 3)**

5 Marks

1. The radius of a circular plate is measured as 12.65 cm instead of the actual length 12.5 cm. find the following in calculating the area of the circular plate: (i) Absolute error (ii) Relative error (iii) Percentage error **Ex. 8. 1– 4**
2. Find the f_x, f_y and show that $f_{xy} = f_{yx}$. (i) $f(x,y) = \frac{3x}{y + \sin x}$ **(EX 8. 4 – 2)**
3. Find the f_x, f_y and show that $f_{xy} = f_{yx}$. (ii) $f(x,y) = \tan^{-1}\left(\frac{x}{y}\right)$ **(EX 8. 4 – 2)**
4. If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$. **(EG 8.22)**
5. If $v(x,y) = \log\left(\frac{x^2+y^2}{\sqrt{x+y}}\right)$, prove that $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = 1$. **(EX 8.7 – 5)**
6. If $w(x,y,z) = \log\left(\frac{5x^3y^4 + 7y^2xz^4 - 75y^3z^4}{x^2+y^2}\right)$, find $x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} + z\frac{\partial w}{\partial z}$. **(EX 8.7 – 6)**
7. Using Euler's theorem $u = tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$.**(creative)**
8. Verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ if $u = \frac{x}{y^2} - \frac{y}{x^2}$**(creative)**
9. Verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ if $u = sin\,3x\cos 4y$**(creative)**

Chapter 9
Applications of Integration

2 Marks

1. Find an approximate value of $\int_1^{1.5} x\,dx$ by applying the left-end rule with the partition $\{1.1,\ 1.2,\ 1.3,\ 1.4,\ 1.5\}$. **(EX 9.1 – 1)**
2. Find an approximate value of $\int_1^{1.5} x^2 dx$by applying the right-end rule with the partition $\{1.1,\ 1.2,\ 1.3,\ 1.4,\ 1.5\}$. **(EX 9.1 – 2)**
3. Find an approximate value of $\int_1^{1.5} (2-x)dx$by applying the mid-point rule with the partition $\{1.1,\ 1.2,\ 1.3,\ 1.4,\ 1.5\}$. **(EX 9.1 – 3)**
4. Evaluate: $\int_0^1 [2x]dx$ where $[\cdot]$ is the greatest integer function. **Eg. 9. 7**
5. Show that $\int_0^{\pi} g(\sin x)dx = 2\int_0^{\frac{\pi}{2}} g(\sin x)dx$ where $g(\sin x)$ is a function of $\sin x$. **(EG 9.20)**
6. Show that $\int_0^{2\pi} g(\cos x)dx = 2\int_0^{\pi} g(\cos x)dx$, where $g(\cos x)$ is a function of $\cos x$. **(EG 9.22)**
7. If $f(x) = f(a+x)$, then $\int_0^{2a} f(x)dx = 2\int_0^{a} f(x)dx$. **(EG 9.23)**
8. Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\cos x\,dx$. **Eg. 9.24**
9. Evaluate: $\int_{-\log 2}^{\log 2} e^{-|x|}dx$. **Eg. 9.25**

10. Evaluate: $\int_3^4 \frac{dx}{x^2-4}$ **(EX 9.3 – 1)**
11. Evaluate the integrals using properties of $\int_{-5}^{5} x\cos\left(\frac{e^x-1}{e^x+1}\right)dx$ **(EX 9.3 – 2 i)**
12. Evaluate $\int_0^{\pi} x^2 \cos nx\, dx$ where n is a positive integer. **(EG 9.31)**
13. Evaluate: $\int_0^{2\pi} x^2 \sin nx\, dx$, where n is a positive integer. **(EG 9.33)**
14. Evaluate: $\int_0^{\infty} \frac{1}{a^2+x^2}dx$, $a > 0, b \in \mathbb{R}$. **Eg. 9.35**
15. Evaluate: $\int_0^{\frac{\pi}{2}}(\sin^2 x + \cos^4 x)dx$ **Eg. 9.37**
16. Evaluate: $\int_0^{\frac{\pi}{2}} \begin{vmatrix} \cos^4 x & 7 \\ \sin^5 x & 3 \end{vmatrix} dx$. **Eg. 9.38**
17. Find the values of $\int_0^{\frac{\pi}{2}} \sin^5 x \cos^4 x\, dx$ **Eg. 9.39(I)**
18. Evaluate: $\int_0^1 x^3(1-x)^4 dx$. **Eg. 9.42**
19. Evaluate $\int_0^{\frac{\pi}{4}} \sin^6 2x\, dx$ **(EX 9.6 – 1 iii)**
20. Evaluate $\int_0^{2\pi} \sin^7 \frac{x}{4} dx$ **(EX 9.6 – 1 vi)**
21. Evaluate: $\int_0^{\infty} e^{-ax}x^n dx$, where $a > 0$. **Eg. 9.44**
22. Evaluate: $\int_0^{\infty} x^5 e^{-3x} dx$ **Ex. 9. 7 – 1(i)**

3 Marks

1. Estimate the value of $\int_0^{0.5} x^2 dx$ using the Riemann sums corresponding to 5 subintervals of equal width and applying (i) left-end rule (ii) right-end rule (iii) the mid-point rule. **(EG 9.1)**
2. Evaluate $\int_1^4 (2x^2+3)dx$, as the limit of a sum. **(EG 9.4)**
3. Evaluate: $\int_0^1 \left(\frac{2x+7}{5x^2+9}\right)dx$. **(EG 9.6)**
4. Evaluate: $\int_0^{\frac{\pi}{3}} \frac{\sec x \tan x}{1+\sec^2 x}dx$. **(EG 9.8)**
5. Evaluate: $\int_0^9 \frac{1}{x+\sqrt{x}}dx$. **(EG 9.9)**
6. Evaluate: $\int_1^2 \frac{x}{(x+1)(x+2)}dx$. **Eg. 9.10**
7. Evaluate: $\int_0^{1.5} [x^2]dx$, where $[x]$ is the greatest integer function. **(EG 9.14)**
8. Evaluate: $\int_{-4}^{4} |x+3| dx$ **Eg.9.15**
9. Show that $\int_0^{\frac{\pi}{2}} \frac{dx}{4+5\sin x} = \frac{1}{3}\log_e 2$. **(EG 9.16)**
10. Prove that $\int_0^{\frac{\pi}{4}} \frac{\sin 2x dx}{\sin^4 x+\cos^4 x} = \frac{\pi}{4}$. **(EG 9.17)**
11. Evaluate: $\int_0^{\pi} \frac{xdx}{1+\sin x}$. **(EG 9.21)**
12. Evaluate: $\int_0^{a} \frac{f(x)}{f(x)+f(a-x)}dx$. **(EG 9.26)**
13. Prove that $\int_0^{\frac{\pi}{4}} \log(1+\tan x)\, dx = \frac{\pi}{8}\log 2$. **(EG 9.27)**
14. Show that $\int_0^1 (\tan^{-1} x + \tan^{-1}(1-x))dx$ **Eg. 9.28**
15. Evaluate: $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}dx$ **Eg. 9.29**
16. Evaluate: $\int_0^1 \sqrt{\frac{1-x}{1+x}}dx$ **(EX 9.3 -1 iii)**
17. valuate the integrals using properties of $\int_0^1 |5x-3|dx$ **(EX 9.3 – 2 vi)**

18. Evaluate the integrals using properties of $\int_0^{\pi} \frac{x \sin x}{1+\sin x} dx$ **(EX 9.3 – 2 ix)**
19. Evaluate the integrals using properties of $\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{dx}{1+\sqrt{\tan x}}$ **(EX 9.3 – 2 x)**
20. Evaluate: $\int_{-1}^{1} e^{-\lambda x}(1-x^2)dx$. **(EG 9.34)**
21. Evaluate $\int_0^{\frac{1}{\sqrt{2}}} \frac{e^{\sin^{-1}x}\sin^{-1}x}{\sqrt{1-x^2}} dx$ **(EX 9.4 – 3)**
22. Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{5+4\sin^2 x}$ **(EX 9.5 – 1)**
23. Evaluate: $\int_0^{2a} x^2\sqrt{2a-x^2}dx$. **(EG 9.40)**
24. Prove that $\int_0^{\infty} e^{-x}x^n dx = n!$, where n is a positive integer. **Eg. 9.43**
25. Show that $\Gamma(n) = 2\int_0^{\infty} e^{-x^2}x^{2n-1}dx$. **Eg. 9.45**
26. Evaluate: $\int_0^{\infty} \frac{x^n}{n^x} dx$, where n is a positive integer ≥ 2. **(EG 9.46)**
27. Find the area of the region bounded by the line $6x + 5y = 30$, x – axis and the lines $x = -1$ and $x = 3$. **Eg. 9.47**
28. Find the area of the region bounded by the line $7x - 5y = 35$, x – axis and the lines $x = -2$ and $x = 3$. **Eg. 9.48**
29. Find the area of the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. **Eg. 9.49**
30. Find the area of the region bounded between the parabola $y^2 = 4ax$ and its latus rectum. **Eg. 9.50**
31. Find the area of the region bounded by x – axis, the sine curve $y = \sin x$, the lines $x = 0$ and $x = 2\pi$. **Eg. 9.52**
32. Find the area of the region bounded by x – axis, the curve $y = |\cos x|$, the lines $x = 0$ and $x = \pi$. **Eg. 9.53**
33. Find the area of region bounded between the parabola $x^2 = y$ and the curve $y = |x|$. **Eg. 9.55**
34. Find the volume of a sphere of radius a. **(EG 9.62)**
35. Find the volume of a right-circular cone of base radius r and height h. **(EG 9.63)**
36. Find the volume of the solid formed by revolving the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$ about the major axis. **(EG 9.66)**
37. Find, by integration, the volume of the solid generated by revolving about y –axis the region bounded by the curves $y = \log x$, $y = 0, x = 0$ and $y = 2$. **(EG 9.69)**
38. Find, by integration, the volume of the solid generated by revolving about the x –axis, the region enclosed by $y = e^{-2x}, y = 0, x = 0$ and $x = 1$. **(EX 9.9 – 2)**

5 Marks

1. Evaluate $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$ **(EX 9.3 – 2)**
2. Find the area of the region bounded between the parabolas $y^2 = 4x$ and $x^2 = 4y$. **Eg. 9.54**
3. Find the area of the region bounded by $y = \cos x$, $y = \sin x$, the lines $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$. **Eg. 9.56**
4. The region enclosed by the circle $x^2 + y^2 = a^2$ is divided into two segments by the line $x = h$. Find the area of the smaller segment. **Eg. 9.57**
5. Find the area of the region in the first quadrant bounded by the parabola $y^2 = 4x$, the line $x + y = 3$ and y – axis. **Eg. 9.58**
6. Find, by integration, the area of the region bounded by the lines $5x - 2y = 15, x + y + 4 = 0$ and the x –axis **Eg. 9.59**.
7. Using integration find the area of the region bounded by triangle ABC, whose vertices A, B, and C are $(-1,1)$, $(3,2)$, and $(0,5)$ respectively. **Eg. 9.60**

8. Using integration, find the area of the region which is bounded by x –axis, the tangent and normal to the circle $x^2 + y^2 = 4$ drawn at $(1, \sqrt{3})$. **Eg. 9.61**
9. Find the area of the region bounded by the curve $2 + x - x^2 + y = 0$, x –axis, $x = -3$ and $x = 3$. **Ex. 9. 8– 3**
10. Find the area bounded by the line $y = 2x + 5$ and the parabola $y = x^2 - 2x$. **Ex. 9. 8– 4**
11. Find the area of the region bounded between the curves $y = \sin x$ and $y = \cos x$ and the lines $x = 0$ and $x = \pi$. **Ex. 9. 8– 5**
12. Find the area of bounded by $y = \tan x$, $y = \cot x$ and the lines $x = 0$, $x = \frac{\pi}{2}$, $y = 0$. **Ex. 9. 8– 6**
13. Find the area of the region bounded by the parabola $y^2 = x$ and the line $y = x - 2$. **Ex. 9. 8– 7**
14. Father of a family wishes to divide his square field bounded by $x = 0$, $x = 4$, $y = 4$ and $y = 0$ along the curve $y^2 = 4x$ and $x^2 = 4y$ into three equal parts for his wife, daughter and son. Is it possible to divide? If so, find the area to be divided among them. **Ex. 9. 8– 8**
15. The curve $y = (x - 2)^2 + 1$ has a minimum point at P. A point Q on the curve is such that the slope of PQ is 2. Find the area bounded by the curve and the chord PQ. **Ex. 9. 8– 9**
16. Find the area of the common to the circle $x^2 + y^2 = 16$ and the parabola $y^2 = 6x$. **Ex. 9. 8– 10**
17. Find the common area enclosed by the parabolas $y^2 = x$ and $x^2 = y$. **(Creative)**

Chapter 10
ORDINARY DIFFFERNTIAL EQUAIONS

2 Marks

1. Determine the order and degree (if exists) of

   $(iii)\ \frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^2y}{dx^2}\right)$ $\quad (iv)\ 3\left(\frac{d^2y}{dx^2}\right) = \left(4 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}$

   $(v)\ dy + (xy - \cos x)dx = 0$ **(EG 10.1)**
2. Determine the order and degree (if exists) of

   $(ii)\ \left(\frac{d^3y}{dx^3}\right)^{\frac{2}{3}} + 3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4 = 0$ $\ (v)\ y\frac{dy}{dx} = \frac{x}{\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^3}$ $\quad \left(\frac{d^2y}{dx^2}\right)^3 = \sqrt{1 + \frac{dy}{dx}}$

   $(viii)\ \frac{d^2y}{dx^2} = xy + \cos\left(\frac{dy}{dx}\right)$ $(ix)\ \frac{d^2y}{dx^2} + 5\frac{dy}{dx} + \int y dx = x^3$ $\quad (x)\ x = e^{xy\left(\frac{dy}{dx}\right)}$ **(EX 10.1 – 1)**

3. Express each of the following physical statements in the form of differential equation.
   (i) Radium decays at a rate proportional to the amount Q present. **(EX 10.2 – 1(i))**
   (ii) The population P of a city increases at a rate proportional to the product of population and to the difference between 5,00,000 and the population. **(EX 10.2 – 1(ii))**
   (iii) For a certain substance, the rate of change of vapor pressure P with respect to temperature T is proportional to the vapor pressure and inversely proportional to the square of the temperature. **(EX 10.2 – 1(iii))**
   (iv)A saving amount pays 8% interest per year, compounded continuously. In addition, the income from another investment is credited to the amount continuously at the rate of ` 400 per year. **(EX 10.2 – 1(iv))**
4. Find the differential equation for the family of all straight lines passing through the origin. **(EG 10.2)**
5. Form the differential equation by eliminating the arbitrary constants A and B from $y = A\cos x + B\sin x$. **(EG 10.3)**
6. Find the differential equation of the family of parabolas $y^2 = 4ax$, where a is an arbitrary constant. **(EG 10.5)**

7. Show that $y = mx + \frac{7}{m}, m \neq 0$ is a solution of the differential equation $xy' + 7\frac{1}{y'} - y = 0$. **(EG 10.8)**

3 Marks

1. Find the differential equation of the family of circles passing through thepoints $(a, 0)$ and $(-a, 0)$. **(EG 10.4)**
2. Find the differential equation of the family of all ellipses having foci on the x −axis and centre at the origin. **(EG 10.6)**
3. Find the differential equation of the family of circles passing through the origin and having their centres on the x −axis. **(EX 10.3 – 3)**
4. Find the differential equation of the family of parabolas with vertex at $(0, -1)$ and having axis along the y −axis. **(EX 10.3 – 5)**
5. Find the differential equation corresponding to the family of curves represented by the equation $y = Ae^{8x} + Be^{-8x}$, where A and B are arbitrary constants. **(EX 10.3 – 7)**
6. Show that $y = ae^{-3x} + b$, where a and b are arbitrary constants, is a solution of the differential equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} = 0$. **(EX 10.4 – 6)**
7. Solve $(1 + x^2)\frac{dy}{dx} = 1 + y^2$. **(EG 10.11)**
8. The velocity v, of a parachute falling vertically satisfies the equation $v\frac{dv}{dx} = g\left(1 - \frac{v^2}{k^2}\right)$, where g and k are constants. If v and x are both initially zero, find v in terms of x. **(EX 10.5 – 2)**
9. Find the equation of the curve whose slope is $\frac{y-1}{x^2+x}$ and which passes through the point (1,0). **(EX 10.5 – 3)**
10. Solve the following differential (ii) $ydx + (1 + x^2)\tan^{-1}x\,dy = 0$ **(EX 10.5 – 4 ii)**
11. Solve the following differential (iii) $\sin\frac{dy}{dx} = a, y(0) = 1$ **(EX 10.5 – 4 iii)**
12. Solve: $\frac{dy}{dx} + 2y = e^{-x}$. **(EG 10.22)**
13. Solve: $\frac{dy}{dx} + 2y\cot x = 3x^2\,cosec^2\,x$. **(EG 10.24)**
14. Solve: $\cos x\frac{dy}{dx} + y\sin x = 1$ **(EX 10.7 – 1)**
15. $(1 - x^2)\frac{dy}{dx} - xy = 1$ **(EX 10.7 – 2)**
16. The engine of a motor boat moving at 10 m/s is shut off. Given that the retardation at any subsequent time (after shutting off the engine) equal to the velocity at that time. Find the velocity after 2 seconds of switching off the engine. **(EX 10.8 – 4)**
17. Suppose a person deposits 10,000 Indian rupees in a bank account at the rate of 5% per annum compounded continuously. How much money will be in his bank account 18 months later? **(EX 10.8 – 5)**

5 Marks

1. Solve $\frac{dy}{dx} = \tan^2(x + y)$ **(EX 10.5 – 4 x)**
2. If F is the constant force generated by the motor of an automobile of mass M, its velocity V is given by $M\frac{dV}{dt} = F - kV$, where k is a constant. Express V in terms of t given that $V = 0$ when $t = 0$. **(EX 10.5 – 1)**
3. Solve $(y^2 - 2xy)dx = (x^2 - 2xy)dy$ **(EX 10.6 – 5)**
4. Solve $(x^2 + y^2)dy = xydx$. It is given that $y(1) = 1$ and $y(x_0) = e$. Find the value of x_0. **(EX 10.6 – 8)**
5. Solve: $(1 + x^3)\frac{dy}{dx} + 6x^2y = 1 + x^2$. **(EG 10.25)**

6. $(x^2+1)\frac{dy}{dx}+2xy=\sqrt{x^2+4}$ **(EX 10.7 – 4)**
7. $\left(y-e^{\sin^{-1}x}\right)\frac{dx}{dy}+\sqrt{1-x^2}=0$ **(EX 10.7 – 7)**
8. The growth of a population is proportional to the number present. If the population of a colony doubles in 50 years, in how many years will the population become triple? **(EG 10.27)**
9. A radioactive isotope has an initial mass 200 mg, which two years later is 50 mg. Find the expression for the amount of the isotope remaining at any time. What is its half-life? (half-life means the time taken for the radioactivity of a specified isotope to fall to half its original value). **(EG 10.28)**
10. In a murder investigation, a corpse was found by a detective at exactly 8 $p.m.$ Being alert, the detective also measured the body temperature and found it to be 70°F. Two hours later, the detective measured the body temperature again and found it to be 60°F. If the room temperature is 50°F, and assuming that the body temperature of the person before death was 98.6°F, at what time did the murder occur? **(EG 10.29)**
11. A tank contains 1000 litres of water in which 100 grams of salt is dissolved. Brine (Brine is a high-concentration solution of salt (usually sodium chloride) in water) runs in a rate of 10 litres per minute, and each litre contains 5 grams of dissolved salt. The mixture of the tank is kept uniform by stirring. Brine runs out at 10 litres per minute. Find the amount of salt at any time t. **(EG 10.30)**
12. The rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number present. Given that the number triples in 5 hours, find how many bacteria will be present after 10 hours? **(EX 10.8 – 1)**
13. Find the population of a city at any time t, given that the rate of increase of population is proportional to the population at that instant and that in a period of 40 years the population increased from 3,00,000 to 4,00,000. **(EX 10.8 – 2)**
14. The equation of electromotive force for an electric circuit containing resistance and selfinductance is $E=Ri+L\frac{di}{dt}$, where E is the lectromotive force is given to the circuit, R the resistance and L, the coefficient of induction. Find the current i at time t when $E=0$. **(EX 10.8 – 3)**
15. Assume that the rate at which radioactive nuclei decay is proportional to the number of such nuclei that are present in a given sample. In a certain sample 10% of the original number of radioactive nuclei have undergone disintegration in a period of 100 years. What percentage of the original radioactive nuclei will remain after 1000 years? **(EX 10.8 – 6)**
16. Water at temperature 100°C cools in 10 minutes to 80°C in a room temperature of 25°C . Find (i) The temperature of water after 20 minutes (ii) The time when the temperature is 40°C. $\left(\log_e\frac{11}{15}=-0.3101, \log_e 5=1.6094\right)$ **(EX 10.8 – 7)**
17. A pot of boiling water at 100°C is removed from a stove at time $t=0$ and left to cool in the kitchen. After 5 minutes, the water temperature has decreased to 80°C , and another 5 minutes later it has dropped to 65°C . Determine the temperature of the kitchen. **(EX 10.8 – 9)**
18. A tank initially contains 50 litres of pure water. Starting at time $t=0$ a brine containing with 2 grams of dissolved salt per litre flows into the tank at the rate of 3 litres per minute. The mixture is kept uniform by stirring and the well-stirred mixture simultaneously flows out of the tank at the same rate. Find the amount of salt present in the tank at any time $t>0$. **(EX 10.8 – 10)**
19. The sum of Rs. 1000 is compounded continuously, the nominal rate of interest being four percent per annum. In how many years will the amount be twice the original principal? ($log2 = 06931$) **(creative)**

20. Radium disappears at the rate proportional to the amount present. 5% of the amount disappears in 50 years, how much will remain at the end of 100 years *(Take A_0 as the initial amount)* **(creative)**

CHAPTER 11
PROBABILITY DISTRIBUTIONS LAPLACE

2 Marks

1. Suppose two coins are tossed once. If X denotes the number of tails, (i)write down the sample space (ii)find the inverse image of 1 (iii) the values of the random variable and number of elements in its inverse images. **(EG 11.1)**
2. Suppose X is the number of tails occurred when three fair coins are tossed once simultaneously. Find the values of the random variable X and number of points in its inverse images. **(EX 11.1 – 1)**
3. In a pack of 52 playing cards, two cards are drawn at random simultaneously. If the number of black cards drawn is a random variable, find the values of the random variable and number of points in its inverse images. **(EX 11.1 – 2)**
4. Two fair coins are tossed simultaneously (equivalent to a fair coin is tossed twice). Find the probability mass function for number of heads occurred. **(EG 11.5)**
5. A pair of fair dice is rolled once. Find the probability mass function to get the number of fours. **(EG 11.6)**
6. Three fair coins are tossed simultaneously. Find the probability mass function for number of heads occurred. **(EX 11.2 – 1)**
7. Find the probability mass function and cumulative distribution function of number of girl child in families with 4 children, assuming equal probabilities for boys and girls. **(EX 11.2 – 3)**
8. The probability density function of X is given by
$$f(x) = \begin{cases} kxe^{-2x}, & for\ x > 0 \\ 0, & for\ x \le 0 \end{cases}$$
Find the value of k . **(EX 11.3 – 1)**
9. Find the mean and variance of a random variable X , whose probability density function is $f(x) = \begin{cases} \lambda e^{-\lambda x}, & for\ x \ge 0 \\ 0, & Otherwise \end{cases}$ **(EG 11.18)**
10. Find the mean and variance. $f(x) = \begin{cases} \frac{4-x}{6}, x = 1,2,3 \end{cases}$ **(EX 11.4 – 1)**
11. Four fair coins are tossed once. Find the probability mass function, mean and variance for number of heads occurred. **(EX 11.4 – 4)**
12. The time to failure in thousands of hours of an electronic equipment used in a manufactured computer has the density function $f(x) = \begin{cases} 3e^{-3x}, & x > 0 \\ 0, & elsewhere \end{cases}$
Find the expected life of this electronic equipment. **(EX 11.4 – 6)**
13. A lottery with 600 tickets gives one prize of `200, four prizes of `100, and six prizes of ` 50. If the ticket costs is ` 2, find the expected winning amount of a ticket. **(EX 11.4 – 8)**

14. Find the binomial distribution function for each of the following. (i) Five fair coins are tossed once and X denotes the number of heads. (ii) A fair die is rolled 10 times and X denotes the number of times 4 appeared. **(EG 11.19)**
15. Compute $P(X = k)$ for the binomial distribution, $B(n, p)$ where

    (i) $n = 6, p = \frac{1}{3}, k = 3$ **(EX 11.5 – 1)**
16. If $X \sim B(n, p)$ such that $4P(X = 4) = P(x = 2)$ and $n = 6$. Find the distribution, mean and standard deviation. **(EX 11.5 – 8)**

3 Marks

1. Suppose a pair of unbiased dice is rolled once. If X denotes the total score of two dice, write down (i) the sample space (ii) the values taken by the random variable X, (iii) the inverse image of 10, and (iv) the number of elements in inverse image of X. **(EG 11.2)**
2. An urn contains 2 white balls and 3 red balls. A sample of 3 balls are chosen at random from the urn. If X denotes the number of red balls chosen, find the values taken by the random variable X and its number of inverse images. **(EG 11.3)**
3. Two balls are chosen randomly from an urn containing 6 white and 4 black balls. Suppose that we win ` 30 for each black ball selected and we lose ` 20 for each white ball selected. If X denotes the winning amount, then find the values of X and number of points in its inverse images. **(EG 11.4)**
4. Two balls are chosen randomly from an urn containing 6 red and 8 black balls. Suppose that we win ` 15 for each red ball selected and we lose ` 10 for each black ball selected. X denotes the winning amount, then find the values of X and number of points in its inverse images. **(EX 11.1 – 4)**
5. A six sided die is marked '2' on one face, '3' on two of its faces, and '4' on remaining three faces. The die is thrown twice. If X denotes the total score in two throws, find the values of the random variable and number of points in its inverse images. **(EX 11.1 – 5)**
6. If the probability mass function $f(x)$ of a random variable X is

x	1	2	3	4
$f(x)$	$\frac{1}{12}$	$\frac{5}{12}$	$\frac{5}{12}$	$\frac{1}{12}$

   find (i) its cumulative distribution function, hence find (ii) $P(X \leq 3)$ and, (iii) $P(X \geq 2)$ **(EG 11.7)**
7. Find the probability mass function $f(x)$ of the discrete random variable X whose cumulative distribution function $F(x)$ is given by

$$F(x) = \begin{cases} 0, & -\infty < x < 2 \\ 0.25, & -2 \leq x < -1 \\ 0.60, & -1 \leq x < 0 \\ 0.90, & 0 \leq x < 1 \\ 1, & 1 \leq x < \infty \end{cases}$$

   Also find (i) $P(X < 0)$ and (ii) $P(X \geq -1)$. **(EG 11.9)**
8. Suppose a discrete random variable can only take the values 0, 1, and 2.

   The probability mass function is defined by

$f(x) = \begin{cases} \frac{x^2+1}{k}, & for\ x = 0,1,2 \\ 0, & \text{otherwise} \end{cases}$. Find (i) the value of k (ii) cumulative distribution function (iii) $P(X \geq 1)$. **(EX 11.2 – 4)**

9. A random variable X has the following probability mass function.

x	1	2	3	4	5
$f(x)$	k^2	$2k^2$	$3k^2$	$2k$	$3k$

Find (i) the value of k (ii) $P(2 \leq X < 5)$ (iii) $P(3 < X)$ **(EX 11.2 – 6)**

10. The cumulative distribution function of a discrete random variable is given by

$$F(x) = \begin{cases} 0, & -\infty < x < 0 \\ \frac{1}{2}, & 0 \leq x < 1 \\ \frac{3}{5}, & 1 \leq x < 2 \\ \frac{4}{5}, & 2 \leq x < 3 \\ \frac{9}{10}, & 3 \leq x < 4 \\ 1, & 4 \leq x < \infty \end{cases}$$

Find (i) the probability mass function (ii) $P(X < 3)$ and (iii) $P(X \geq 2)$. **(EX 11.2 – 7)**

11. Find the constant C such that the function $f(x) = \begin{cases} Cx^2, & 1 < x < 4 \\ 0, & \text{Otherwise} \end{cases}$ is a density function, and compute (i) $P(1.5 < X < 3.5)$ (ii) $P(X \leq 2)$ (iii) $P(3 < X)$. **(EG 11.11)**

12. If X is the random variable with distribution function $F(x)$ given by,

$$F(x) = \begin{cases} 0, & x < 0 \\ x, & 0 \leq x < 1 \\ 1, & 1 \leq x \end{cases}$$

then find (i) the probability density function $f(x)$ (ii) $P(0.2 \leq X \leq 0.7)$. **(EG 11.13)**

13. Suppose that $f(x)$ given below represents a probability mass function, Find (i) the value of c (ii) Mean and variance. **(EG 11.16)**

14. Two balls are chosen randomly from an urn containing 8 white and 4 black balls. Suppose that we win Rs 20 for each black ball selected and we lose Rs10 for each white ball selected. Find the expected winning amount and variance. **(EG 11.17)**

15. find the mean and variance.

(i) $f(x) = \begin{cases} \frac{1}{10}, & x = 2,5 \\ \frac{1}{5}, & x = 0,1,3,4 \end{cases}$ **(EX 11.4 – 1)**

16. Two balls are drawn in succession without replacement from an urn containing four red balls and three black balls. Let X be the possible outcomes drawing red balls. Find the probability mass function and mean for X. **(EX 11.4 – 2)**

17. If μ and σ^2 are the mean and variance of the discrete random variable X, and $E(X+3) = 10$ and $E(X+3)^2 = 116$, find μ and σ^2. **(EX 11.4 – 3)**

18. A multiple choice examination has ten questions, each question has four distractors with exactly one correct answer. Suppose a student answers by guessing and if X denotes the

number of correct answers, find (i) binomial distribution (ii) probability that the student will get seven correct answers(iii) the probability of getting at least one correct answer. **(EG 11.20)**

19. The mean and variance of a binomial variate X are respectively 2 and 1.5. Find (i) $P(X = 0)$ (ii) $P(X = 1)$ (iii) $P(X \geq 1)$ **(EG 11.21)**
20. On the average, 20% of the products manufactured by ABC Company are found to be defective. If we select 6 of these products at random and X denote the number of defective products find the probability that (i) two products are defective (ii) at most one product is defective (iii) at least two products are defective. **(EG 11.22)**
21. The probability that $Mr.Q$ hits a target at any trial is $\frac{1}{4}$. Suppose he tries at the target 10 times. Find probability that he hits the target (i) exactly 4 times (ii) at least one time. **(EX 11.5 – 2)**
22. The probability that a certain kind of component will survive a electrical test is $\frac{3}{4}$. Find the probability that exactly 3 of the 5 components tested survive. **(EX 11.5 – 4)**
23. A retailer purchases a certain kind of electronic device from a manufacturer. The manufacturer indicates that the defective rate of the device is 5%. The inspector of the retailer randomly picks 10 items from a shipment. What is the probability that there will be (i) at least one defective item (ii) exactly two defective items. **(EX 11.5 – 5)**
24. The mean and standard deviation of a binomial variate X are respectively 6 and 2. Find (i) the probability mass function (ii) $P(X = 3)$ (iii) $P(X \geq 2)$. **(EX 11.5 – 7)**
25. In a binomial distribution consisting of 5 independent trials, the probability of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the mean and variance of the distribution. **(EX 11.5 – 9)**

5 Marks

1. A six sided die is marked '1' on one face, '2' on two of its faces, and '3' on remaining three faces. The die is rolled twice. If X denotes the total score in two throws. (i) Find the probability mass function. (ii) Find the cumulative distribution function. (iii) Find $P(3 \leq X < 6)$$(iv)$ Find $P(X \geq 4)$. **(EG 11.8)**
2. A random variable X has the following probability mass function.

x	1	2	3	4	5	6
$f(x)$	k	$2k$	$6k$	$5k$	$6k$	$10k$

Find (i) $P(2 < X < 6)$ (ii) $P(2 \leq X \leq 5)$ (iii) $P(X \leq 4)$ (iv) $P(3 < X)$ **(EG 11.10)**

3. A six sided die is marked '1' on one face, '3' on two of its faces, and '5' on remaining three faces. The die is thrown twice. If X denotes the total score in two throws, find (i) the probability mass function (ii) cumulative distribution function (iii) $P(4 \leq X < 10)$ (iv) $P(X \geq 6)$ **(EX 11.2 – 2)**
4. If X is the random variable with probability density function $f(x)$ given by, $f(x) = \begin{cases} x-1, & 1 \leq x < 2 \\ -x+3, & 2 \leq x < 3 \\ 0, & Otherwise \end{cases}$ find (i) the distribution function $F(x)$ (ii) $P(1.5 \leq X \leq 2.5)$ **(EG 11.12)**
5. The probability density function of random variable X is given by $f(x) = \begin{cases} k, & 1 \leq x \leq 5 \\ 0, & Otherwise \end{cases}$. Find (i) Distribution function (ii) $P(X < 3)$ (iii) $P(2 < X < 4)$ (iv) $P(3 \leq X)$ **(EG 11.14)**
6. Let X be a random variable denoting the life time of an electrical equipment having probability density function $f(x) = \begin{cases} ke^{-2x}, & for\ x > 0 \\ 0, & for\ x \leq 0 \end{cases}$ Find (i) the value of k (ii) Distribution function (iii) $P(X < 2)$

(iv) calculate the probability that X is at least for four unit of time (v) $P(X = 3)$. **(EG 11.15)**

7. If the probability that a fluorescent light has a useful life of at least 600 hours is 0.9, find the probabilities that among 12 such lights
   (i) exactly 10 will have a useful life of at least 600 hours;
   (ii) at least 11 will have a useful life of at least 600 hours;
   (iii) at least 2 will not have a useful life of at least 600 hours. **(EX 11.5 – 6)**
8. Suppose the amount of milk sold daily at a milk booth is distributed with a minimum of 200 litres and a maximum of 600 litres with probability density function $f(x) = \begin{cases} k, & 200 \le x \le 600 \\ 0, & \text{Otherwise} \end{cases}$
   Find (i) the value of k (ii) the distribution function (iii) the probability that daily sales will fall between 300 litres and 500 litres? **(EX 11.3 – 3)**
9. 4. The probability density function of X is given by
   $f(x) = \begin{cases} ke^{\frac{-x}{3}}, & for\ x > 0 \\ 0, & for\ x \le 0 \end{cases}$ Find (i) the value of k (ii) the distribution function (iii) $P(X < 3)$ (iv) $P(5 \le X)$ (v) $P(X \le 4)$. **(EX 11.3 – 4)**
10. If X is the random variable with probability density function $f(x)$ given by,
   $$f(x) = \begin{cases} x+1, & -1 \le x < 0 \\ -x+1, & 0 \le x < 1 \\ 0, & Otherwise \end{cases}$$
   then find (i) the distribution function $F(x)$ (ii) $P(-0.5 \le X \le 0.5)$ **(EX 11.3 – 5)**

x	1	2	3	4	5	6
$f(x)$	c^2	$2c^2$	$3c^2$	$4c^2$	c	$2c$

11. On the average, 20% of the products manufactured by ABC Company are found to be defective. If we select 6 of these products at random and X denote the number of defective products find the probability that (i) two products are defective (ii) at most one product is defective (iii) at least two products are defective. **(EG 11.22)**

CHAPTER 12
DISCRETE MATHEMATICS

2 Marks

1. Examine the binary operation (closure property) of the following operations on the respective sets (if it is not, make it binary): (i) $a * b = a + 3ab - 5b^2, \forall a, b \in \mathbb{Z}$ $(ii) a * b = \left(\frac{a-1}{b-1}\right), \forall a, b \in \mathbb{Q}$ **(EG**
2. **Uniqueness of Identity):** In an algebraic structure the identity element (if exists) must be unique. **(TH 12.1)**
3. **(Uniqueness of Inverse):** In an algebraic structure the inverse of an element (if exists) must be unique. **(TH 12.2)**
4. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ be any two Boolean matrices of the same type. Find $A \vee B$ and $A \wedge B$. **(EG 12.8)**
5. Determine whether $*$ is a binary operation on the sets given below.
   (i) $a * b = a.|b|$ on $\mathbb{R}$ (ii) $a * b = min\,(a, b)$ on $A = \{1,2,3,4,5\}$
   (iii) $a * b = a\sqrt{b}$ is binary on $\mathbb{R}$. **(EX 12.1 – 1)**
6. On $\mathbb{Z}$, define $\otimes$ by $(m \otimes n) = m^n + n^m : m, n \in \mathbb{Z}$. Is $\otimes$ binary on $\mathbb{Z}$? **(EX 12.1 – 2)**

7. Let $A = \{a + \sqrt{5}b : a, b \in \mathbb{Z}\}$. Check whether the usual multiplication is a binary operation on A. **(EX 12.1 – 4)**
8. Fill in the following table so that the binary operation $*$ on $A = \{a, b, c\}$ is commutative.

$*$	a	b	c
a	b		
b	c	b	a
c	a		c

(EX 12.1 – 6)

9. Consider the binary operation $*$ defined on the set $A = \{a, b, c, d\}$ by the following table: Is it commutative and associative?

$*$	a	b	c	d
a	a	c	b	d
b	d	a	b	c
c	c	d	a	a
d	d	a	a	c

(EX 12.1 – 7)

10. Write the statements in words corresponding to $\neg p, p \wedge q, p \vee q$ and $q \vee \neg p$, where p is 'It is cold' and q is 'It is raining.' **(EG 12.12)**
11. How many rows are needed for following statement formulae?
    $(i)\ p \vee \neg t \wedge (p \vee \neg s)$ $(ii)((p \wedge q) \vee (\neg r \vee \neg s)) \wedge (\neg t \wedge v)$ **(EG 12.13)**
12. Consider $p \rightarrow q$: If today is Monday, then $4 + 4 = 8$. **(EG 12.14)**
13. Construct the truth table for the following statements. $(i)\ \neg p \wedge \neg q$ $(ii)\ \neg(p \wedge \neg q)$ $(iii)\ (p \vee q) \vee \neg q$ $(iv)\ (\neg p \rightarrow r) \wedge (p \leftrightarrow q)$ **(EX 12.2 – 6)**

3 Marks

1. Verify the (i) closure property, (ii) commutative property (iii) associative property (iv) existence of identity and (v) existence of inverse for the arithmetic operation $+$ on $\mathbb{Z}$. **(EG 12.2)**
2. Verify the (i) closure property, (ii) commutative property, (iii) associative property (iv) existence of identity and (v) existence of inverse for the arithmetic operation $+$ on $\mathbb{Z}_e$ = the set of all even integers. **(EG 12.4)**
3. Verify (i) closure property (ii) commutative property, and (iii) associative property of the following operation on the given set. $(a * b) = a^b; \forall\, a, b \in \mathbb{N}$. (exponentiation property) **(EG 12.6)**
4. Let $*$ be defined on $\mathbb{R}$ by $(a * b) = a + b + ab - 7$. Is $*$ binary on $\mathbb{R}$? If so, find $3 * \left(\frac{-7}{15}\right)$. **(EX 12.1 – 3)**
5. Write down the (i) conditional statement (ii) converse statement (iii) inverse statement, and (iv) contrapositive statement for the two statements p and q given below. p: The number of primes is infinite. q: Ooty is in Kerala. **(EG 12.15)**
6. Construct the truth table for $(p \veebar q) \wedge (p \veebar \neg q)$. **(EG 12.16)**
7. Establish the equivalence property: $p \rightarrow q \equiv \neg p \vee q$ **(EG 12.17)**
8. Establish the equivalence property connecting the bi-conditional with conditional: $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$. **(EG 12.18)**
9. Using the equivalence property, S.T. $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$. **(EG 12.19)**
10. Show that $p \rightarrow q$ and $q \rightarrow p$ are not equivalent **(EX 12.2 – 10)**
11. Check whether the statement $p \rightarrow (q \rightarrow p)$ is a tautology or a contradiction without using the truth table. **(EX 12.2 – 12)**
12. Prove $p \rightarrow (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$ without using truth table. **(EX 12.2 – 14)**
13. Construct the truth table for $(p \veebar q) \wedge (p \veebar \neg q)$. **(EG 12.16)**

14. Establish the equivalence property: $p \to q \equiv \neg p \vee q$ **(EG 12.17)**
15. Establish the equivalence property connecting the bi-conditional with conditional: $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$. **(EG 12.18)**
16. Using the equivalence property, S.T. $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$. **(EG 12.19)**
17. Show that $p \to q$ and $q \to p$ are not equivalent **(EX 12.2 – 10)**
18. Check whether the statement $p \to (q \to p)$ is a tautology or a contradiction without using the truth table. **(EX 12.2 – 12)**
19. Prove $p \to (q \to r) \equiv (p \wedge q) \to r$ without using truth table. **(EX 12.2 – 14)**

5 Marks

1. Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for following operation on the given set. $m * n = m + n - mn;\ m, n \in \mathbb{Z}$. **(EG 12.7)**
2. Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for the operation X_{11} on a subset $A = \{1,3,4,5,9\}$ of the set of remainders $\{0,1,2,3,4,5,6,7,8,9,10\}$. **(EG 12.10)**
3. Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for the operation $+5$ on $\mathbb{Z}_5$ using table corresponding to addition modulo 5. **(EG 12.9)**
4. (i) Define an operation$*$ on $\mathbb{Q}$ as follows: $(a * b) = \frac{a+b}{2}$, $a, b \in \mathbb{Q}$. and existence of identity and the existence of inverse for the operation $*$ on $\mathbb{Q}$. **(EX 12.1 – 5)**
5. (i) Define an operation$*$ on $\mathbb{Q}$ as follows: $(a * b) = \frac{ab}{3}$, $a, b \in \mathbb{Q}$. and existence of identity and the existence of inverse for the operation $*$ on $\mathbb{Q}$. **(creative)**
6. Let $A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}, B = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}, C = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$ be any three Boolean matrices of the same type. Find (i) $A \vee B$ (ii) $A \wedge B$ (iii) $(A \vee B) \wedge C$ (iv) $(A \wedge B) \vee C$. **(EX 12.1 – 8)**
7. (i) Let $M = \left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in \mathbb{R} - \{0\} \right\}$ and let $*$ be the matrix multiplication. Determine whether M is closed under $*$. If so, examine the commutative and associative and examine the existence of identity, existence of inverse properties for the operation $*$ on M . **(EX 12.1 – 9)**
8. Let $M = \left\{ \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} : x \in \mathbb{R} - \{0\} \right\}$ and let $*$ be the matrix multiplication. Determine whether M is closed under $*$. If so, examine the commutative and associative and examine the existence of identity, existence of inverse properties for the operation $*$ on M . **(creative)**
9. (i) Let A be $\mathbb{Q}\backslash\{1\}$. Define $*$ on A by $x * y = x + y - xy$. Is $*$ binary on A? If so, examine the commutative and associative andexamine the existence of identity, existence of inverse properties for the operation $*$ on A. **(EX 12.1 – 10)**
10. Let A be $\mathbb{Q}\backslash\{-1\}$. Define $*$ on A by $x * y = x + y + xy$. Is $*$ binary on A? If so, examine the commutative and associative andexamine the existence of identity, existence of inverse properties for the operation $*$ on A. **(creative)**
11. Prove $p \to (q \to r) \equiv (p \wedge q) \to r$ without using truth table. **(EX 12.2 – 14)**
12. Prove that $p \to (\neg q \vee r) \equiv \neg p \vee (\neg q \vee r)$ using truth table. **(EX 12.2 – 15)**