

**Class 11**

**2023-24**



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# **FORMULAE AND THEOREMS**

**SUBJECT:**

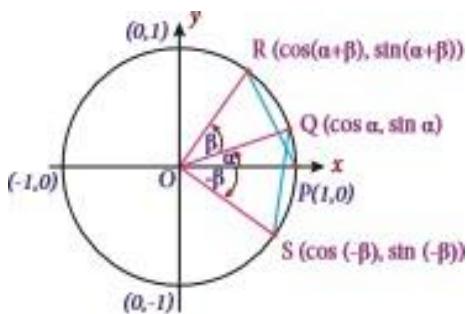
**MATH**  
**MR. SS PRITHVI**

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**UNIT – 3****TRIGONOMETRY****IDENTITY 3.1**

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

**Proof**

- Consider the unit circle with centre at O. Let P = P(1,0).
- Let Q, R and S be points on the unit circle such that  $\angle POQ = \alpha$ ,  $\angle POR = \alpha + \beta$  and  $\angle POS = -\beta$
- Clearly, angles  $\alpha$ ,  $\alpha + \beta$  and  $-\beta$  are in standard positions.
- Now, the points Q, R and S are given by Q( $\cos \alpha, \sin \alpha$ ), R( $\cos(\alpha + \beta), \sin(\alpha + \beta)$ ) and S( $\cos(-\beta), \sin(-\beta)$ ).

Since  $\Delta POR$  and  $\Delta SOQ$  are congruent. So,  $PR = SQ$  which gives  $PR^2 = SQ^2$

$$\text{Thus, } [\cos(\alpha + \beta) - 1]^2 + \sin^2(\alpha + \beta) = [\cos \alpha - \cos(-\beta)]^2 + [\sin \alpha - \sin(-\beta)]^2$$

$$-2\cos(\alpha + \beta) + 2 = 2 - 2\cos \alpha \cos \beta + 2\sin \alpha \sin \beta$$

$$\text{Hence, } \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta.$$

**IDENTITY 3.2**

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

**Proof**

We know that,  $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$

$$\cos(\alpha - \beta) = \cos[\alpha + (-\beta)]$$

$$= \cos \alpha \cos (-\beta) - \sin \alpha \sin (-\beta)$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta.$$

### IDENTITY 3.3

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

#### Proof

$$\sin(\alpha + \beta) = \cos \left[ \frac{\pi}{2} - (\alpha + \beta) \right]$$

$$= \cos \left[ \left( \frac{\pi}{2} - \alpha \right) - \beta \right]$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$= \cos \left( \frac{\pi}{2} - \alpha \right) \cos \beta + \sin \left( \frac{\pi}{2} - \alpha \right) \sin \beta$$

$$[\cos(90^\circ - \theta) = \sin \theta]$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

### IDENTITY 3.4

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

#### Proof

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha + (-\beta)) = \sin \alpha \cos(-\beta) + \cos \alpha \sin(-\beta)$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

### IDENTITY 3.5

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

**Proof**

$$\begin{aligned}
 \tan(\alpha + \beta) &= \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} \\
 &= \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\cos \alpha \cos \beta - \sin \alpha \sin \beta} \\
 &= \frac{\frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\cos \alpha \cos \beta}}{\frac{\cos \alpha \cos \beta - \sin \alpha \sin \beta}{\cos \alpha \cos \beta}} \\
 \tan(\alpha + \beta) &= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}
 \end{aligned}$$

**IDENTITY 3.6**

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

**Proof**

$$\begin{aligned}
 \tan(\alpha - \beta) &= \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)} \\
 &= \frac{\sin \alpha \cos \beta - \cos \alpha \sin \beta}{\cos \alpha \cos \beta + \sin \alpha \sin \beta} \\
 &= \frac{\frac{\sin \alpha \cos \beta - \cos \alpha \sin \beta}{\cos \alpha \cos \beta}}{\frac{\cos \alpha \cos \beta + \sin \alpha \sin \beta}{\cos \alpha \cos \beta}} \\
 \tan(\alpha - \beta) &= \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}
 \end{aligned}$$

**IDENTITY 3.7**

$$\sin 2A = 2 \sin A \cos A$$

**Proof**

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

Taking  $\alpha = \beta = A$ ,

$$\sin(A + A) = \sin A \cos A + \sin A \cos A$$

$$\sin 2A = 2 \sin A \cos A.$$

**IDENTITY 3.8**

$$\cos 2A = \cos^2 A - \sin^2 A$$

**Proof**

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

Taking  $\alpha = \beta = A$ ,

$$\cos(A + A) = \cos A \cos A - \sin A \sin A$$

$$\cos 2A = \cos^2 A - \sin^2 A.$$

**IDENTITY 3.9**

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

**Proof**

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

Taking  $\alpha = \beta = A$

$$\tan(A + A) = \frac{\tan A + \tan A}{1 - \tan A \tan A}$$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

**IDENTITY 3.10**

$$\sin 2A = \frac{2 \tan A}{1 + \tan^2 A}$$

**Proof**

$$\begin{aligned} \sin 2A &= 2 \sin A \cos A \\ &= \frac{2 \sin A \cos A}{\sin^2 A + \cos^2 A} \\ &= \frac{2 \sin A \cos A}{\frac{\cos^2 A}{\sin^2 A + \cos^2 A}} \\ &= \frac{2 \tan A}{1 + \tan^2 A} \end{aligned}$$

**IDENTITY 3.11**

$$\cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

**Proof**

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= \frac{\cos^2 A - \sin^2 A}{\cos^2 A + \sin^2 A} \\ &= \frac{\frac{\cos^2 A - \sin^2 A}{\cos^2 A}}{\frac{\cos^2 A + \sin^2 A}{\cos^2 A}} \\ &= \frac{1 - \tan^2 A}{1 + \tan^2 A} \end{aligned}$$

**IDENTITY 3.12**

$$\sin 3A = 3 \sin A - 4 \sin^3 A$$

**Proof**

$$\sin 3A = \sin (2A + A)$$

$$\begin{aligned}
 &= \sin 2A \cos A + \cos 2A \sin A \\
 &= 2 \sin A \cos^2 A + (1 - 2\sin^2 A) \sin A \\
 &= 2 \sin A (1 - \sin^2 A) + (1 - 2\sin^2 A) \sin A \\
 \text{Sin } 3A &= 3 \sin A - 4 \sin^3 A
 \end{aligned}$$

**IDENTITY 3.13**

$$\cos 3A = 4\cos^3 A - 3\cos A$$

**Proof**

$$\begin{aligned}
 \text{Cos } 3A &= \cos (2A + A) \\
 &= \cos 2A \cos A - \sin 2A \sin A \\
 &= (2 \cos^2 A - 1) \cos A - 2 \sin A \cos A \sin A \\
 &= (2 \cos^2 A - 1) \cos A - 2 \cos A (1 - \cos^2 A) \\
 \text{Cos } 3A &= 4 \cos^3 A - 3 \cos A.
 \end{aligned}$$

**IDENTITY 3.14**

$$\tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$$

**Proof**

$$\begin{aligned}
 \tan 3A &= \tan (2A + A) \\
 &= \frac{\tan 2A + \tan A}{1 - \tan 2A \tan A} \\
 &= \frac{\frac{2 \tan A}{1 - \tan^2 A} + \tan A}{1 - \frac{2 \tan A}{1 - \tan^2 A} \tan A}
 \end{aligned}$$

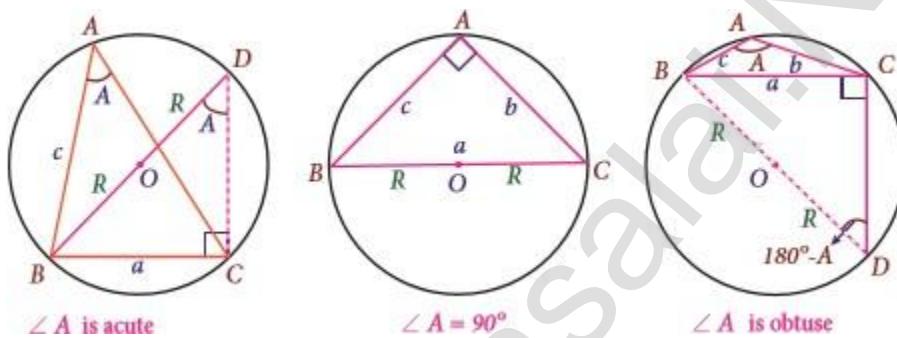
$$\tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$$

**Theorem 3.1 (Law of Sines)**

In any triangle, the lengths of the sides are proportional to the sines of the opposite angles. That is, in  $\Delta ABC$ ,  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$  where  $R$  is the circumradius of the triangle

**Proof**

The angle  $A$  of the  $\Delta ABC$  is either acute or right or obtuse. Let  $O$  be the centre of the circumcircle of  $\Delta ABC$  and  $R$ , its radius.



**Case I:**  $\angle A$  is acute.

Produce  $BO$  to meet the circle at  $D$ .

$$\angle BDC = \angle BAC = A$$

$$\angle BCD = 90^\circ$$

$$\sin \angle BDC = \frac{BC}{BD} \text{ or } \sin A = \frac{a}{2R} \Rightarrow \frac{a}{\sin A} = 2R$$

**Case II:**  $\angle A$  is right angle.

In this case  $O$  must be on the side  $BC$  of the  $\Delta ABC$ .

$$\text{Now, } \frac{a}{\sin A} = \frac{BC}{\sin 90^\circ} = \frac{2R}{1} = 2R \Rightarrow \frac{a}{\sin A} = 2R$$

**Case III:**  $\angle A$  is obtuse .

Produce BO to meet the circle at D.

$$\angle BDC + \angle BAC = 180^\circ$$

$$\angle BDC = 180^\circ - \angle BAC = 180^\circ - A$$

$$\angle BCD = 90^\circ$$

$$\sin \angle BDC = \frac{BC}{BD} \text{ or } \sin(180^\circ - A) = \sin A = \frac{a}{2R} \Rightarrow \frac{a}{\sin A} = 2R$$

$$\text{In each case, we have } \frac{a}{\sin A} = 2R$$

$$\text{Thus, } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

### Napier's Formula Theorem 3.2

In  $\Delta ABC$ , we have

$$(i) \tan \frac{A-B}{2} = \frac{a-b}{a+b} \cot \frac{C}{2} \quad (ii) \tan \frac{B-C}{2} = \frac{b-c}{b+c} \cot \frac{A}{2} \quad (iii) \tan \frac{C-A}{2} = \frac{c-a}{c+a} \cot \frac{B}{2}$$

### Proof

We know the sine formula:  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$

$$\begin{aligned} \text{Now, } \frac{a-b}{a+b} \cot \frac{C}{2} &= \frac{2R \sin A - 2R \sin B}{2R \sin A + 2R \sin B} \cot \frac{C}{2} \\ &= \frac{\sin A - \sin B}{\sin A + \sin B} \cot \frac{C}{2} \\ &= \frac{2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}}{2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}} \cot \frac{C}{2} \\ &= \cot \frac{A+B}{2} \tan \frac{A-B}{2} \cot \frac{C}{2} \\ &= \cot(90^\circ - \frac{C}{2}) \tan \frac{A-B}{2} \cot \frac{C}{2} \end{aligned}$$

$$\begin{aligned}
 &= \tan \frac{C}{2} \tan \frac{A-B}{2} \cot \frac{C}{2} \\
 &= \tan \frac{A-B}{2}
 \end{aligned}$$

### Theorem 3.3 (The Law of Cosines)

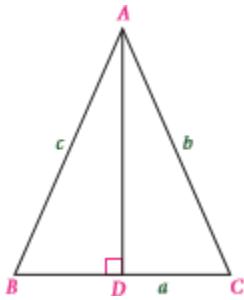
In  $\Delta ABC$ , we have

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{c^2 + a^2 - b^2}{2ca}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Proof.



In  $\Delta ABC$ , draw  $AD \perp BC$ .

In  $\Delta ABD$ , we have  $AB^2 = AD^2 + BD^2 \Rightarrow c^2 = AD^2 + BD^2$ .

find the values of  $AD$  and  $BD$  in terms of the elements of  $\Delta ABC$

$$\frac{AD}{AC} \sin C \Rightarrow AD = b \sin C$$

$$BD = BC - DC = a - b \cos C$$

$$\begin{aligned}
 c^2 &= (b \sin C)^2 + (a - b \cos C)^2 \\
 &= b^2 \sin^2 C + a^2 + b^2 \cos^2 C - 2 ab \cos C \\
 &= a^2 + b^2 (\sin^2 C + \cos^2 C) - 2 ab \cos C \\
 &= a^2 + b^2 - 2 ab \cos C
 \end{aligned}$$

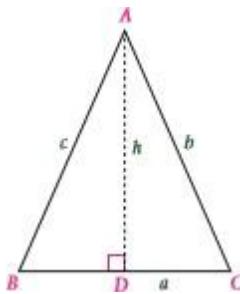
$$\text{Thus, } c^2 = a^2 + b^2 - 2ab \cos C \text{ or } \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

### Projection Formula Theorem 3.4

In a  $\Delta ABC$ , we have

$$(i) a = b \cos C + c \cos B \quad (ii) b = c \cos A + a \cos C \quad (iii) c = a \cos B + b \cos A$$

#### Proof



In  $\Delta ABC$ , we have  $a = BC$ . Draw  $AD \perp BC$ .

$$\begin{aligned} a &= BC = BD + DC \\ &= \frac{BD}{AB} AB + \frac{DC}{AC} AC \\ &= (\cos B) c + (\cos C) b \\ a &= b \cos C + c \cos B \end{aligned}$$

### Heron's Formula Theorem 3.7

In  $\Delta ABC$ ,  $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$  where  $s$  is the semi-perimeter of  $\Delta ABC$ .

#### Proof

$$\begin{aligned} \Delta &= \frac{1}{2} ab \sin C = \frac{1}{2} ab \left(2 \sin \frac{C}{2} \cos \frac{C}{2}\right) \\ &= ab \sqrt{\frac{(s-a)(s-b)}{ab}} \sqrt{\frac{s(s-c)}{ab}} \\ &= \sqrt{s(s-a)(s-b)(s-c)} \end{aligned}$$

**UNIT 4****COMBINATORICS AND MATHEMATICAL INDUCTION****PROPERTY 4**

$${}^n C_r + {}^n C_{r-1} = {}^{n+1} C_r$$

**Proof**

Using the expressions for the “combination”,

$$\begin{aligned} {}^n C_r + {}^n C_{r-1} &= \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-(r-1))!} \\ &= \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!} \\ &= \frac{n!}{r \cdot (r-1)!(n-r)!} + \frac{n!}{(r-1)!(n-r)!(n-r+1)} \\ &= \frac{n!}{(r-1)!(n-r)!} \times \left(1 + \frac{1}{r(n-r+1)}\right) \\ &= \frac{n!}{(r-1)!(n-r)!} \times \frac{(n-r+1+r)}{r(n-r+1)} \\ &= \frac{n!}{(r-1)!(n-r)!} \times \frac{(n+1)}{r(n-r+1)} \\ &= \frac{(n+1)!}{r!(n+1-r)!} \\ &= {}^{n+1} C_r \end{aligned}$$

**PROPERTY 5**

$${}^n C_r = \frac{n}{r} \times {}^{(n-1)} C_{(r-1)}$$

**Proof**

$$\begin{aligned} \frac{n}{r} \times {}^{(n-1)} C_{(r-1)} &= \frac{n}{r} \frac{(n-1)!}{(r-1)! \times ((n-1)-(r-1))!} \\ &= \frac{n(n-1)!}{r(r-1)!(n-r)!} = {}^n C_r \end{aligned}$$

**UNIT 5****BINOMIAL THEOREM, SEQUENCES AND SERIES****THEOREM 5.2**

If AM and GM denote the arithmetic mean and the geometric mean of two nonnegative numbers, then  $AM \geq GM$ . The equality holds if and only if the two numbers are equal.

**Proof**

Let a and b be any two nonnegative numbers. Then

$$AM = \frac{a+b}{2} \text{ and } GM = \sqrt{ab}$$

$$(a+b)^2 - 4ab = (a-b)^2 \geq 0$$

Thus,  $(a+b)^2 - 4ab \geq 0$  which gives  $(a+b) \geq 2\sqrt{ab}$

$$\frac{a+b}{2} \geq \sqrt{ab}$$

$$AM \geq GM.$$

The equality holds if and only if  $(a+b)^2 - 4ab = 0$ . This holds if and only if  $(a-b)^2 = 0$  which holds if and only if  $a = b$ .

Thus  $AM = GM$  if and only if  $a = b$ .

**THEOREM 5.3**

If GM and HM denote the geometric mean and the harmonic mean of two nonnegative numbers, then  $GM \geq HM$ . The equality holds if and only if the two numbers are equal

**Proof**

Let a and b be any two positive numbers. Then

$$GM = \sqrt{ab}$$

$$HM = \frac{2ab}{a+b}$$

$$GM - HM = \sqrt{ab} - \frac{2ab}{a+b}$$

$$\begin{aligned} &= \frac{\sqrt{ab}(a+b) - 2ab}{a+b} \\ &= \frac{\sqrt{ab}(a+b) - 2\sqrt{ab}}{a+b} \\ &= \frac{\sqrt{ab}(\sqrt{a} - \sqrt{b})^2}{a+b} \geq 0 \end{aligned}$$

Thus  $GM - HM \geq 0$  and hence  $GM \geq HM$ .

**UNIT 7****MATRICES AND DETERMINANTS****THEOREM 7.1**

**For any square matrix A with real number entries,  $A + A^T$  is a symmetric matrix and  $A - A^T$  is a skew-symmetric matrix.**

**Proof**

Let  $B = A + A^T$ .

$$B = (A + A^T)^T = A^T + (A^T)^T = A^T + A = A + A^T = B$$

This implies  $A + A^T$  is a symmetric matrix.

Let  $C = A - A^T$

$$C^T = (A + (-A^T))^T = A^T + (-A^T)^T = A^T - (A^T)^T = A^T - A = -(A - A^T) = -C$$

This implies  $A - A^T$  is a skew-symmetric matrix.

**THEOREM 7.2**

**Any square matrix can be expressed as the sum of a symmetric matrix and a skew-symmetric matrix.**

**Proof**

Let A be a square matrix. Then, we can write

$$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$$

$(A + A^T)$  and  $(A - A^T)$  are symmetric and skew-symmetric matrices.

Since  $(kA)^T = kA^T$  it follows that  $\frac{1}{2}(A + A^T)$  and  $\frac{1}{2}(A - A^T)$  are symmetric and skew-symmetric matrices, respectively.

### THEOREM 7.3 (FACTOR THEOREM)

If each element of a matrix A is a polynomial in x and if  $|A|$  vanishes for  $x = a$ , then  $(x - a)$  is a factor of  $|A|$ .

### PROPERTIES OF DETERMINANTS

#### Property 1

The determinant of a matrix remains unaltered if its rows are changed into columns and columns into rows. That is,  $|A| = |A^T|$

#### Property 2

If any two rows / columns of a determinant are interchanged, then the determinant changes in sign but its absolute value remains unaltered.

#### Property 3

If there are n interchanges of rows (columns) of a matrix A then the determinant of the resulting matrix is  $(-1)^n |A|$ .

#### Property 4

If two rows (columns) of a matrix are identical, then its determinant is zero.

#### Property 5

If a row (column) of a matrix A is a scalar multiple of another row (or column) of A, then its determinant is zero.

#### Property 6

If each element in a row (or column) of a matrix is multiplied by a scalar k, then the determinant is multiplied by the same scalar k.

**Property 7**

If each element of a row (or column) of a determinant is expressed as sum of two or more terms then the whole determinant is expressed as sum of two or more determinants.

$$\begin{vmatrix} a_1 + m_1 & b_1 & c_1 \\ a_2 + m_2 & b_2 & c_2 \\ a_3 + m_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} m_1 & b_1 & c_1 \\ m_2 & b_2 & c_2 \\ m_3 & b_3 & c_3 \end{vmatrix}$$

**Property 8**

If, to each element of any row (column) of a determinant the equi-multiples of the corresponding entries of one or more rows (columns) are added or subtracted, then the value of the determinant remains unchanged.

UNIT 8

# VECTOR ALGEBRA

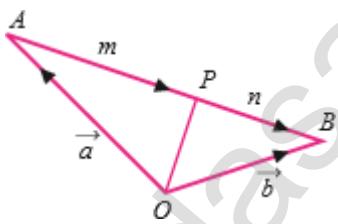
## Theorem 8.1 (Section Formula - Internal Division)

## Statement

Let O be the origin. Let A and B be two points. Let P be the point which divides the line segment AB internally in the ratio  $m : n$ . If  $\vec{a}$  and  $\vec{b}$  are the position vectors of A and B, then the position vector  $\overrightarrow{OP}$  of P is given by

$$\overrightarrow{OP} = \frac{n\vec{a} + m\vec{b}}{n+m}$$

## Diagram



## Proof

**Given:** Since O is the origin, If  $\vec{a}$  and  $\vec{b}$  are the position vectors of A and B

$$\overrightarrow{OA} = \vec{a} \text{ and } \overrightarrow{OB} = \vec{b}$$

Let  $\overrightarrow{OP} = \mathbf{r}$

Since P divides the line segment AB internally in the ratio  $m : n$ , we have,

$$\frac{|\overrightarrow{AP}|}{|\overrightarrow{PB}|} = \frac{m}{n}$$

$$n|\overrightarrow{AP}| = m|\overrightarrow{PB}|$$

But the vectors  $|\overrightarrow{AP}|$  and  $|\overrightarrow{PB}|$  have the same direction. Thus

But  $\vec{AP} = \vec{OP} - \vec{OA} = \vec{r} - \vec{a}$ ,  $\vec{PB} = \vec{OB} - \vec{OP} = \vec{b} - \vec{r}$

Substituting this in (1), we get

$$n(\vec{r} - \vec{a}) = m(\vec{b} - \vec{r})$$

$$(n + m)\vec{r} = n\vec{a} + m\vec{b}$$

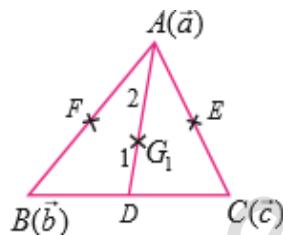
$$\vec{OP} = \frac{n\vec{a} + m\vec{b}}{n+m}$$

### THEOREM 8.3

**Statement**

**The medians of a triangle are concurrent**

**Diagram:**



**Proof**

**Given:** Let ABC be a triangle and let D, E, F be the mid points of its sides BC, CA and AB respectively.

**To prove:** Medians AD, BE, CF are concurrent.

**Proof**

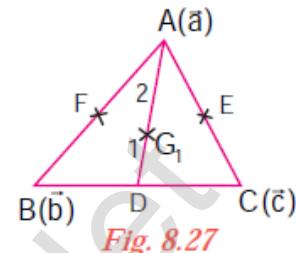
Let  $ABC$  be a triangle and let  $D, E, F$  be the mid points of its sides  $BC, CA$  and  $AB$  respectively. We have to prove that the medians  $AD, BE, CF$  are concurrent.

Let  $O$  be the origin and  $\vec{a}, \vec{b}, \vec{c}$  be the position vectors of  $A, B$ , and  $C$  respectively.

The position vectors of  $D, E$ , and  $F$  are respectively

$$\frac{\vec{b} + \vec{c}}{2}, \frac{\vec{c} + \vec{a}}{2}, \frac{\vec{a} + \vec{b}}{2}.$$

Let  $G_1$  be the point on  $AD$  dividing it internally in the ratio  $2 : 1$



Therefore, position vector of  $G_1 = \frac{1\overrightarrow{OA} + 2\overrightarrow{OD}}{1+2}$

$$\overrightarrow{OG_1} = \frac{1\vec{a} + 2\left(\frac{\vec{b} + \vec{c}}{2}\right)}{3} = \frac{\vec{a} + \vec{b} + \vec{c}}{3} \quad (1)$$

Let  $G_2$  be the point on  $BE$  dividing it internally in the ratio  $2 : 1$

$$\text{Therefore, } \overrightarrow{OG_2} = \frac{1\overrightarrow{OB} + 2\overrightarrow{OE}}{1+2}$$

$$\overrightarrow{OG_2} = \frac{1\vec{b} + 2\left(\frac{\vec{c} + \vec{a}}{2}\right)}{3} = \frac{\vec{a} + \vec{b} + \vec{c}}{3}. \quad (2)$$

Similarly if  $G_3$  divides  $CF$  in the ratio  $2 : 1$  then

$$\overrightarrow{OG_3} = \frac{\vec{a} + \vec{b} + \vec{c}}{3} \quad (3)$$

From (1), (2), and (3) we find that the position vectors of the three points  $G_1, G_2, G_3$  are one and the same. Hence they are not different points. Let the common point be  $G$ .

Therefore the three medians are concurrent and the point of concurrence is  $G$ .

**THEOREM 8.4****Statement**

A quadrilateral is a parallelogram if and only if its diagonals bisect each other.

**Diagram**

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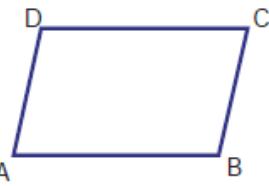
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**Proof**

Let  $A, B, C, D$  be the vertices of a quadrilateral with diagonals  $AC$  and  $BD$ . Let  $\vec{a}, \vec{b}, \vec{c}$  and  $\vec{d}$  be the position vectors of  $A, B, C$ , and  $D$  respectively with respect to  $O$ .

Let the quadrilateral  $ABCD$  be a parallelogram. Then

$$\overline{AB} = \overline{DC} \Rightarrow \overline{OB} - \overline{OA} = \overline{OC} - \overline{OD} \Rightarrow \vec{b} - \vec{a} = \vec{c} - \vec{d} \Rightarrow \vec{b} + \vec{d} = \vec{a} + \vec{c}$$

**Fig. 8.28**

and hence 
$$\frac{\vec{b} + \vec{d}}{2} = \frac{\vec{a} + \vec{c}}{2}.$$

This shows that the position vectors of the midpoint of the line segments  $AC$  and  $BD$  are the same. In other words, the diagonals bisect each other.

Conversely let us assume that the diagonal bisects each other. Thus the position vectors of the midpoint of the line segments  $AC$  and  $BD$  are the same. Thus

$$\frac{\vec{a} + \vec{c}}{2} = \frac{\vec{b} + \vec{d}}{2} \Rightarrow \vec{a} + \vec{c} = \vec{b} + \vec{d} \Rightarrow \vec{c} - \vec{d} = \vec{b} - \vec{a}.$$

This implies that  $\overline{OC} - \overline{OD} = \overline{OB} - \overline{OA}$  and hence  $\overline{DC} = \overline{AB}$ . This shows that the lines  $AB$  and  $DC$  are parallel. From  $\vec{a} + \vec{c} = \vec{b} + \vec{d}$  we see that  $\vec{a} - \vec{d} = \vec{b} - \vec{c}$  which shows that the lines  $AD$  and  $BC$  are parallel. Hence  $ABCD$  is a parallelogram.

**UNIT 9****DIFFERENTIAL CALCULUS – LIMITS AND CONTINUITY****THEOREM 9.4**

$$\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}$$

**Proof**

We know that

$$\begin{aligned} x^n - a^n &= (x - a)(x^{n-1} + x^{n-2}a + x^{n-3}a^2 + \dots + xa^{n-2} + a^{n-1}) \\ \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} &= \lim_{x \rightarrow a} \frac{(x-a)(x^{n-1} + x^{n-2}a + x^{n-3}a^2 + \dots + xa^{n-2} + a^{n-1})}{(x-a)} \\ &= \lim_{x \rightarrow a} (x^{n-1} + x^{n-2}a + x^{n-3}a^2 + \dots + xa^{n-2} + a^{n-1}) \\ &= a^{n-1} + a^{n-1} + \dots + a^{n-1} \quad (\text{n times}) \\ \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} &= na^{n-1} \end{aligned}$$

It is also true for any rational number n.

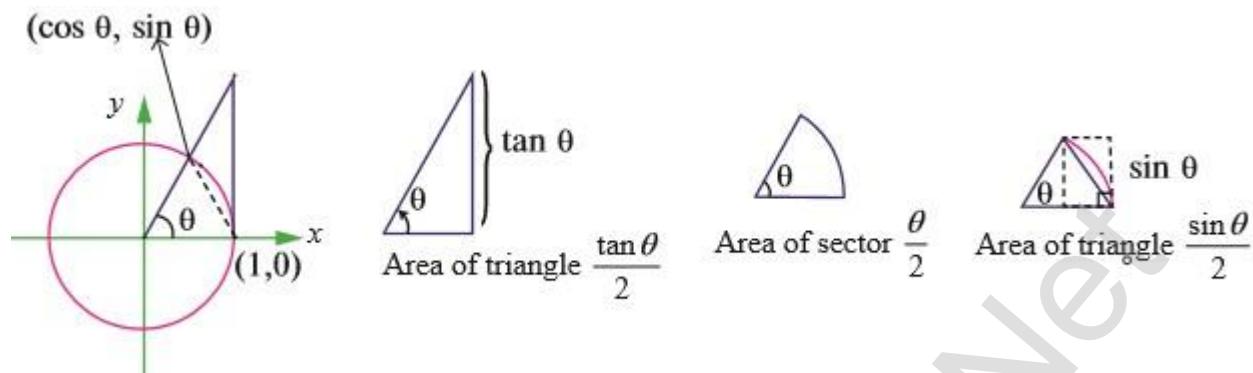
**THEOREM 9.5 (SANDWICH THEOREM)**

If  $f, g, h : I \subseteq \mathbb{R} \rightarrow \mathbb{R}$  such that  $g(x) \leq f(x) \leq h(x)$  for all  $x$  in a deleted neighbourhood of  $x_0$  contained in  $I$ , and if

$$\lim_{x \rightarrow x_0} g(x) = \lim_{x \rightarrow x_0} h(x) = l, \text{ then } \lim_{x \rightarrow x_0} f(x) = l$$

**RESULT 9.1**

$$(a) \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$$

**Proof**

We use a circular sector to prove the result. Consider the circle with centre  $(0, 0)$  and radius 1. Any point on this circle is  $P(\cos\theta, \sin\theta)$ .

- By area property  $\frac{\tan\theta}{2} \geq \frac{\theta}{2} \geq \frac{\sin\theta}{2}$
- Multiplying each expression by  $\frac{2}{\sin\theta}$  produces  $\frac{1}{\cos\theta} \geq \frac{\theta}{\sin\theta} \geq 1$
- Taking reciprocals  $\cos\theta \leq \frac{\sin\theta}{\theta} \leq 1$

Because  $\cos(-\theta) = \cos\theta$  and  $\frac{\sin(-\theta)}{\theta} = \frac{\sin\theta}{\theta}$  one can conclude that this inequality is valid for all non-zero  $\theta$  in the open interval  $(-\frac{\pi}{2}, \frac{\pi}{2})$

We know that  $\lim_{\theta \rightarrow 0} \cos\theta = 1$ ,  $\lim_{\theta \rightarrow 0} (1) = 1$  and applying sandwich theorem we get  $\lim_{\theta \rightarrow 0} \frac{\sin\theta}{\theta} = 1$

$$(b) \lim_{\theta \rightarrow 0} \frac{1-\cos\theta}{\theta} = 0$$

**Proof**

$$\begin{aligned} 1 - \cos\theta &= 2\sin^2 \frac{\theta}{2} \\ \frac{1-\cos\theta}{\theta} &= (\sin \frac{\theta}{2}) \frac{\sin(\frac{\theta}{2})}{(\frac{\theta}{2})} \\ \lim_{\theta \rightarrow 0} \frac{1-\cos\theta}{\theta} &= \lim_{\theta \rightarrow 0} (\sin \frac{\theta}{2}) \cdot \lim_{\theta \rightarrow 0} \frac{\sin(\frac{\theta}{2})}{(\frac{\theta}{2})} \\ &= 0 \times 1 = 0 \end{aligned}$$

## OTHER IMPORTANT LIMITS

**Result 9.2**

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$$

**Result 9.3**

$$\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log a, a > 0$$

**Result 9.4**

$$\lim_{x \rightarrow 0} \frac{\log(1+x)}{x} = 1$$

**Result 9.5**

$$\lim_{x \rightarrow 0} \frac{\sin^{-1}x}{x} = 1$$

**Result 9.6**

$$\lim_{x \rightarrow 0} \frac{\tan^{-1}x}{x} = 1$$

**Result 9.7**

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x \text{ exists and this limit is e.}$$

**Result 9.8**

$$\lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e$$

**Result 9.9**

$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$  exists and this limit is  $e$ .

**Unit 11 integral calculus**

**Integrals of the form**  $\int \frac{dx}{a^2 \pm x^2}$ ,  $\int \frac{dx}{x^2 - a^2}$ ,  $\int \frac{dx}{\sqrt{a^2 \pm x^2}}$ ,  $\int \frac{dx}{\sqrt{x^2 - a^2}}$

$$(i) \quad \int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + c$$

$$(ii) \quad \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c$$

$$(iii) \quad \int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) + c$$

$$(iv) \quad \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \left( \frac{x}{a} \right) + c$$

$$(v) \quad \int \frac{dx}{\sqrt{x^2 - a^2}} = \log \left| x + \sqrt{x^2 - a^2} \right| + c$$

$$(vi) \quad \int \frac{dx}{\sqrt{x^2 + a^2}} = \log \left| x + \sqrt{x^2 + a^2} \right| + c$$

**TYPE II**

**Integral of the form**  $\int \frac{dx}{ax^2 + bx + c}$  and  $\int \frac{dx}{\sqrt{ax^2 + bx + c}}$

**Proof**

The following rule is used to express the expression  $ax^2+bx+c$  as a sum or difference of two square terms.

(1) Make the coefficient of  $x^2$  as unity.

(2) Completing the square by adding and subtracting the square of half of the coefficient of  $x$ .

$$\begin{aligned}\text{That is, } ax^2 + bx + c &= a \left[ x^2 + \frac{b}{a}x + \frac{c}{a} \right] \\ &= a \left[ \left( x + \frac{b}{2a} \right)^2 + \frac{4ac - b^2}{4a^2} \right]\end{aligned}$$

**TYPE III**

**Integrals of the form  $\int \frac{px+q}{ax^2+bx+c} dx$  and  $\int \frac{px+q}{\sqrt{ax^2+bx+c}} dx$**

**Proof**

To evaluate the above integrals,

$$px + q = A \frac{d}{dx}(ax^2 + bx + c) + B$$

$$px + q = A(2ax + b) + B$$

Calculate the values of A and B, by equating the coefficients of like powers of x on both sides

- (i) The given first integral can be written as

$$\begin{aligned} \int \frac{px+q}{ax^2+bx+c} dx &= \int \frac{A(2ax+b)+B}{ax^2+bx+c} dx \\ &= A \int \frac{2ax+b}{ax^2+bx+c} dx + B \int \frac{1}{ax^2+bx+c} dx \end{aligned}$$

The first integral is of the form  $\int \frac{f'(x)}{f(x)} dx$

$$= A \log|ax^2 + bx + c| + B \int \frac{1}{ax^2+bx+c} dx$$

The second term on the right hand side can be evaluated using the previous types.

(ii) The given second integral can be written as

$$\begin{aligned} \int \frac{px+q}{\sqrt{ax^2+bx+c}} dx &= \int \frac{A(2ax+b)+B}{\sqrt{ax^2+bx+c}} dx \\ &= A \int \frac{2ax+b}{\sqrt{ax^2+bx+c}} dx + B \int \frac{1}{\sqrt{ax^2+bx+c}} dx \end{aligned}$$

The first integral is of the form  $\int f'(x)[f(x)]^n dx$

$$= A(2\sqrt{ax^2+bx+c}) + B \int \frac{1}{\sqrt{ax^2+bx+c}} dx$$

The second term on the right hand side can be evaluated using the previous types.

#### TYPE IV

##### Type IV

Integrals of the form  $\int \sqrt{a^2 \pm x^2} dx, \int \sqrt{x^2 - a^2} dx$

##### Result 11.3

$$(1) \int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) + c$$

$$(2) \int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log \left| x + \sqrt{x^2 - a^2} \right| + c$$

$$(3) \int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log \left| x + \sqrt{x^2 + a^2} \right| + c$$

**Proof.**

i

**UNIT 12****INTRODUCTION TO PROBABILITY THEORY****Theorem 12.11 (Bayes' Theorem)**

If  $A_1, A_2, A_3, \dots, A_n$  are mutually exclusive and exhaustive events

$P(A_i) > 0$ ,  $i = 1, 2, 3, \dots, n$  and  $B$  is any event in which  $P(B) > 0$ , then

$$P(A_i/B) = \frac{P(A_i)P(B/A_i)}{P(A_1)P(B/A_1) + P(A_2)P(B/A_2) + \dots + P(A_n)P(B/A_n)}$$

**Proof**

By the law of total probability of  $B$

$$P(B) = P(A_1). P(B/A_1) + P(A_2). P(B/A_2) + \dots + P(A_n). P(B/A_n)$$

By multiplication theorem

$$P(A_i \cap B) = P(B/A_i)P(A_i)$$

By the definition of conditional probability,

$$P(A_i/B) = \frac{P(A_i \cap B)}{P(B)}$$

**TRIGONOMETRY****SUM AND DIFFERENCE IDENTITIES (Ptolemy Identities)**

- $\sin(\alpha \pm \beta) = \sin\alpha \cos\beta \pm \cos\alpha \sin\beta$
- $\cos(\alpha \pm \beta) = \cos\alpha \cos\beta \mp \sin\alpha \sin\beta$
- $\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$
- $\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \tan\beta}$

**DOUBLE, TRIPLE AND HALF ANGLE IDENTITIES****SINE FORMULAS**

i)

$$\frac{2 \tan A}{1 + \tan^2 A} \longleftrightarrow \begin{matrix} \sin 2A \\ \uparrow \\ 2 \sin A \cos A \end{matrix}$$

ii)  $\sin 3A = 3 \sin A - 4 \sin^3 A$

iii)

$$\begin{matrix} \sin \theta \\ \uparrow \\ - - - \end{matrix} \longleftrightarrow \frac{2 \tan \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}}$$

**COSINE FORMULAS**

$$\begin{matrix} 2 \cos^2 A - 1 \\ \uparrow \\ \cos 2A \\ \uparrow \\ \cos^2 A - \sin^2 A \end{matrix} \longleftrightarrow \begin{matrix} \downarrow \\ 1 - 2 \sin^2 A \\ \downarrow \\ \frac{1 - \tan^2 A}{1 + \tan^2 A} \end{matrix}$$

$$\cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2} \quad \xleftarrow{\text{Up}} \quad \cos \theta \quad \xrightarrow{\text{Up}} \quad 2\cos^2 \frac{\theta}{2} - 1$$

$$\cos 3A = 4\cos^3 A - 3\cos A$$

### TANGENT FORMULA

I.  $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

II.  $\tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3\tan^2 A}$

III.  $\tan \theta = \frac{2 \tan \frac{\theta}{2}}{1 - \tan^2 \frac{\theta}{2}}$

Trigonometric Equation	General solution
$\sin \theta = \sin \alpha$ where $\alpha \in [-\frac{\pi}{2}, \frac{\pi}{2}]$	$\theta = n\pi + (-1)^n \alpha, n \in \mathbb{Z}$
$\cos \theta = \cos \alpha$ , where $\alpha \in [0, \pi]$	$\theta = 2n\pi \pm \alpha, n \in \mathbb{Z}$
$\tan \theta = \tan \alpha$ , where $\alpha \in (-\frac{\pi}{2}, \frac{\pi}{2})$	$\theta = n\pi + \alpha, n \in \mathbb{Z}$

LAW OF SINE	LAW OF COSINE	LAW OF TANGENT
$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$	$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$	$\tan \frac{A - B}{2} = \frac{a - b}{a + b} \cot \frac{C}{2}$
	$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$	$\tan \frac{B - C}{2} = \frac{b - c}{b + c} \cot \frac{A}{2}$
	$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$	$\tan \frac{C - A}{2} = \frac{c - a}{c + a} \cot \frac{B}{2}$

In a  $\triangle$  ABC, we have,

### PROJECTION FORMULA

- i)  $a = b \cos C + c \cos B$
- ii)  $b = c \cos A + a \cos C$
- iii)  $c = a \cos B + b \cos A$

### AREA OF TRIANGLE

$$\text{O} = \frac{1}{2}ab \sin C = \frac{1}{2}bc \sin A = \frac{1}{2}ac \sin B$$

$$\text{O} = \sqrt{s(s-a)(s-b)(s-c)}$$

### HALF ANGLE FORMULA

- i)  $\sin \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$
- ii)  $\cos \frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$
- iii)  $\tan \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$

where s is the semi-perimeter of  $\triangle$  ABC given by  $s = \frac{a+b+c}{2}$

## TRIGONOMETRY TABLE

Function → Degree ↓	cos	sin	tan	sec	csc	cot
0°	1	0	0	1	undefined	undefined
30°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{3}$	$\frac{2\sqrt{3}}{3}$	2	$\sqrt{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$	2	$\frac{2\sqrt{3}}{3}$	$\frac{\sqrt{3}}{3}$
90°	0	1	undefined	undefined	1	0
120°	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\sqrt{3}$	-2	$\frac{2\sqrt{3}}{3}$	$-\frac{\sqrt{3}}{3}$
135°	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1	$-\sqrt{2}$	$\sqrt{2}$	-1
150°	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{3}$	$-\frac{2\sqrt{3}}{3}$	2	$-\sqrt{3}$
180°	-1	0	0	-1	undefined	undefined
210°	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{3}$	$-\frac{2\sqrt{3}}{3}$	-2	$\sqrt{3}$
225°	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1	$-\sqrt{2}$	$-\sqrt{2}$	1
240°	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\sqrt{3}$	-2	$-\frac{2\sqrt{3}}{3}$	$\frac{\sqrt{3}}{3}$
270°	0	-1	undefined	undefined	-1	0
300°	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\sqrt{3}$	-2	$\frac{2\sqrt{3}}{3}$	$-\sqrt{3}$
315°	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	$\sqrt{2}$	$\sqrt{2}$	-1
330°	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{3}$	$\frac{2\sqrt{3}}{3}$	2	$-\frac{\sqrt{3}}{3}$
360°	1	0	0	1	undefined	undefined

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**BINOMIAL THEOREM, SEQUENCE AND SERIES**

**Binomial theorem** for any  $n \in \mathbb{N}$

$$(a + b)^n = {}^nC_0 a^n b^0 + {}^nC_1 a^{n-1} b^1 + \dots + {}^nC_n a^0 b^n$$

	AP	GP	AGP
<b><math>n^{\text{th}}</math> - Term</b>	$T_n = a + (n - 1)d$	$T_n = ar^{n-1}$	$T_n = (a + (n - 1)d)r^{n-1}$
<b>Sum of n Terms</b>	$S_n = \frac{n}{2}(2a + (n - 1)d)$	$S_n = \frac{a(1-r^n)}{1-r}$ for $r \neq 1$	$S_n = \frac{a-(a+(n-1)d)r^n}{1-r} + dr\left(\frac{1-r^{n-1}}{(1-r)^2}\right)$ for $r \neq 1$

For any positive numbers a and b, we have

$$\text{AM} = \frac{a+b}{2}, \quad \text{GM} = \sqrt{ab}, \quad \text{HM} = \frac{2ab}{a+b}$$

**Binomial theorem for rational exponent**

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots \text{ for all real numbers } x \text{ satisfying } |x| < 1$$

$$(1 + x)^{-1} = 1 - x + x^2 - x^3 + \dots$$

$$(1 - x)^{-1} = 1 + x + x^2 + x^3 + \dots$$

$$(1 - x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \dots$$

$$(1 + x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \dots$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

$$e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

$$\frac{e^x + e^{-x}}{2} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} \dots \text{ and } \frac{e^x - e^{-x}}{2} = \frac{x}{1!} + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$$

$$\log(1 + x) = x - \frac{x^2}{2!} + \frac{x^3}{3!} - \frac{x^4}{4!} + \dots \text{ for all values of } x \text{ satisfying } |x| < 1$$

$$\log(1 - x) = -x - \frac{x^2}{2!} - \frac{x^3}{3!} - \frac{x^4}{4!} - \dots \text{ for all values of } x \text{ satisfying } |x| < 1$$

$$\log\left(\frac{1+x}{1-x}\right) = 2 \left[ x + \frac{x^3}{3} + \frac{x^5}{5} + \dots \right]$$

**TWO DIMENSIONAL ANALYTICAL GEOMETRY**

<b>TYPES</b>	<b>EQUATION</b>
Slope (m) and y – intercept (b)	$y = mx + c$
Slope (m) and point $(x_1, y_1)$	$y - y_1 = m(x - x_1)$
Two points $(x_1, y_1)$ and $(x_2, y_2)$	$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$
$x$ – intercept (a) and $y$ – intercept (b)	$\frac{x}{a} + \frac{y}{b} = 1$
Normal length (p), angle ( $\alpha$ )	$x \cos \alpha + y \sin \alpha = p$
Parametric form (r)	$\frac{x - x_1}{\cos \theta} = \frac{y - y_1}{\sin \theta} = r$
The general equation	$ax + by + c = 0$

<b>Form of lines</b>	<b>Condition for parallel</b>	<b>Condition for perpendicular</b>
$y = m_1x + c_1$ and $y = m_2x + c_2$	$m_2 = m_1$	$m_1m_2 = -1$
$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$	$a_1b_2 = a_2b_1$	$a_1a_2 + b_1b_2 = 0$

The distance from a point  $P(x_1, y_1)$  to a line  $ax + by + c = 0$  is 
$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

The distance between two parallel lines  $a_1x + b_1y + c_1 = 0$  and  $a_2x + b_2y + c_2 = 0$  is 
$$\frac{|c_2 - c_1|}{\sqrt{a^2 + b^2}}$$

The line parallel to  $ax + by + c = 0$  through a point  $(x_1, y_1)$  is  $ax + by = ax_1 + by_1$  and perpendicular line is  $bx - ay = bx_1 - ay_1$

The condition that the general degree equation  $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$  should represent a pair of straight line in  $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

DERIVATIVES	ANTIDERIVATIVES
$\frac{d}{dx}(c) = 0$ where c is constant	$\int 0 \, dx = c$ , where c is constant
$\frac{d}{dx}(kx) = k$ , where k is constant	$\int k \, dx = kx + c$ , where c is constant
$\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n$	$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$ , $n \neq -1$
$\frac{d}{dx} \log x \left(\frac{1}{x}\right)$	$\int \frac{1}{x} \, dx = \log x  + c$
$\frac{d}{dx}(-\cos x) = \sin x$	$\int \sin x \, dx = -\cos x + c$
$\frac{d}{dx}(\sin x) = \cos x$	$\int \cos x \, dx = \sin x + c$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\int \sec^2 x \, dx = \tan x + c$
$\frac{d}{dx}(-\cot x) = \operatorname{cosec}^2 x$	$\int \operatorname{cosec}^2 x \, dx = -\cot x + c$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\int \sec x \tan x \, dx = \sec x + c$
$\frac{d}{dx}(-\operatorname{cosec} x) = \operatorname{cosec} x \cot x$	$\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + c$
$\frac{d}{dx}(e^x) = e^x$	$\int e^x \, dx = e^x + c$
$\frac{d}{dx}\left(\frac{a^x}{\log a}\right) = a^x$	$\int a^x \, dx = \frac{a^x}{\log a} + c$
$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x + c$
$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2} \, dx = \tan^{-1} x + c$

- $\int \tan x \, dx = \log |\sec x| + c$
- $\int \cot x \, dx = \log |\sin x| + c$
- $\int \operatorname{cosec} x \, dx = \log |\operatorname{cosec} x - \cot x| + c$
- $\int \sec x \, dx = \log |\sec x + \tan x| + c$

### Bernoulli's formula for integration by parts

If u and v are functions of x, then the Bernoulli's rule is

$$\int u \, dv = uv - u'v_1 + u''v_2 - \dots$$

Where  $u'$ ,  $u''$ ,  $u'''$ , ... are successive derivatives of u and

$v$ ,  $v_1$ ,  $v_2$ ,  $v_3$ , ... are successive integral of dv

$$\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2+b^2} [a \sin bx - b \cos bx] + c$$

$$\int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2+b^2} [a \cos bx + b \sin bx] + c$$

