



Standard 10

MATHEMATICS

Time: 3.00 Hrs.

Marks: 100

I. Answer all the questions:

14 × 1 = 14

- 1) If $f(x) = x^2$ and $g(x) = \frac{1}{3x}$, then $f \circ g$ is
- a) $\frac{1}{3x^2}$ b) $\frac{2}{3x^2}$ c) $\frac{1}{9x^2}$ d) $\frac{1}{6x^2}$
- 2) If the ordered pairs $(a+2, 4)$ and $(5, 2a+b)$ are equal the (a, b) is
- a) $(2, -2)$ b) $(5, 1)$ c) $(2, 3)$ d) $(3, -2)$
- 3) Given $a_1 = -1$ and $a_n = \frac{a_{n-1}}{n+2}$ then a_4 is
- a) $-\frac{1}{20}$ b) $-\frac{1}{4}$ c) $-\frac{1}{840}$ d) $-\frac{1}{120}$
- 4) An A.P consists of 31 terms. If its 16th term is m , then the sum of all the terms of this A.P is
- a) $16m$ b) $62m$ c) $31m$ d) $31/2m$
- 5) $y^2 + \frac{1}{y^2}$ is not equal to
- a) $\frac{y^4 + 1}{y^2}$ b) $\left(y + \frac{1}{y}\right)^2$ c) $\left(y - \frac{1}{y}\right)^2 + 2$ d) $\left(y + \frac{1}{y}\right)^2 - 2$
- 6) If A is a 2×3 matrix and B is 3×4 matrix, how many columns does AB have
- a) 3 b) 4 c) 2 d) 5
- 7) If in triangles ABC and EDF , $\frac{AB}{DE} = \frac{BC}{FD}$ then they will be similar, when
- a) $\angle B = \angle E$ b) $\angle A = \angle D$ c) $\angle B = \angle D$ d) $\angle A = \angle E$
- 8) If $(0, 0)$, $(a, 0)$ and $(0, b)$ are collinear, then
- a) $a = b$ b) $a+b = 0$ c) $ab = 0$ d) $a \neq b$
- 9) The point of intersection of $3x - y = 4$ and $x + y = 8$ is
- a) $(5, 3)$ b) $(2, 4)$ c) $(3, 5)$ d) $(4, 4)$
- 10) The value of $\sin^2 \theta + \frac{1}{1 + \tan^2 \theta}$ is equal to
- a) $\tan^2 \theta$ b) 1 c) $\cot^2 \theta$ d) 0
- 11) Find the radius of a sphere whose surface area is $100\pi \text{ cm}^2$.
- a) 25 cm b) 100 cm c) 5 cm d) 10 cm
- 12) A spherical ball of radius r_1 units is melted to make 8 new identical balls each of radius r_2 units. Then $r_1 : r_2$ is
- a) 2:1 b) 1:2 c) 4:1 d) 1:4
- 13) Which of the following is not a measure of dispersion?
- a) Range b) Standard c) Deviation d) Arithmetic mean
- 14) If the mean and coefficient of variation of a data are 4 and 87.5% then the standard deviation is
- a) 3.5 b) 3 c) 4.5 d) 2.5

II. Answer any 10 questions: (Q.No. 28 is compulsory)

10 × 2 = 20

- 15) If $B \times A = \{(-2, 3), (-2, 4), (0, 3), (0, 4), (3, 3), (3, 4)\}$ find A and B .
- 16) Show that the function $f: \mathbb{N} \rightarrow \mathbb{N}$ defined by $f(m) = m^2 + m + 3$ is one-one function.
- 17) Write an A.P whose first term is 20 and common difference is 8.
- 18) Find the number of terms in the G.P 4, 8, 16, 8192.
- 19) The father's age is six times his son's age. Six years hence the age of father will be four times his son's age. Find the present ages (in years) of the son and father.
- 20) The number of volley ball games that must be scheduled in a league with n teams is given by $G(n) = \frac{n^2 - n}{2}$ where each team plays with every other team exactly once. A league schedules 15 games. How many teams are in the league?

VIOM

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- 21) If $\triangle ABC$ is similar to $\triangle DEF$ such that $BC = 3$ cm, $EF = 4$ cm and area of $\triangle ABC = 54$ cm². Find the area of $\triangle DEF$.
- 22) The line p passes through the points $(3, -2)$, $(12, 4)$ and the line q passes through the points $(6, -2)$ and $(12, 2)$. Is p parallel to q ?
- 23) Find the equation of a line whose inclination is 30° , and making an intercept -3 on the y axis.
- 24) From the top of a rock $50\sqrt{3}$ m high, the angle of depression of a car on the ground is observed to be 30° . Find the distance of the car from the rock.
- 25) The volume of a solid right circular cone is 11088 cm³. If its height is 24 cm then find the radius of the cone.
- 26) How many liters of water will a hemispherical tank of diameter 4.2 m hold?
- 27) Find the standard deviation of first 21 natural numbers.
- 28) Prove that $\tan^2\theta - \sin^2\theta = \tan^2\theta \cdot \sin^2\theta$.

III. Answer any 10 questions: (Q.No. 42 is compulsory)**10×5=50**

- 29) A function f is defined by $f(x) = 2x - 3$.

- (i) find $\frac{f(0) + f(1)}{2}$ (ii) find x such that $f(x) = 0$. (iii) find x such that $f(x) = x$.
(iv) find x such that $f(x) = f(1-x)$.

- 30) If the function $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = \begin{cases} 2x + 7 & ; \quad x < -2 \\ x^2 - 2 & ; \quad -2 \leq x < 3 \\ 3x - 2 & ; \quad x \geq 3 \end{cases}$ then find the value

- of (i) $f(4)$ (ii) $f(-2)$ (iii) $f(4) + 2f(1)$ (iv) $\frac{f(1) - 3f(4)}{f(-3)}$.

- 31) If a, b, c are three consecutive terms of an A.P. and x, y, z are three consecutive terms of a G.P. then prove that $x^{b-c} \cdot y^{c-a} \cdot z^{a-b} = 1$.
- 32) Find the sum of the following: $10^3 + 11^3 + 12^3 + \dots + 20^3$
- 33) If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ show that $A^2 - (a+d)A = (bc-ad)I_2$.
- 34) If one root of the equation $2y^2 - ay + 64 = 0$ is twice the other then find the values of a .
- 35) State and prove - Basic Proportionality Theorem.
- 36) If the points $A(2, 2)$, $B(-2, -3)$, $C(1, -3)$ and $D(x, y)$ form a parallelogram the find the value of x and y .
- 37) Find the equation of the lines, whose sum and product of intercepts are 1 and -6 respectively.
- 38) Two ships are sailing in the sea on either sides of a lighthouse. The angle of elevation of the top of the lighthouse as observed from the ships are 30° and 45° respectively. If the lighthouse is 200 m high, find the distance between the two ships. ($\sqrt{3} = 1.732$)
- 39) A container open at the top is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends are 8 cm and 20 cm respectively. Find the cost of milk which can completely fill a container at the rate of Rs. 40 per litre.
- 40) A cylindrical bucket of height 32 cm and radius 18 cm is completely filled with sand. This sand is dumped on the ground and arranged in the shape of a cone. Find the radius and slant height of the cone if its height is 24 cm?
- 41) The number of televisions sold in each day of a week are $13, 8, 4, 9, 7, 12, 10$. Find its standard deviation.
- 42) A coin is tossed thrice. Find the probability of getting exactly two heads or at least one tail or two consecutive heads.

IV. Answer the following:**2×8=16**

- 43) Construct a $\triangle PQR$ in which $PQ = 8$ cm, $\angle R = 60^\circ$ and the median RG from R to PQ is 5.8 cm. Find the length of the altitude from R to PQ .
Construct a triangle similar to a given triangle LMN with its equal to $4/5$ of the corresponding sides of the triangle LMN (scale factor $4/5 < 1$). (OR)
- 44) Draw the graph of $y = x^2 + 3x - 4$ and hence use it to solve $x^2 + 3x - 4 = 0$. (OR)
A bus is travelling at a uniform speed of 50 km/hr. Draw the distance-time graph and hence find,
i) the constant of variation ii) how far will it travel in 90 minutes?
iii) the time required to cover a distance of 100 km from the graph.

- 1) (c) $\frac{1}{9x^2}$
- 2) (d) (3, -2)
- 3) (d) $\frac{-1}{120}$
- 4) (a) 0
- 5) (b) $(y + \frac{1}{y})^2$
- 6) (b) 4
- 7) (c) $LB = LP$
- 8) (b) $ab = 0$
- 9) (c) (3, 5)
- 6) (b) 1
- 11) (c) 5 or 6
- 12) (a) 2:1
- 13) (c) $5 \times 6 \times 8 \times 8 \times 8 \times 8$
- 14) (a) 3.5

$2'' = 2''$
 $n = 12$

19) $\log_{10} x = x$
 $\log_{10} 5 = 5 = 6x$
 $6x + 6 = 4(x + 6)$
 $x = 9$
 $\therefore \log_{10} 9 = 9, \log_{10} 9 = 54$

20) $\frac{x^2 - n}{2} = 15$
 $n^2 - n - 30 = 0$
 $(n - 6)(n + 5) = 0$
 $n = 6, n = -5$

21) $\frac{\Delta ABC}{\Delta DEF} = \frac{BC^2}{EF^2}$
 $\frac{54}{\Delta DEF} = \frac{3^2}{4^2}$
 $\therefore \Delta DEF = 96 \text{ cm}^2$

22) $m_1 = \frac{4+2}{12-3} = \frac{6}{9} = \frac{2}{3}$
 $m_2 = \frac{2+2}{12-6} = \frac{4}{6} = \frac{2}{3}$
 $m_1 = m_2$
 $\Rightarrow P \parallel Q$

23) $m = \tan 30^\circ = \frac{1}{\sqrt{3}}$
 $y = \frac{1}{\sqrt{3}}x - 3$
 $x - \sqrt{3}y - 3\sqrt{3} = 0$

24)
 $\tan 30^\circ = \frac{50}{x}$
 $\frac{1}{\sqrt{3}} = \frac{50}{x}$
 $x = 150 \text{ cm}$

25) $\frac{1}{3}\pi r^2 h = 11088$
 $r^2 = 441$
 $r = 21$

$\frac{20 \times 21}{3} \times \frac{22}{7} \times (2.1)^5$
 $= 19.404 \text{ L}^3$
 $= 19404 \text{ Litres}$
 $1 \text{ L}^3 = 1000 \text{ ml}$

27) $\sigma = \sqrt{\frac{21^2 - 1}{12}} = \sqrt{36.67}$
 $\sigma = 6.06$

28) $\cos A = \frac{\sin^2 \theta}{\cos^2 \theta} = \sin^2 \theta$
 $= \sin^2 \theta (\sec^2 \theta - 1)$
 $= \sin^2 \theta \times \tan^2 \theta$

29) (i) $\frac{-3 + (-1)}{2} = -2$
 (ii) $2x - 3 = 0 \Rightarrow x = \frac{3}{2}$
 (iii) $2x - 3 = x \Rightarrow x = 3$
 (iv) $2x - 3 = 2(1 - x) - 3$
 $x = \frac{1}{2}$

30) (i) $3(4) - 2 = 10$
 (ii) $(-2)^2 - 2 = 2$
 (iii) $10 + 2(-1) = 8$
 (iv) $-1 + 3(10) = 31$

31) $a = a, b = a + d, c = a + 2d$
 $x = x, y = x^2, z = x^3$
 $LHS = x(a+d) - (a+2d)$
 $= xy - a - (a+d)$
 $= x^2 - a - (a+d)$
 $= x^2 - 2a - d$
 $= x^2 - 2a - d$

32) $(\frac{20 \times 21}{2}) - (\frac{1 \times 10}{2})$
 $= 210^2 - 45^2$
 $= 44100 - 2025$
 $= 42075$

33) $A^2 = \begin{bmatrix} a^2 + bc & ab + bd \\ ca + dc & cb + d^2 \end{bmatrix}$
 $(a+d)A = \begin{bmatrix} a^2 + ad & ab + bd \\ ac + dc & ab + d^2 \end{bmatrix}$
 $(bc - ad)I_2 = \begin{bmatrix} bc - ad & 0 \\ 0 & bc - ad \end{bmatrix}$

34) $a + p = \frac{a}{2}, q + b = 32$
 Given: $x, 2q$
 $a = 4, a = 12 + 4$

35) $\frac{2+1}{2}, \frac{2-3}{2} = (\frac{2+x-3+y}{2}, \frac{2}{2})$
 $x = 5, y = 2$

37) $a + b = 1 \Rightarrow b = 1 - a$
 $ab = -b \Rightarrow a^2 - a - b = 0$
 $(a-3)(a+2) = 0$
 $a = 3, a = -2$
 $b = -2, b = 3$
 (i) $2x - 3y - b = 0$
 (ii) $3x - 2y + b = 0$

38)
 $\frac{1}{\sqrt{3}} = \frac{200}{x}$
 $x = 346.4$
 $1 = \frac{200}{y} \Rightarrow y = 200$
 $\therefore x + y = 546.4 \text{ cm}$

39) $\sqrt{22 \times 16} \sqrt{20^2 + 8^2 + 20 \times 8}$
 $= \frac{22 \times 16}{7 \times 3} [20^2 + 8^2 + 20 \times 8]$
 $= 10459.43 \text{ cm}^3$
 $\approx 10459.43 \text{ cm}^3$
 ≈ 418.36

40) $\pi \times 18 \times 18 \times 32 = \frac{1}{3} \pi \times r^2 \times 24$
 $r^2 = 1296$
 $r = 36$
 $\therefore x = \sqrt{36^2 + 24^2}$
 $= \sqrt{1872}$
 $x = 12\sqrt{13} \text{ cm}$

41) $\sigma = \sqrt{\frac{\sum x^2}{n} - (\frac{\sum x}{n})^2}$
 $\sigma = \sqrt{\frac{623}{7} - (\frac{63}{7})^2}$
 $\sigma = \sqrt{8} = 2.83$

42) $n(S) = 8$
 $A = \{HHH, HTH, THH\}$
 $P(A) = \frac{3}{8}$
 $B = \{HHT, HTH, HTT, THT, THT, TTH, TTT\}$
 $P(B) = \frac{7}{8}$
 $C = \{HHH, HHT, THH\}$
 $P(C) = \frac{3}{8}$
 $A \cap B = \{HHT, HTH, THH\}$
 $P(A \cap B) = \frac{3}{8}$
 $B \cap C = \{HHT, THH\}$
 $P(B \cap C) = \frac{2}{8}$
 $A \cap C = \{HHH, THH\}$
 $P(A \cap C) = \frac{2}{8}$
 $A \cap B \cap C = \{HHT, THH\}$
 $P(A \cap B \cap C) = \frac{2}{8}$

$\therefore P(A \cup B \cup C) = 1$

43.a
 P
 6
 8
 A
 B

43.b
 L
 4
 5
 M
 N

44.a $y = x^2 + 3x - 4$

x	-5	-4	-3	-2	-1	0	1
y	6	0	-4	-6	-6	-4	0

2	3	4
6	14	24

$x = \{-4, 1\}$

44.b

x	60	120	180	240	300
y	50	100	150	200	250

$k = \frac{5}{6}$
 $y = kx$
 $y = \frac{5}{6}x$

(i) $k = \frac{5}{6}$
 (ii) $x = 90 \Rightarrow y = 75$
 (iii) $y = 300 \Rightarrow x = 600$